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Hydrodynamic limit of mean zero asymmetric zero range processes in infinite volume

by

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ABSTRACT. – We prove the hydrodynamic behaviour of mean zero, asymmetric zero range processes evolving on the infinite lattice $\mathbb{Z}^d$. The proof relies on a bound, uniform in the volume, for the entropy production of processes in large finite volume. Such an entropy production bound, uniform in the volume, was first proved by Fritz in [6] to extend to infinite volume the proof of Guo Papanicolaou and Varadhan of hydrodynamic behaviour of interacting particle systems. Our approach follows a method introduced by Yau in [12].

Key words: Particle systems, hydrodynamic limit.

RÉSUMÉ. – Nous prouvons le comportement hydrodynamique des processus de zero range asymétriques de moyenne nulle en volume infini. La démonstration repose sur une borne, uniforme par rapport au volume, de la production d'entropie du processus en volume fini. Une telle borne sur l'entropie a déjà été démontré par Fritz dans [6] pour étendre au volume infini la démonstration de Guo, Papanicolaou et Varadhan sur le comportement hydrodynamique des systèmes de particules en interaction. Notre approche suit une méthode introduite par Yau dans [12].

Mots clés : Systèmes de particules, limite hydrodynamique.

INTRODUCTION

The major problem in the theory of hydrodynamic limit of interacting particle systems consists in describing the macroscopic time evolution of a gas from the microscopic interaction between molecules. Consider, for instance, a gas evolving on a d-dimensional volume V and assume that all equilibrium states of the system are characterized by a macroscopic variable p (the density, the temperature, etc.).

If the system is not in equilibrium, due to the interaction between molecules, we expect the process to be near equilibrium in small neighborhoods of each macroscopic point u of the volume V. This local equilibrium will be characterized by a parameter $p(u)$, possibly different at each point u.

We expect this local equilibrium state to change smoothly in time, that is, we expect the system, at any time t and around any point u, to be close to a new equilibrium state characterized by a parameter $p(t, u)$. This parameter $p(t, u)$ should evolve smoothly in time according to a differential equation, the so-called hydrodynamic equation.

Although physically well understood, this passage from microscopic dynamics to macroscopic behaviour still presents in the general case important mathematical problems. The interacting particle systems introduced by Spitzer constitute a class of stochastic models, complex enough, on the one hand, to present interesting macroscopic behaviour and relatively simple, on the other hand, to allow rigorous mathematical proofs.

Until the break through of Guo, Papanicolaou and Varadhan [7], where the intensive use of large deviation techniques led to a robust proof of the hydrodynamic behaviour of a large class of finite volume gradient systems with one conserved quantity, most methods to derive the hydrodynamic limit relied on specific properties of each model (*cf.* [8], [3], [4], [5], [9], [11] or [10] for a complete list of references).

Investigating the time evolution of the entropy of the state of the process with respect to some reference equilibrium measure, Guo, Papanicolaou and Varadhan proved a weak version of the conservation of local equilibrium described above: they showed that the density of particles in small macroscopic neighborhoods of a space point u at time t converges in probability to the solution of the hydrodynamic equation.

Later, Fritz [6] extended this method to infinite volume Ginzburg-Landau models proving a bound, uniform in the volume, for the entropy production of processes in large finite volume. Yau in [12] gave a new proof of this uniform entropy production bound for Ginzburg-Landau type models.

In this paper, following Yau's approach, we prove the hydrodynamic behaviour of a class of discrete spin systems in infinite volume, the so-called mean zero asymmetric zero range processes. In section 1 we introduce the notation and describe the hydrodynamic behaviour of this class of processes in infinite volume. In section 2 we prove the main result of this article. We consider mean zero asymmetric zero range processes in large but finite volume. Here large should be understood as large with respect to N, where N^{-1} denotes the distance between particles. We prove that the entropy production, that is the time derivative of the entropy, is bounded by the entropy minus the N^2 times the Dirichlet form. This inequality provides a bound, uniform in the volume, of the entropy and of the time integral of the Dirichlet form. By lower semicontinuity, these estimates extend to the infinite volume dynamics. This proves the hydrodynamic behaviour of mean zero asymmetric zero range processes in infinite volume. Details are given in section 3.

1. NOTATION AND RESULTS

In this section we introduce the notation and state the main theorem of this article.

The mean zero asymmetric zero range processes can be informally described as follows. Consider indistinguishable particles moving on the d-dimensional integers $\mathbb{Z}^d$. Let $g: \mathbb{N} \to \mathbb{R}_+$ be a non negative function with $g(0) = 0$ and $P(x, y)$ transition probabilities on $\mathbb{Z}^d$. Suppose that there are n particles on a site x of $\mathbb{Z}^d$. These particles, independently from particles on other sites, wait a mean $1/g(n)$ - exponential time at the end of which one of them jumps to y with probability $P(x, y)$.

The state space of the process $\mathbb{N}^{\mathbb{Z}^d}$ is denoted by $\mathcal{X}$ and the configurations by greek letters η and ξ. In this way, for $x \in \mathbb{Z}^d$, $\eta(x) \in \mathbb{N}$ represents the number of particles at site x for the configuration η.

The zero-range process $(\eta_t)_{t\geq 0}$, informally described above, is the Markov process on $\mathcal{X}$ whose generator acts on functions that depend only on a finite number of coordinates as

$$(Lf)(\eta) = \sum_{x,y\in\mathbb{Z}^d} g(\eta(x))P(x,y)[f(\eta^{x,y}) - f(\eta)]. \tag{1.1}$$

Here for configurations η such that $\eta(x) \geq 1$, $\eta^{x,y}$ is the configuration obtained from η letting a particle jump from x to y:

$$(\eta^{x,y})(z) = \begin{cases} \eta(z) & \text{if } z \neq x, y \\ \eta(x) - 1 & \text{if } z = x \\ \eta(y) + 1 & \text{if } z = y. \end{cases} \tag{1.2}$$

$P(x, y)$ is a family of transition probabilities on $\mathbb{Z}^d$ that we shall assume to be translation invariant, of finite range and to have mean drift equal to 0:

$$P(x, y) = P(0, y - x) =: p(y - x), \qquad \sum_x x\, p(x) = 0$$

and

$$\text{there exists } A_0 \text{ such that } p(x) = 0 \text{ if } |x| \geq A_0.$$

The rate jump $g : \mathbb{N} \to \mathbb{R}_+$ vanishes at 0 and is strictly positive on $\mathbb{N}^*$: $g(0) = 0$, $g(k) > 0$ for $k \geq 1$.

