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SIMPLE MARKOV CHAINS

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A Markov chain is d-dimensional iff S, its state space, can be written as the union $\bigcup_{i \in \mathbb{Z}^d} S_i$ such that

(1) $i \neq j \Longleftrightarrow S_i \cap S_j = \emptyset$,

(2) $\sup \{\mathrm{Card}\ (S_i)\} < \infty$,

and

(3) if $x, y \in S_i \cup S_j$, than

$$|i - j| \leq 1 \Longleftrightarrow$$

$\exists n \in \mathbb{N},\ \exists \{z_k : k \in \{1, \ldots n\}\} \subset S_i \cup S_j,\ p(x,z_1)\ p(z_1, z_2) \ldots p(z_n, y) > 0$

(n can equal 0, in which case $p(x,y) > 0$).

The property "$\forall i$, $S_i \neq \emptyset$" is a by-product of (1). Setting j=i in (3), we note that any point in S_i can be arrived at from any other point in S_i, without getting out of S_i ; and letting i and j be neighbors in $\mathbb{Z}^d$, we see we can pass from S_i to S_j with no other S_k as a go between. (3) implies also the irreducibility of the Markov chain.

A Markov chain with a state space S is fair iff

$$x, y \in S,\ p(x,y) > 0 \Longrightarrow$$

$$p(y,x) > 0 \quad \text{and} \quad p(x,y) = \frac{1 - p(x,x)}{\mathrm{Card}\ (\{z : p(x,z) > 0\} \setminus \{x\})}$$

Note that this definition implies that for every x in S, Card $(\{y : p(x, y) > 0\}) < \infty$, a condition which is automatically satisfied by a d-dimensional Markov chain if $d < \infty$.

Theorem. A 1-dimensional fair Markov chain is recurrent (i.e., any state in its state space is recurrent).

The proof is based on ideas already used in [1].

If, for the one-dimensional case, we had replaced (2) by

$$(2') \qquad \left[\liminf_{i \nearrow \infty} \text{Card}(S_i)\right] \left[\liminf_{i \searrow -\infty} \text{Card}(S_i)\right] < \infty,$$

that we could find examples of transient fair Markov chains satisfying (1), (2') and (3).

Conjecture. A d-dimensional fair Markov chain is recurrent iff $d \leqslant 2$.

[1] Omer ADELMAN, Some use of some "symmetries" of some random process, Ann. inst. Henri Poincarré, Vol. XII, N°2 , 1976, p. 193.