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Regularity of constraints and reduction in the Minkowski space Yang-Mills-Dirac theory

by

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ABSTRACT. – The constraint equations for Yang-Mills and Dirac fields are investigated for the extended phase space consisting of the Cauchy data $A \in H^2(\mathbb{R}^3)$, $E \in H^1(\mathbb{R}^3)$, and $\Psi \in H^2(\mathbb{R}^3)$. The solution set is a smooth submanifold of a dense subspace of the extended phase space. It is a principal fibre bundle over the reduced phase space with structure group consisting of the gauge symmetries approaching the identity at infinity. © Elsevier, Paris

Key words: Banach manifolds, constraints, non-linear partial differential equations, reduction, Yang-Mills fields.

RÉSUMÉ. – Les équations de contraintes pour les champs de Yang-Mills et Dirac sont examinées dans l'espace de phases prolongé composé de données de Cauchy $A \in H^2(\mathbb{R}^3)$, $E \in H^1(\mathbb{R}^3)$, et $\Psi \in H^2(\mathbb{R}^3)$. L'ensemble de solutions est une sous-variété d'un sous-espace dense de l'espace de phases prolongé. C'est un espace fibré ayant pour base l'espace de phase reduit et pour groupe structural le groupe de symétries de jauge convergeant vers l'identité à l'infini. © Elsevier, Paris

1. INTRODUCTION

This paper is second in a series devoted to a systematic study of a classical phase space for minimally interacting Yang-Mills and Dirac fields

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in the Minkowski space-time. The structure of such a phase space plays an important role in the quantization of the theory, independently of the quantization techniques employed. An analysis of a possible phase space for the theory involves:

1. A determination of a space of Cauchy data admitting solutions to the evolution component of the field equations,
2. An analysis of the constraint equation in the chosen space of the Cauchy data,
3. A determination of the reduced phase space and an analysis of its structure,
4. An analysis of the physical consequences of the given choice of the phase space.

There are several papers devoted to the first point, that is a determination of various spaces of Cauchy data admitting unique solutions of the field equations, Ref. [1-7]. A complete analysis of the constraint equation was given by Moncrief, [8]. However, there are no existence and uniqueness theorems for the space of the Cauchy data used in [8].

We consider here the Yang-Mills-Dirac theory for the internal symmetry group G with the Lie algebra $\mathfrak{g}$ admitting an Ad-invariant positive definite metric. The Yang-Mills potential A_μ describes a connection in the principal fibre bundle $\mathbb{R}^4 \times G$ with respect to the trivialization given by the product structure. It is a $\mathfrak{g}$-valued 1-form on $\mathbb{R}^4$. The curvature form of the connection given by A_μ is described by the Yang-Mills field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$, where $[.,.]$ denotes the Lie bracket in $\mathfrak{g}$. The 3+1 splitting of the space-time leads to a decomposition of the Yang-Mills potential A_μ into its time component A_0 and the spatial component $A = (A_1, A_2, A_3)$. Similarly, the Yang-Mills field $F_{\mu\nu}$ can be decomposed into its electric components $E = (F_{01}, F_{02}, F_{03})$ and the magnetic components $B = (F_{23}, F_{31}, F_{12})$. From the point of view of the evolution equations it is convenient to consider A, E and B as $\mathfrak{g}$-valued time-dependent vector fields on $\mathbb{R}^3$, and A_0 as a $\mathfrak{g}$-valued time dependent function on $\mathbb{R}^3$. The vector potential A describes the induced connection in $\mathbb{R}^3 \times G$. The matter is described by a Dirac spinor field Ψ, that is a time dependent map from $\mathbb{R}^3$ to $\mathbb{C}^4 \otimes V$, where V is the space of a representation of G. The field equations split into the evolution equations

$$\begin{aligned} \dot{A} &= E + grad\ A_0 - [A_0, A] \\ \dot{E} &= -curl\ B - [A\times, B] - [A_0, E] + \Psi^\dagger \gamma^0 \gamma T_a \Psi T^a \\ \dot{\Psi} &= -\gamma^0(\gamma^j \partial_j + im + \gamma^0 A_0 + \gamma^j A_j)\Psi \end{aligned} \tag{1}$$

and the constraint equation

$$div\ E = [A; E] + \Psi^{\dagger} T_a \Psi T^a \tag{2}$$

where $[.;.]$ is the Lie bracket in $\mathfrak{g}$ combined with the Euclidean scalar product of vector fields in $\mathbb{R}^3$, $\{T^a\}$ is an orthonormal basis in $\mathfrak{g}$, and the Latin indices are lowered in terms of the Ad-invariant metric in $\mathfrak{g}$.

