ANNALES SCIENTIFIQUES
DE L'UNIVERSITÉ DE CLERMONT-FERRAND 2
Série Mathématiques

JOSEPH GLOVER

Markov processes and their last exit distributions

Annales scientifiques de l'Université de Clermont-Ferrand 2, tome 71, série *Mathématiques*, n° 20 (1982), p. 107

<http://www.numdam.org/item?id=ASCFM_1982__71_20_107_0>

MARKOV PROCESSES AND THEIR LAST EXIT DISTRIBUTIONS

Joseph GLOVER
University of ROCHESTER (U.S.A.)

In a forthcoming paper [1], we prove the following result.

Theorem. Let $(X(t), P^x)$ and $(Y(t), Q^x)$ be two (canonically defined) transient Hunt processes on E. Assume for each compact set K contained in E that $P^x(f(X_{L(K)-}); L(K)>0) = Q^x(f(Y_{L(K)-}); L(K)>0)$ for all bounded functions f on E, where L(K) is the last time the process is in K. Then Y is equivalent to a time change of X by the inverse of a strictly increasing continuous additive functional.

Let (q_k) be a collection of points dense in E, and let $B_r(q_k)$ be the open ball of radius r about q_k. If we let L(r,k) be the last time the process is in $B_r(q_k)$, and if we set $A(t) = \sum_j 2^{-j} \int_0^1 1_{\{0<L(r,k)\leq t\}} dr$, then A(t) is a raw additive functional of X and Y and the hypothesis of the theorem implies that $P^x \int f(X_{t-})\, dA(t) = Q^x \int f(Y_{t-})\, dA(t)$. Let B(t) (resp. C(t)) denote the dual predictable projection of A(t) for the process $(X(t), P^x)$ (resp. $(Y(t), Q^x)$. It is not difficult to show that B(t) (resp. C(t)) is a strictly increasing continuous additive functional of X (resp. Y), which implies that $P^x \int f(X(t))dB(t) = Q^x \int f(Y(t))\, dC(t)$. Thus if we let T(t) (resp. S(t)) denote the right continuous inverse of B(t) (resp. C(t)), then $P^x \int f(X(T(t)))\, dt = Q^x \int f(Y(S(t)))\, dt$. Therefore, the resolvents of the processes $(X(T(t)), P^x)$ and $(Y(S(t)), Q^x)$ agree, so the processes have the same joint distributions. The main result follows.

This result can also be interpreted (with natural auxiliary hypotheses) as a statement in potential theory involving equilibrium measures.

[1] J. Glover. Markov Processes with Identical Last Exit Distributions. (to appear, Zeitschrift fur Wahrscheinlichkeitstheorie verw. Geb.).