For each pair of sites (x, y) we denote by $L_{x,y}$ the piece of the generator corresponding to jumps from x to y and by $L^s_{x,y}$ the symmetric part of $L_{x,y} + L_{y,x}$:

$$\begin{aligned} (L_{x,y} f)(\eta) &= P(x, y)(T_{x,y} f)(\eta) \\ (L^s_{x,y} f)(\eta) &= (1/2)\{P(x, y) + P(y, x)\}\{(T_{x,y} f)(\eta) + (T_{y,x} f)(\eta)\}. \end{aligned} \tag{1.3}$$

In this formula, for two sites x and y, $T_{x,y}$ stands for the operator defined by $(T_{x,y} f)(\eta) = g(\eta(x))[f(\eta^{x,y}) - f(\eta)]$. Notice that $L_{x,y} \neq L_{y,x}$ and that $L^s_{x,y} = L^s_{y,x}$. We now introduce the invariant measures of the process. Denote by $Z: \mathbb{R}_+ \to \mathbb{R}_+$ the partition function defined by

$$Z(\varphi) = \sum_{k \geq 0} \frac{\varphi^k}{g(1) \cdots g(k)} \cdot$$

It is clear that $Z(\cdot)$ is an increasing function. Denote by φ^* the radius of convergence of Z. We shall assume that the partition function diverges as φ approaches the boundary of its domain of definition:

$$\lim_{\varphi \uparrow \varphi^*} Z(\varphi) = \infty.$$

For $0 \leq \varphi < \varphi^*$, let $\bar{\nu}_\varphi$ be the product measure on $\mathcal{X}$ with marginals given by:

$$\bar{\nu}_\varphi\{\eta; \eta(x) = j\} = \begin{cases} \frac{1}{Z(\varphi)} \frac{\varphi^j}{g(1) \cdots g(j)} & \text{if } j \geq 1 \\ \frac{1}{Z(\varphi)} & \text{if } j = 0. \end{cases}$$

Let $\rho(\varphi)$ be the expected number of particles under the measure $\bar{\nu}_\varphi$:

$$\rho(\varphi) = \bar{\nu}_\varphi[\eta(0)].$$

Since we assumed the partition function to diverge at the boundary of its domain of definition, it is easy to check that the density ρ is a smooth increasing bijection from $[0, \varphi^*)$ to $\mathbb{R}_+$. Moreover, since $\rho(\varphi)$ has a physical meaning as the density of particles, instead of parameterizing the above family of measures by φ, we use the density ρ as the parameter and we write:

$$\nu_\rho = \bar{\nu}_{\varphi(\rho)} \quad \text{for } \rho \geq 0. \tag{1.4}$$

With this convention, it is easy to check that

$$\varphi(\rho) = \nu_\rho[g(\eta(0))], \quad \rho \geq 0. \tag{1.5}$$

We shall assume throughout this article that the jump rate $g(\cdot)$ satisfies the following two assumptions.

(H1) $\sup_k |g(k+1) - g(k)| < \infty$.

(H2) For each $\varphi > 0$, there exists $\varepsilon(\varphi) > 0$ such that

$$E_{\bar{\nu}_\varphi}[\exp\{\varepsilon(\varphi) W(g(\eta(0)))\}] < \infty$$

where $W(u) = u(\log u)^2$.

Assumption (H1) guarantees the existence of the Markov process on $\mathbb{N}^{\mathbb{Z}^d}$ with generator L (cf. [1]). On the other hand, assumption (H2) excludes the case of independent random walks. It is satisfied by zero range processes with jump rate $g(\cdot)$ such that $C_0 k/\log k \leq g(k) \leq C_1 k/\log k$ for some finite positive constant $C_0 < C_1$.

We now define the two main ingredients needed in the proof of hydrodynamic limit of interacting particles systems: the entropy and the Dirichlet form of a measure on $\mathcal{X}$ with respect to some reference measure ν_ρ.

Fix once for all an invariant measure ν_ρ. For each subset Λ of $\mathbb{Z}^d$, denote by $\nu_{\rho,\Lambda}$ the product measure on $\mathbb{N}^\Lambda$ with marginals equal to the marginals of ν_ρ. When Λ is equal to $\{-n, \dots, n\}^d$ for some positive integer n, we shall denote $\mathbb{N}^\Lambda$ and $\nu_{\rho,\Lambda}$ simply by $\mathcal{X}_n$ and $\nu_{\rho,n}$. Moreover, for each measure μ on $\mathcal{X}$, we denote by μ_n the marginal of μ on $\mathcal{X}_n$:

$$\mu_n(\xi) = \mu\{\eta; \eta(x) = \xi(x) \text{ for } |x| \leq n\}, \quad \text{for each } \xi \in \mathcal{X}_n.$$

In this article $|\cdot|$ stands for the max norm of $\mathbb{R}^d$. For each positive integer n and each measure λ on $\mathcal{X}_n$, we denote by $H_n(\lambda)$ the relative entropy of λ with respect to $\nu_{\rho,n}$:

$$H_n(\lambda) = \sup_{f \in C_b(\mathcal{X}_n)} \Big\{ \int f d\lambda - \log \int e^f d\nu_{\rho,n} \Big\}.$$

