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O. ADELMAN

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SIMPLE MARKOV CHAINS

O. ADELMAN, UNIVERSITE PARIS VI

A Markov chain is d-dimensional iff S , its state space, can be written as the union $\bigcup_{i \in \mathbb{Z}^d} S_i$ such that

(1) $i \neq j \iff S_i \cap S_j = \emptyset$,

(2) $\sup \{\text{Card } (S_i)\} < \infty$,

and

(3) if $x, y \in S_i \cup S_j$, then

$$|i - j| \leq 1 \iff$$

$$\exists n \in \mathbb{N}, \exists \{z_k : k \in \{1, \dots, n\}\} \subset S_i \cup S_j, p(x, z_1) p(z_1, z_2) \dots p(z_n, y) > 0$$

(n can equal 0, in which case $p(x, y) > 0$).

The property " $\forall i, S_i \neq \emptyset$ " is a by-product of (1). Setting $j=i$ in (3), we note that any point in S_i can be arrived at from any other point in S_i , without getting out of S_i ; and letting i and j be neighbors in \mathbb{Z}^d , we see we can pass from S_i to S_j with no other S_k as a go between. (3) implies also the irreducibility of the Markov chain.

A Markov chain with a state space S is fair iff

$$x, y \in S, p(x, y) > 0 \implies$$

$$p(y, x) > 0 \quad \text{and} \quad p(x, y) = \frac{1 - p(x, x)}{\text{Card } (\{z : p(x, z) > 0\} \setminus \{x\})}$$

Note that this definition implies that for every x in S , $\text{Card} (\{y : p (x, y) > 0\}) < \infty$, a condition which is automatically satisfied by a d -dimensional Markov chain if $d < \infty$.

Theorem. A 1-dimensional fair Markov chain is recurrent (i.e., any state in its state space is recurrent).

The proof is based on ideas already used in [1].

If, for the one-dimensional case, we had replaced (2) by

$$(2') \quad \left[\lim_{i \nearrow \infty} \inf \text{Card} (S_i) \right] \left[\lim_{i \searrow -\infty} \inf \text{Card} (S_i) \right] < \infty,$$

that we could find examples of transient fair Markov chains satisfying (1), (2') and (3).

Conjecture. A d -dimensional fair Markov chain is recurrent iff $d \leq 2$.

[1] Omer ADELMAN, Some use of some "symmetries" of some random process, Ann. inst. Henri Poincaré, Vol. XII, N°2 , 1976, p. 193.