ANNALES DE L'I. H. P., SECTION B

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Correction to "Domains of attraction in several dimensions"

Annales de l'I. H. P., section B, tome 17, nº 1 (1981), p. 143-145
<http://www.numdam.org/item?id=AIHPB_1981__17_1_143_0>

Annales Inst. Henri Poincaré, Section B :
Vol. XVII, n° 1, 1981, p. 143-145. *Calcul des Probabilités et Statistique*.

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by

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For $p > 2$ let X be a l_p-valued random variable defined as follows:

$$X = \varepsilon \Sigma_{N^2 < k \leq N^2 + N} e_k \tag{1}$$

where $\{ e_k \}$ is the canonical basis of l_p, ε is a Bernoulli random variable with parameter 1/2 and N is a random variable independent of ε and with distribution

$$P\{ N = n \} = c/n^{1+2/p} \text{ for } n \in [2, 2^2) \cup [2^{2^2}, 2^{2^3}) \cup \ldots \cup [2^{2^{2k}}, 2^{2^{2k+1}}) \cup \ldots$$
$$P\{ N = n \} = 0 \qquad \text{otherwise}.$$

In example 3.5 of [*2*] it is claimed that X is in the domain of partial attraction of only one law (one type of laws), the centered Gaussian law γ with the covariance of X, which exists, but that nevertheless X is not in its domain of attraction. To the best of our knowledge this statement is true, however the proof given in [*2*] of the fact that X is actually in the domain of partial attraction of γ is not correct. In this note we give a (hopefully) correct proof of this.

The notation will be as in [*2*]; in particular, if $x \in l_p$, $x_{(k)}$ will denote its k-th coordinate in the canonical basis.

We need the following

PROPOSITION. — Assume that X is a l_p-valued symmetric random variable.

$p \geq 2$. Let $\{X_i\}$ be independent identically distributed copies of X, and let $n_k \uparrow \infty$, $n_k \in \mathbb{N}$. Then, in order that the sequence

$$(2) \qquad \{\mathscr{L}(\Sigma_{i=1}^{n_k} X_i/n_k^{1/2})\}_{k=1}^{\infty}$$

converge weakly to a Gaussian law it is (necessary and) sufficient that the following conditions hold:

i) X is pregaussian,
ii) for every $\delta > 0$, $\lim_{k\to\infty} n_k P\{\|X\| > \delta n_k^{1/2}\} = 0$,
iii) $\lim_{R\to\infty} \sup_k n_k^{1-p/2}\Sigma_{m=R+1}^{\infty} E\,|X_{(m)}|^p I_{[\|X\|\leq n_k^{1/2}]} = 0$.

This proposition follows directly from Theorem 3.1 in [*3*]. See also exercises 13 and 17, p. 205-206, in [*1*].

We will prove that the variable X defined in (1) satisfies conditions (*i*)-(*iii*) of the previous proposition for the sequence $\{n_k\}$ given by

$$(3) \qquad n_k = [2^{2^{2k+3/2(2/p)}}], \qquad k \in \mathbb{N}$$

where [ ] denotes « integer part of ».

It is already proved in [*2*] that X is pregaussian. So we need only prove that X verifies properties (*ii*) and (*iii*).

Proof of ii). — Obviously, given $\delta > 0$ there exists k_δ such that for $k > k_\delta$

$$2^{2^{2k+1}} < \delta^p n_k^{p/2}.$$

Since $\|X\| = N^{1/p}$ it follows that for $k > k_\delta$,

$$\begin{aligned} n_k P\{\|X\| > \delta n_k^{1/2}\} &= n_k P\{N > \delta^p n_k^{p/2}\} \\ &\leq c 2^{2^{2k+3/2(2/p)}} \Sigma_{n=2^{2^{2k+2}}}^{\infty} n^{-1-2/p} \approx 2^{-1} c p 2^{2^{2k}(2/p)(2^{3/2}-4)} \\ &\to 0 \quad \text{as } k \to \infty. \end{aligned}$$

Proof of iii). — Let us first observe that

$$(4) \quad \Sigma_{m=R+1}^{\infty} E\,|X_{(m)}|^p I_{[\|X\|\leq n_k^{1/2}]} = \Sigma_{m=R+1}^{\infty} P\{N^2 < m \leq N^2 + N,\ N \leq n_k^{p/2}\}$$
$$\leq \Sigma_{s=[R^{1/2}]}^{\infty} sP\{N = s, N \leq n_k^{p/2}\} = \begin{cases} 0 & \text{if } [R^{1/2}] > n_k^{p/2} \\ \Sigma_{s=[R^{1/2}]}^{n_k^{p/2}} sP\{N = s\} & \text{otherwise}. \end{cases}$$

Let $r \in \mathbb{N}$ be such that $2^{2^{2r}} \leq [R^{1/2}] < 2^{2^{2r+2}}$. By (4) the limit in (*iii*) is then bounded above by

$$\begin{aligned} &\lim_{R\to\infty} \sup_{k\geq r} c n_k^{1-p/2} \int_{[2^{2^{2r}}-1,2^{2^{2r+1}}]\cup\ldots\cup[2^{2^{2k}}-1,2^{2^{2k+1}}]} x^{-2/p}dx \\ &\qquad \leq \lim_{r\to\infty} \sup_{k\geq r} c(1-2/p)^{-1} n_k^{1-p/2} k 2^{2^{2k+1}(1-2/p)} \\ &\qquad = \lim_{r\to\infty} c(1-2/p)^{-1} r 2^{2^{2r+1}(1-2/p)(1-2^{1/2})} = 0. \end{aligned}$$

The proof that X satisfies (*ii*) and (*iii*) is thus completed. Hence, X is in the domain of partial attraction of the centered Gaussian law which has its covariance.

There is a trivial error in [*2*] which we also correct now. The last two lines of the proof of Theorem 3.1, part (2) should read: « ... therefore, if $\mu\{\,\|x\| > 0\,\} = 0$ we have $P\{\,\|X\| > a_{n_i}\,\}/a_{n_i}^{-2}E\,\|X\|^2 I_{\{\|X\| \leq a_{n_i}\}} \to 0$ as ... ».

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[*3*] E. GINÉ, V. MANDREKAR, J. ZINN, On sums of independent random variables with values in L_p, $2 \leq p < \infty$. *Lectures Notes in Math.*, t. **709**, 1979, p. 111-124.

(Manuscrit reçu le 11 septembre 1980).