In this formula $C_b(\mathcal{X}_n)$ stands for the space of bounded continuous functions on $\mathcal{X}_n$. Notice that all measures on $\mathcal{X}_n$ are absolutely continuous with respect to $\nu_{\rho,n}$ since the latter gives a positive probability to each configuration. Moreover, it is well known that the entropy is equal to:

$$H_n(\lambda) = \int \log\left\{\frac{d\lambda}{d\nu_{\rho,n}}\right\} d\lambda. \tag{1.6}$$

Denote also by $D_n(\lambda)$ the Dirichlet form of λ with respect to $\nu_{\rho,n}$:

$$D_n(\lambda) = -\sum_{x,y\in\Lambda_n} \left\langle \sqrt{\frac{d\lambda}{d\nu_{\rho,n}}}, L_{x,y}\sqrt{\frac{d\lambda}{d\nu_{\rho,n}}} \right\rangle_{\rho,n}.$$

In this formula, $< \cdot,\cdot >_{\rho,n}$ stands for expectation with respect to the measure $\nu_{\rho,n}$, $L_{x,y}$ for the piece of generator associated to jumps from site x to site y defined in (1.3) and Λ_n for the cube of length $2n+1$ centered at the origin:

$$\Lambda_n = \{-n,\dots,n\}^d.$$

We are now ready to define the entropy and the Dirichlet form of a measure μ on $\mathcal{X}$ with respect to ν_ρ. Fix once for all $\theta > 0$. For a measure μ on $\mathcal{X}$ define the entropy of μ with respect to ν_ρ by

$$H(\mu) \ := \ N^{-1}\sum_{n\geq 1} H_n(\mu_n)\, e^{-\theta(n/N)}.$$

Similarly, define the Dirichlet form of μ with respect to ν_ρ by

$$D(\mu) \ := \ N^{-1}\sum_{n\geq 1} D_n(\mu_n)\, e^{-\theta(n/N)}.$$

For a measure μ on $\mathcal{X}$, denote by $P_\mu = P_\mu^N$ the probability measure on the path space $D(\mathbb{R}_+, \mathcal{X})$ corresponding to the Markov process η_t with generator accelerated by N^2 and starting from μ and by E_μ expectation with respect to P_μ.

THEOREM 1.1. – *Consider a sequence of measures μ^N on $\mathcal{X}$ associated to a continuous profile $\rho_0\colon \mathbb{R}^d \to \mathbb{R}_+$ in the following sense:*

$$\lim_{N\to\infty} \mu^N\left[\left|N^{-d}\sum_{x\in\mathbb{Z}^d} G(x/N)\eta(x) \ - \ \int G(u)\rho_0(u)du\right| > \delta\right] = 0$$

for all continuous function $G : \mathbb{R}^d \to \mathbb{R}$ with compact support and all $\delta > 0$. Assume that μ^N has entropy bounded by $C_0 N^d$ for some finite constant C_0:

$$H(\mu^N) \le C_0 N^d.$$

Then, for all $t \ge 0$,

$$\lim_{N\to\infty} P_{\mu^N}\Big[\Big|N^{-d}\sum_{x\in\mathbb{Z}^d} G(x/N)\eta_t(x) - \int G(u)\rho(t,u)du\Big| > \delta\Big] = 0$$

for all continuous function $G : \mathbb{R}^d \to \mathbb{R}$ with compact support and all $\delta > 0$. Here $\rho(t,u)$ is the unique weak solution of the parabolic equation

$$\begin{cases} \partial_t \rho(t,u) = (1/2)\Delta\varphi(\rho) \\ \rho(0\,,\,\cdot\,) = \rho_0(\,\cdot\,). \end{cases} \tag{1.7}$$

We present below two classes of initial measures satisfying assumptions of Theorem 1.1.

(a) *Deterministic initial profiles.* – Consider a sequence of configurations η^N in $\mathbb{N}^{\mathbb{Z}^d}$ associated to some profile $\rho_0 : \mathbb{R}^d \to \mathbb{R}_+$ in the sense that

$$\lim_{N\to\infty} N^{-d}\sum_x G(x/N)\eta^N(x) = \int G(u)\rho_0(u)du$$

for every continuous function G with compact support. Assume furthermore that $\eta^N(x)$ does not increase, as $|x| \uparrow \infty$ faster than exponentially:

$$\eta^N(x) \le C_1 e^{C_2|x|/N}$$

for some finite positive constants C_1, C_2 and all x in $\mathbb{Z}^d$.

(b) *Product initial measures.* – Consider a continuous profile $\rho_0 : \mathbb{R}^d \to \mathbb{R}_+$ such that $|\rho_0(u)| \le C_1 \exp\{C_2|u|\}$ for some finite positive constants C_1, C_2 and all u in $\mathbb{R}^d$. For each $N \ge 1$, let μ^N be a product measure on $\mathbb{N}^{\mathbb{Z}^d}$ with marginals given by

$$\mu^N\{\eta;\ \eta(x) = k\} = \nu_{\rho_0(x/N)}\{\eta;\ \eta(x) = k\}$$

for all x in $\mathbb{Z}^d$ and k in $\mathbb{N}$.

The previous two examples satisfy assumptions of Theorem 1.1 for θ sufficiently large.

We shall prove Theorem 1.1 in two steps. In the next section, we shall consider zero range processes on large finite volumes. Large means here volumes of length M for some $M = M(N) \gg N$. For these finite volume processes, following the approach presented by Yau in [12], we shall prove a bound for the entropy and for the time integral of the Dirichlet form uniform in the volume. This is the main result of the article. The lower semicontinuity of the entropy and of the Dirichlet form permits to extend these bounds to the infinite volume process. This concludes the proof of Theorem 1.1, for these estimates and an uniqueness result of weak solutions of (1.7) (proved in [2]) are the unique ingreedients needed to prove the hydrodynamical behaviour of interacting particle systems.