In the preceding paper [9] we have proved the existence and uniqueness theorems for the evolution component of the field equations, Eq. (1), with the Cauchy data (A, E) for the Yang-Mills field in the space

$$P_{_{YM}} = H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3) \times H^1(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3),$$

and the Cauchy data Ψ for the Dirac field in

$$P_D = H^2(\mathbb{R}^3, \mathbb{C}^4 \otimes V).$$

Here, for any vector space W, we use $H^k(\mathbb{R}^3, W)$ to denote the Sobolev space of maps from the source space $\mathbb{R}^3$ to the target space W which are square integrable together with their derivatives up to the order k. In [9] we omitted the target spaces in our notation for the sake of brevity. In the present paper we have several intermediate target spaces and use the full notation for Sobolev spaces in order to avoid a confusion.

The aim of this paper is to study the structure of the constraint set, that is the solution set of Eq. (2). The main results obtained here are summarized in the theorems below.

THEOREM 1. – *For minimally interacting Yang-Mills and Dirac fields, the constraint set*

$$C = \{(A, E, \Psi) \in P \mid divE + [A; E] = \Psi^{\dagger}(I \otimes T^a)\Psi T_a\}$$

is a C^∞-submanifold of the Banach space

$$P^0 = \{(A, E, \Psi) \in P \mid div\ E \in L^{6/5}(\mathbb{R}^3, \mathfrak{g})\} \tag{3}$$

with the norm

$$||(A, E, \Psi)||_{P^0} = ||(A, E, \Psi)||_P + ||div\ E||_{L^{6/5}(\mathbb{R}^3, \mathfrak{g})}. \tag{4}$$

Similarly, in absence of Dirac fields the set of the Cauchy data (A, E) satisfying the constraint equation is a C^∞-submanifold of

$$\{(A, E) \in P_{YM} \mid div\ E \in L^{6/5}(\mathbb{R}^3, \mathfrak{g})\}.$$

Using the Implicit Function Theorem for Banach manifolds, [10], and the results of Eardley and Moncrief, [4], one can show that Theorem 1 is a consequence of

THEOREM 2. – *For each* $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, *the operator*

$$Div_A e = div\ e + [A; e]$$

maps

$$D = \{e \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3) | div\ e \in L^{6/5}(\mathbb{R}^3, \mathfrak{g})\},$$

onto the Banach space

$$B = H^1(\mathbb{R}^3, \mathfrak{g}) \cap L^{6/5}(\mathbb{R}^3, \mathfrak{g}).$$

Outline of Proof. – It follows from the results of Ref. [4], that the Laplace operator restricted to the space $\{\phi \in H^3(\mathbb{R}^3, \mathfrak{g}) | \Delta\phi \in L^{6/5}(\mathbb{R}^3, \mathfrak{g})\}$ is onto B. Hence, taking $e = grad\ \phi$, we see that the divergence operator div maps D onto B. Now, the covariant divergence operator $Div_A e = div\ e + [A; e]$ is a perturbation of the ordinary divergence div by a multiplication operator $e \mapsto [A; e]$. We show that, for $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, the operator $D \to B : e \mapsto [A; e]$ is compact. Since $div\ e$ is onto and $[A; e]$ is compact it follows that Div_A is semi-Fredholm, which implies that it has closed range, [11]. Next we show that the annihilator of the range of Div_A vanishes which implies that Div_A is onto B. This will complete the proof.