2. UNIFORM UPPER BOUND ON THE ENTROPY PRODUCTION FOR PROCESSES IN FINITE VOLUME

We consider in this section zero range processes in large finite volume and prove a bound on the entropy production, uniform in the volume. We fix a positive integer M, large with respect to N, and consider the restriction of the processes on $\mathcal{X}_M = \mathbb{N}^{\Lambda_M}$. The generator L_M of this process is given by

$$L_M = \sum_{x,y \in \Lambda_M} L_{x,y},$$

where $L_{x,y}$ is the piece of generator corresponding to jumps from site x to site y and is defined in (1.3). By extension we define the generator L_n for $1 \le n \le M$. We shall denote by $S_t^{M,N}$ the semigroup of the Markov process on $\mathcal{X}_M$ with generator L_M accelerated by N^2.

Fix a density ρ and recall from section 1 the definition of the product measure $\nu_{\rho,n}$ on $\mathcal{X}_n$. Consider a measure μ on $\mathcal{X}_M$ and denote by $\mu(t)$ the state at time t of the process that started from μ: $\mu(t) = \mu S_t^{M,N}$. For each $1 \le n \le M$ and measure μ on $\mathcal{X}_M$, denote by μ_n the marginal of μ on $\mathcal{X}_n$ and by $H_n(\mu_n)$ and $D_n(\mu_n)$ the entropy and the Dirichlet form of μ_n with respect to $\nu_{\rho,n}$. These functionals were defined in full detail in section 1. Moreover, we shall denote by $\mu_n(t)$ the marginal of $\mu(t)$ on $\mathcal{X}_n$.

Let $R : \mathbb{R}_+ \to \mathbb{R}_+$ be a continuous positive function with support contained in $[0, (M-N)/N]$. For a measure μ on $\mathcal{X}_M$, define, respectively, the entropy $\mathcal{H}(\mu) = \mathcal{H}_{M,R}(\mu)$ and the Dirichlet form $\mathcal{D}(\mu) = \mathcal{D}_{M,R}(\mu)$ by

$$\mathcal{H}(\mu) = N^{-1} \sum_{n=1}^{M} H_n(\mu_n)\, R(n/N),$$

$$\mathcal{D}(\mu) = N^{-1} \sum_{n=1}^{M} D_n(\mu_n)\, R(n/N).$$

We are now ready to state the main result of this article.

THEOREM 2.1. – *There exists positive and finite constants K_1, K_2 and K_3 that depend only on* $\max_{n\geq 0} N|R((n+1)/N) - R(n/N)|$ *and ρ such that for all probability μ on $\mathcal{X}_M$,*

$$\partial_t \mathcal{H}(\mu(t)) \leq -K_1 N^2 \mathcal{D}(\mu(t)) + K_2 \mathcal{H}(\mu(t)) + K_3 N^d.$$

Proof. – To keep notation simple and to detach the main arguments, we shall prove this theorem for nearest neighbour symmetric zero range processes in dimension 1. We indicate at the end of this section the modifications required to extend the proof to mean zero asymmetric zero range processes in higher dimensions.

Fix a measure μ on $\mathcal{X}_M$ and an integer $1 \leq n \leq M - N$. Denote by $f_n(t)$ the density of $\mu_n(t)$ with respect to $\nu_{\rho,n}$. A simple computation shows that in the nearest neighbour case,

$$\partial_t f_n(t) = N^2 \langle L^*_{n+1} f_{n+1}(t) \rangle_{n+1}. \tag{2.1}$$

Here L^*_n denotes the adjoint operator of L_n in $L^2(\nu_{\rho,M})$. In the symmetric case L_n is self-adjoint and $L^*_n = L_n$. Moreover, for subsets $\Omega \subset \Lambda$ of $\mathbb{Z}^d$, and a function g in $L^1(\nu_{\rho,\Lambda})$, $\langle g \rangle_\Omega$ indicates that we are integrating g over the coordinates $\{\eta(x),\ x \in \Omega\}$ with respect to $\nu_{\rho,\Lambda}$. When $\Omega = \Lambda_{n+1} - \Lambda_n$, we shall denote this expectation simply by $\langle g \rangle_{n+1}$. Notice that $f_n(t) = \langle f_{n+1}(t) \rangle_{n+1}$ because ν_ρ is a product measure.

From the explicit formula for the relative entropy given in (1.6), from identity (2.1) and since $\nu_{\rho,M}$ is an invariant state we have that

$$\begin{aligned}\partial_t H_n(t) &= \partial_t \int f_n(t) \log f_n(t)\, d\nu_{\rho,n} \\ &= N^2 \int L^*_{n+1} f_{n+1}(t) \log f_n(t)\, d\nu_{\rho,M}.\end{aligned}$$

We shall decompose the generator L_{n+1} as the sum of two terms, the first one corresponding to jumps in the "interior" of Λ_{n+1} and the second to jumps at the boundary: $L_{n+1} = L_n + \{L^s_{n,n+1} + L^s_{-n-1,-n}\}$. To keep notation simple we shall denote the second part by $(\partial L)_{n+1}$. With this notation, we may write the time derivative of the entropy $H_n(t)$ as

$$N^2 \int f_{n+1}(t) L_n \log f_n(t) d\nu_{\rho,M} + N^2 \int f_{n+1}(t) (\partial L)_{n+1} \log f_n(t) d\nu_{\rho,M}.$$

Since time t is fixed, we shall omit from now on the time dependence of the density $f_n(t)$. The first term, after, by now, standard manipulations, is shown to be bounded above by $-N^2 D_n(\mu_n(t))$ while the second, after a change of variables, can be rewritten as the sum of two similar terms. The first one, which correspond to jumps over the bond $\{n, n+1\}$, is equal to

$$-\varphi\frac{\varphi(\rho)N^2}{2}\int\{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1} - f_n(\eta+\delta_n)\}$$
$$\times\{\log f_n(\eta) - \log f_n(\eta+\delta_n)\}\, d\nu_{\rho,n}. \quad (2.2)$$

The second term, which corresponds to jumps at the left boundary $\{-n-1, -n\}$, is handled in the same way as the one above. Here, for an integer x, δ_x is the configuration with no particles but one at site x and addition of two configurations is taken coordinate by coordinate.

Recall that $f_n = \langle f_{n+1}\rangle_{n+1}$ and, more generally, that $f_n = \langle f_m\rangle_{\{n+1,\dots,m\}}$ for all $n+1 \le m \le M$. We may thus rewrite the last line as

$$\frac{\varphi(\rho)N^2}{2}\int\{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1} - f_n(\eta+\delta_n)\}$$
$$\times\{\log f_n(\eta+\delta_n) - \log f_n(\eta)\}\, d\nu_{\rho,n}.$$

Since for positive reals a, b and c we have that $(c-b)(\log b - \log a)$ is negative unless $\min\{a,c\} \le b \le \max\{a,c\}$, we may introduce in the last integral the indicator function of the set E_n defined by

$$E_n = \{\eta;\ \min\{f_n(\eta), \langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}\} \le f_n(\eta+\delta_n)$$
$$\le \max\{f_n(\eta), \langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}\}\}.$$

By the elementary inequality $2\alpha\beta \le A^{-1}\alpha^2 + A\beta^2$ and since $(c-b) = (\sqrt{c}-\sqrt{b})(\sqrt{c}+\sqrt{b})$, we have that for every positive A,

$$(c-b)(\log b - \log a) \le \frac{1}{2A}(\sqrt{c}-\sqrt{b})^2 + \frac{A}{2}(\sqrt{c}+\sqrt{b})^2(\log b - \log a)^2.$$

In particular the last integral is bounded above by

$$\frac{\varphi(\rho)N^3}{4A}\int\{\sqrt{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}} - \sqrt{f_n(\eta+\delta_n)}\}^2 d\nu_{\rho,n}$$
$$+\frac{\varphi(\rho)AN}{4}\int\{\sqrt{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}} + \sqrt{f_n(\eta+\delta_n)}\}^2$$
$$\times\{\log f_n(\eta+\delta_n) - \log f_n(\eta)\}^2 \mathbf{1}_{E_n}\, d\nu_{\rho,n}. \quad (2.3)$$

The first line of this expression is bounded above by a piece of the Dirichlet form. Indeed, it is equal to

$$\frac{\varphi(\rho)N^2}{4A}\sum_{m=n+1}^{n+N}\int\{\sqrt{\langle f_m(\eta+\delta_{n+1})\rangle_{\{n+1,\dots,m\}}} - \sqrt{\langle f_m(\eta+\delta_n)\rangle_{\{n+1,\dots,m\}}}\}^2 d\nu_{\rho,n},$$

which, by Schwarz inequality, is bounded above by

$$\frac{\varphi(\rho)N^2}{4A}\sum_{m=n+1}^{n+N}\int\langle\{\sqrt{f_m(\eta+\delta_{n+1})}-\sqrt{f_m(\eta+\delta_n)}\}^2\rangle_{\{n+1,\dots,m\}}d\nu_{\rho,n}$$

$$=\frac{N^2}{2A}\sum_{m=n+1}^{n+N}\langle -L_{n,n+1}\sqrt{f_m(t)},\sqrt{f_m(t)}\rangle_{\rho,M}.$$

We estimate now to the second term of formula (2.3). We consider first the case where $f_n(\eta)\le\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}$. In this case, on the set E_n, we have that $\langle f_{n+1}(\eta+\delta_n)\rangle_{n+1}\le\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}$. In particular, in this region the second term in equation (2.3) is bounded above by

$$\varphi(\rho)AN\int\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}\left\{\log\frac{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}}{f_n(\eta)}\right\}^2 d\nu_{\rho,n}.$$

For sufficiently large b, the function V_b defined by $V_b(u)=u\big(\log u\big)^2 - b\log u$ is convex. With this notation we can rewrite the last integral as

$$\varphi(\rho)AN\int f_n(\eta)\,V_b\left(\frac{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}}{f_n(\eta)}\right)d\nu_{\rho,n}$$
$$+\varphi(\rho)bAN\int f_n(\eta)\log\frac{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}}{f_n(\eta)}\,d\nu_{\rho,n}.$$

The elementary inequality

$$a\big(\log b-\log a\big)\;\le\;-(\sqrt{a}-\sqrt{b})^2+(b-a) \tag{2.4}$$

for positives a and b permits to bound the second term of last formula by

$$-\varphi(\rho)bAN\int\{\sqrt{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}}-\sqrt{f_n(\eta)}\}^2\,d\nu_{\rho,n}$$
$$+\varphi(\rho)bAN\int\{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}-f_n(\eta)\}\,d\nu_{\rho,n}.$$

We consider now the case where $\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1} \le f_n(\eta)$ and call F_n set of configurations satisfying this inequality. On the set $E_n \cap F_n$, we have that $\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1} \le f_n(\eta+\delta_n) \le f_n(\eta)$. From inequality (2.4), on the set $E_n \cap F_n$ we obtain that the second term in formula (2.3) is bounded by

$$
\begin{aligned}
4\varphi(\rho)AN \int \Big\{ \sqrt{f_n(\eta+\delta_n)} \log \frac{\sqrt{f_n(\eta)}}{\sqrt{f_n(\eta+\delta_n)}} \Big\}^2 \mathbf{1}_{E_n \cap F_n}\, d\nu_{\rho,n} \\
\le 4\varphi(\rho)AN \int \{ \sqrt{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}} - \sqrt{f_n(\eta)} \}^2\, d\nu_{\rho,n}.
\end{aligned}
$$

Adding the previous estimates and taking b larger than 4, we obtain that the second term in formula (2.3) is bounded above by

$$
\varphi(\rho)AN \int f_n(\eta) W_b\Big(\frac{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}}{f_n(\eta)} \Big)\, d\nu_{\rho,n}, \tag{2.5}
$$

where $W_b(u)$ is the function defined by $W_b(u) = V_b(u) + b(u-1)$. From now on we shall consider b as a fixed constant larger than 4 and so that $V_b(\cdot)$ is a convex function. Performing the change of variables $\xi = \eta + \delta_{n+1}$, we obtain that $\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}$ is equal to $\langle g_\varphi(\eta(n+1)) f_{n+1}(\eta)\rangle_{n+1}$, where g_φ is the function defined by $g_\varphi(k) = g(k)/\varphi$. Since W_b is a convex function and since f_{n+1}/f_n is a probability density with respect to $\nu_{\rho,\{-n-1,n+1\}}$, by Jensen's inequality, last expression is bounded above by

$$
\varphi(\rho)AN \int f_{n+1}(\eta) W_b(g_\varphi(\eta(n+1)))\, d\nu_{\rho,n+1}.
$$

Applying Lemma 2.2 below, we estimate this expression by

$$
\begin{aligned}
&\frac{\varphi(\rho)AN}{\gamma} \log \int e^{\gamma W_b(g_\varphi(\eta(0)))} \nu_{\rho,0}(d\eta) \\
&\quad + \frac{\varphi(\rho)AN}{\gamma} \{ H_{n+1}(\mu_{n+1}(t)) - H_n(\mu_n(t)) \}.
\end{aligned}
$$

By Assumption (H2), the first term is finite for γ smaller than some $\gamma_0 = \gamma(\rho)$. Therefore, recollecting all previous estimates, we proved that the time derivative of the entropy on the box Λ_n is bounded above by a function of the entropy and of the Dirichlet form:

$$
\begin{aligned}
\partial_t H_n(\mu_n(t)) \le &- N^2 D_n(\mu_n(t)) \\
&+ \frac{N^2}{2A} \sum_{m=n+1}^{n+N} \langle -\partial L_{n+1} \sqrt{f_m(t)}, \sqrt{f_m(t)} \rangle_{\rho,M} \\
&+ C_1(\rho)AN[H_{n+1}(\mu_{n+1}(t)) - H_n(\mu_n(t))] + C_2(\rho)AN.
\end{aligned}
$$

To conclude the proof of the Theorem we just have to multiply both sides of this inequality by $R(n/N)$, sum over $1 \le n \le M - N$ and choose A large enough. Notice that by summation by parts the factor N at the second line disappears. □

LEMMA 2.2. – *Fix $n \geq 1$ and some function $h: \mathbb{N} \to \mathbb{R}$. There exists an universal constant C such that for every $\gamma > 0$,*

$$\sum_{x \in \Lambda_{n+1} - \Lambda_n} \int f_{n+1}(\eta) h(\eta(x)) \nu_{\rho,n+1}(d\eta)$$
$$\leq \frac{CN^{d-1}}{\gamma} \log \int e^{\gamma h(\eta(0))} \nu_{\rho,0}(d\eta) + \frac{1}{\gamma}\{H_{n+1}(\mu_{n+1}(t)) - H_n(\mu_n(t))\}.$$

Proof. – This result is a trivial consequence of the entropy inequality. We may rewrite the integral in the left hand side of the statement as

$$\int f_n(\eta) \Big\langle \sum_{x \in \Lambda_{n+1} - \Lambda_n} h(\eta(x)) \frac{f_{n+1}(\eta)}{f_n(\eta)} \Big\rangle_{n+1} \nu_{\rho,n}(d\eta).$$

Notice that f_{n+1}/f_n, considered as a function of the variables $\{\eta(x), x \in \Lambda_{n+1} - \Lambda_n\}$ is a density with respect to $\nu_{\rho,\Lambda_{n+1}-\Lambda_n}$. In particular, by the entropy inequality,

$$\Big\langle \sum_{x \in \Lambda_{n+1} - \Lambda_n} h(\eta(x)) \frac{f_{n+1}(\eta)}{f_n(\eta)} \Big\rangle_{n+1} \leq \frac{1}{\gamma} H(f_{n+1}/f_n)$$
$$+ \frac{|\Lambda_{n+1} - \Lambda_n|}{\gamma} \log \int e^{\gamma h(\eta(0))} \nu_{\rho,0}(d\eta)$$

for every positive γ because $\nu_{\rho,n}$ is a product measure. In this formula $H(f_{n+1}/f_n)$ stands for the entropy, with respect to the measure $\nu_{\rho,\Lambda_{n+1}-\Lambda_n}$, of the density $f_{n+1}(\eta)/f_n(\eta)$ considered as a function of the variables $\{\eta(x), x \in \Lambda_{n+1} - \Lambda_n\}$ only:

$$H(f_{n+1}/f_n) = \Big\langle \frac{f_{n+1}(\eta)}{f_n(\eta)} \log \Big\{ \frac{f_{n+1}(\eta)}{f_n(\eta)} \Big\} \Big\rangle_{n+1}. \tag{2.6}$$

To conclude the proof of the lemma it remains to recall the definition of f_n and to observe that

$$\int f_n(\eta) \Big\langle \frac{f_{n+1}(\eta)}{f_n(\eta)} \log \Big\{ \frac{f_{n+1}(\eta)}{f_n(\eta)} \Big\} \Big\rangle_{n+1} d\nu_{\rho,n}$$
$$= [H_{n+1}(\mu_{n+1}(t)) - H_n(\mu_n(t))]. \quad \square$$

We conclude this section indicating the elements needed to extend the proof of Theorem (2.1) to mean zero asymmetric zero range processes in $\mathbb{Z}^d$. We start with symmetric processes evolving on $\mathbb{Z}^d$.

Extension to $\mathbb{Z}^d$

The proof in dimension d is essentially the same, with only notational differences. To fix ideas, we consider again the symmetric nearest neighbour case. In this context, when computing the time derivative of the entropy, instead of (2.2), we get the following formula for the boundary term:

$$\sum_{\substack{x\in\Lambda_n, y\notin\Lambda_n\\ |x-y|=1}} \frac{\varphi(\rho)N^2}{2}\int\{< f_{n+1}(\eta+\delta_y) >_{n+1} - < f_{n+1}(\eta+\delta_x) >_{n+1}\}$$
$$\times\{\log < f_{n+1}(\eta+\delta_x) >_{n+1} - \log < f_{n+1}(\eta) >_{n+1}\}\, d\nu_{\rho,n}.$$

The same arguments presented in the one dimensional case lead to the following upper bound for the expression that corresponds to the first term in formula (2.3):

$$\frac{N^2}{2A}\sum_{m=n+1}^{n+N}\{\mathcal{D}_{n+1}(f_m) - \mathcal{D}_n(f_m)\},$$

provided $\mathcal{D}_n$ stands for the restriction of the Dirichlet form to the cube Λ_n:

$$\mathcal{D}_n(f) = \sum_{\substack{x\in\Lambda_n, y\in\Lambda_n\\ |x-y|=1}} \langle -L_{x,y}f, f\rangle_{\rho,M}.$$

With the very same arguments presented in the proof of Theorem 2.1, the expression that corresponds to the second term in formula (2.3) is shown to be bounded above by

$$\varphi(\rho)AN\sum_y\int f_{n+1}(\eta)W_b(g_\varphi(\eta(y)))\, d\nu_{\rho,n+1}.$$

In this formula, summation is carried over all sites y in Λ_n^c that are at distance 1 from Λ_n. By the entropy inequality applied to the function $\sum_y W_b(g_\varphi(\eta(y)))$ and Lemma 2.2, this expression is bounded above by

$$\frac{\varphi(\rho)AN^d}{\gamma}\log\int e^{\gamma W_b(\eta(0))}\nu_{\rho,0}(d\eta)$$
$$+\frac{\varphi(\rho)AN}{\gamma}\{H_{n+1}(\mu_{n+1}(t)) - H_n(\mu_n(t))\}.$$

This estimate together with the arguments presented at the end of the proof of Theorem 2.1 concludes the argument for symmetric processes in any dimension. □

Extension to the mean zero asymmetric case

We consider now the extension of the proof of Theorem 2.1 to mean zero asymmetric processes. To fix ideas, we shall consider the simplest one dimensional example of mean zero asymmetric zero range evolution: the process with generator L defined by (1.1) with transition probability $P(x, y)$ given by

$$P(x, y) = \mathbf{1}\{y = x + 2\} + 2\,\mathbf{1}\{y = x - 1\}.$$

For each site x, denote by L_x^1 the piece of the generator associated to jumps around x:

$$L_x^1 = T_{x-1,x+1} + T_{x,x-1} + T_{x+1,x}.$$

In this formula, for two sites x and y, $T_{x,y}$ is the operator defined by $(T_{x,y}f)(\eta) = g(\eta(x))[f(\eta^{x,y}) - f(\eta)]$. For a positive integer n, denote by L_n the generator L restricted to the cube Λ_n and by $(\partial L)_{n+1}$ the boundary generator:

$$L_n = \sum_{x=-n+1}^{n-1} L_x^1, \quad (\partial L)_{n+1} = L_{-n-1}^1 + L_{-n}^1 + L_n^1 + L_{n+1}^1.$$

The main step in the proof of Theorem 2.1 consists in estimating the expression

$$N^2 \int f_{n+1} (\partial L)_{n+1} \log f_n d\nu_{\rho,M}.$$

In our context, after a change of variables, this integral writes as the sum of two terms. The first one, corresponding to jumps over the right boundary of Λ_n is equal to a constant $(\varphi(\rho)N^2)$ that multiplies

$$\begin{aligned}
&\int f_{n+1}(\eta + \delta_{n-1}) \log \frac{f_n(\eta)}{f_n(\eta + \delta_{n-1})} d\nu_{\rho,M} \\
&+ \int f_{n+1}(\eta + \delta_n) \log \frac{f_n(\eta)}{f_n(\eta + \delta_n)} d\nu_{\rho,M} \\
&+ \int f_{n+1}(\eta + \delta_n) \log \frac{f_n(\eta + \delta_{n-1})}{f_n(\eta + \delta_n)} d\nu_{\rho,M} \\
&+ 2 \int f_{n+1}(\eta + \delta_{n+1}) \log \frac{f_n(\eta + \delta_n)}{f_n(\eta)} d\nu_{\rho,M}.
\end{aligned}$$

The second term, corresponding to jumps over the left boundary is similar and is estimated in the same way. Notice that the sum of all logarithms in last formula vanishes. We may therefore rewrite this expression as

$$\int \{f_n(\eta+\delta_{n-1}) - f_n(\eta+\delta_n)\} \log \frac{f_n(\eta)}{f_n(\eta+\delta_{n-1})} d\nu_{\rho,n}$$
$$+2\int \{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1} - f_n(\eta+\delta_n)\} \log \frac{f_n(\eta+\delta_n)}{f_n(\eta)} d\nu_{\rho,n}. \quad (2.7)$$

The second term is exactly equal to the expression (2.2) obtained in the proof of Theorem 2.1 in the case of reversible dynamics. On the other hand, the first term is negative unless

$$\min\{f_n(\eta), f_n(\eta+\delta_n)\} \le f_n(\eta+\delta_{n-1}) \le \max\{f_n(\eta), f_n(\eta+\delta_n)\}.$$

We have therefore to consider two cases. We start estimating formula (2.7) on the set $f_n(\eta+\delta_n) \le f_n(\eta+\delta_{n-1}) \le f_n(\eta)$. Denote the set of configurations satisfying these inequalities by $\tilde{E}_n$. Assume first that $\langle f_{n+1}(\eta+\delta_{n+1})\rangle_n \le f_n(\eta+\delta_n)$ and denote this set by $\tilde{F}_n^0$. In this case we may bound the first expression in (2.7) by

$$\int \{f_n(\eta+\delta_{n-1}) - \langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}\} \log \frac{f_n(\eta)}{f_n(\eta+\delta_{n-1})} \mathbf{1}\{\tilde{E}_n \cap \tilde{F}_n^0\} d\nu_{\rho,n}.$$

This expression can be estimated exactely in the same way we estimated expression (2.2).

Suppose now that $f_n(\eta+\delta_n) \le \langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1}$ and denote by $\tilde{F}_n^1$ the set of configurations satisfying this inequality. On $\tilde{E}_n \cap \tilde{F}_n^1$, the second term in formula (2.7) is negative and bounded above by

$$2\int \{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1} - f_n(\eta+\delta_n)\} \log \frac{f_n(\eta+\delta_{n-1})}{f_n(\eta)} \mathbf{1}\{\tilde{E}_n \cap \tilde{F}_n^1\} d\nu_{\rho,n}.$$

Adding one half of this expression to the first integral in formula (2.7) restricted to the set $\tilde{E}_n \cap \tilde{F}_n^1$ we obtain

$$\int \{\langle f_{n+1}(\eta+\delta_{n+1})\rangle_{n+1} - f_n(\eta+\delta_{n-1})\} \log \frac{f_n(\eta+\delta_{n-1})}{f_n(\eta)} \mathbf{1}\{\tilde{E}_n \cap \tilde{F}_n^1\} d\nu_{\rho,n}.$$

Again, we may estimate this expression exactely in the same way we bounded (2.2).

The same type of argument goes through in the case where $f_n(\eta) \le f_n(\eta+\delta_{n-1}) \le f_n(\eta+\delta_n)$. This concludes the proof of Theorem 2.1 in this simple example of mean zero asymmetric process. We leave the general case to the reader. □

3. PROOF OF THEOREM 1.1

In view of [7], in order to prove the hydrodynamical behaviour of the infinite system, it is sufficient to obtain a bound on the entropy and on the time integral of the Dirichlet form. More precisely, fix a sequence μ^N satisfying assumptions of Theorem 1.1. Denote by $S_t = S_t^N$ the semigroup of the Markov process with generator L defined by (1.1) accelerated by N^2. We claim that for each $t \geq 0$, there exists a constant $C = C(C_0, t)$ such that

$$H(S_t\mu^N) \; + \; N^2 \int_0^t ds\, D(S_s\mu^N) \; \leq \; CN^d. \tag{3.1}$$

It is by now well known that this estimate and an uniqueness result of weak solutions of equation (1.7) imply the hydrodynamical behaviour of the interacting particle system stated in Theorem 1.1.

To prove inequality (3.1), fix a positive integer $M \geq N^2$ and define the following finite volume approximation of the transition probability:

$$P_M(x,y) = \begin{cases} P(x,y) & \text{if } x, y \in \Lambda_M, \\ 0 & \text{otherwise.} \end{cases}$$

Notice that for this new dynamic, particles outside Λ_M do not move and particles inside Λ_M jump as in the original infinite volume process with the restriction that jumps off Λ_M are suppressed. Denote by $S_t^{M,N}$ the semigroup associated to the generator L defined in (1.1) accelerated by N^2 and with transition probabilities P_M instead of P.

Define the positive continuous function $R = R_{M,N}: \mathbb{R}_+ \to \mathbb{R}_+$ in the following way. For each positive integer $n \leq M - N - 1$, let $R(n/N) = e^{-\theta n/N}$. Set $R(n/N) = 0$ if $n \geq M - N$ and interpolate linearly. Notice that R satisfy the assumptions of Theorem 2.1 and that there exists a constant $C_1 = C_1(\theta)$ such that $N|R((n+1)/N) - R(n/N)| \leq C_1$ because $M \geq N^2$. By Theorem 2.1 and Gronwal inequality for each $t > 0$,

$$\sum_{n=1}^{M-N} \Big\{ H_n((S_t^{M,N}\mu^N)_n) + N^2 \int_0^t ds\, D_n((S_s^{M,N}\mu^N)_n) \Big\} e^{-\theta n/N} \leq CN^d \tag{3.2}$$

for some constant $C = C(C_0, t)$.

For $M - N \leq n \leq M$, by convexity of the entropy and of the Dirichlet form, $H_n((S_t^M\mu^N)_n)$ and $D_n((S_t^M\mu^N)_n)$ are respectively bounded above by $H_M((S_t^M\mu^N)_M)$ and $D_M((S_t^M\mu^N)_M)$.

On the other hand, since for the process with semigroup $S_t^{M,N}$, particles on sites outside Λ_M do not move, for $n \geq M$, $H_n((S_t^M\mu^N)_n)$ decreases in time. Moreover, the usual computation of the entropy production shows that

$$H_n((S_t^{M,N}\mu^N)_n) + N^2 \int_0^t ds\, D_n((S_s^{M,N}\mu^N)_n) \le H_n((\mu^N)_n). \quad (3.3)$$

This previous remark together with (3.2) and (3.3) proves that for each $t \geq 0$, there exists a constant $C = C(C_0, t)$ such that

$$N^{-1}\sum_{n\geq 1}\left\{ H_n((S_t^M\mu^N)_n) + N^2 \int_0^t ds\, \mathcal{D}_n((S_s^M\mu^N)_n)\right\} e^{-\theta n/N} \le C(C_0,t)N^d.$$

It remains to let $M \uparrow \infty$ to obtain (3.1) by the lower semicontinuity of the entropy and of the Dirichlet form.

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