The problem of regularity of the constraint set in the extended phase space P_{YM} was studied in [12]. However, the argument given there was based on an invalid assumption that the range of $grad$ from $H^1(\mathbb{R}^3, \mathbb{R})$ to $L^2(\mathbb{R}^3, \mathbb{R}^3)$ is closed. Therefore, even though the operator $grad + [A, .]$ is a compact perturbation of $grad$, we cannot conclude that it is semi-Fredholm, which would have implied that Div_A were semi-Fredholm. This difficulty is overcome here by the restriction of the target space for our operators by the condition that the divergence is contained in $L^{6/5}(\mathbb{R}^3, \mathfrak{g})$.

The extended phase space

$$P = P_{YM} \times P_D$$

is weakly symplectic, with the weak symplectic form $\omega = d\theta$, where

$$\langle \theta(A, E, \Psi) \mid (a, e, \psi) \rangle = \int_{\mathbb{R}^3} (E \cdot a + \Psi^\dagger \psi) d_3x.$$

The pull-back ω_C of ω to C has involutive kernel $\ker \omega_C$. The reduced phase space $\check{P}$ is defined as the set of equivalence classes of points in C

under the equivalence relation $p \simeq p'$ if and only if there is a piece-wise smooth curve in C with the tangent vector contained in $\ker \omega_C$. If $\ker \omega_C$ is a distribution, it is clearly involutive and the equivalence classes coincide with integral manifolds of $\ker \omega_C$. We denote by $\rho : C \to \check{P}$ the canonical projection associating to each $p \in C$ its equivalence class containing p.

In [9] we have studied the gauge symmetry group $GS(P)$ of the extended phase space P. It consists of time independent gauge transformations, represented by maps Φ from $\mathbb{R}^3$ to $\mathfrak{g}$, such that the action on the Cauchy data (A, E, Ψ) given by

$$A \mapsto \Phi A \Phi^{-1} + \Phi grad \Phi^{-1}, \ E \mapsto \Phi E \Phi^{-1}, \ \Psi \mapsto \Phi \Psi, \tag{5}$$

is continuous in P. We have shown that $\Phi \in GS(P)$ if and only if the Beppo Levi norm

$$\| \phi \|^2_{\mathcal{B}^3} = \int_{B_1} | \phi |^2 \, d_3x + \| grad \phi \|^2_{H^2(\mathbb{R}^3 \otimes \mathbb{R}^3, \mathfrak{g})},$$

where B_1 is the unit ball in $\mathbb{R}^3$ centered at the origin, is finite. With the topology given by this norm, the gauge symmetry group of P is a Hilbert-Lie group.

The action in P of an element ξ of the Lie algebra $gs(P)$ of $GS(P)$ is given by the vector field

$$\xi_P(A, E, \Psi) = (-D_A \xi, -[E, \xi], \Psi^\dagger \xi),$$

where

$$D_A \xi = d\xi + [A, \xi]$$

is the covariant differential of ξ with respect to the connection A. The action of $GS(P)$ in P preserves the 1-form θ. Since $\omega = d\theta$, this action is Hamiltonian with an equivariant momentum map $\mathcal{J}$ such that, for every $\xi \in gs(P)$

$$\langle \mathcal{J}(A, E, \Psi) \mid \xi \rangle = \langle \theta \mid \xi_P(A, E, \Psi) \rangle = \int_{\mathbb{R}^3} (-E \cdot D_A \xi + \Psi^\dagger \xi \Psi) d_3x. \tag{6}$$

The closure, with respect to the norm $\| \Phi \|^2_{\mathcal{B}^3}$, of the group of smooth maps Φ which differ from the identity only in compact sets is a normal subgroup $GS(P)_0$ of the gauge symmetry group $GS(P)$. The constraint manifold C coincides with the zero level of the momentum map $\mathcal{J}$ restricted to the Lie algebra $gs(P)_0$ of $GS(P)_0$,

$$(A, E, \Psi) \in C \iff \langle \mathcal{J}(A, E, \Psi) \mid \xi \rangle = 0 \text{ for all } \xi \in gs(P)_0. \tag{7}$$

The action of $GS(P)_0$ in P is proper, [9].

THEOREM 3. – *The action of $GS(P)_0$ in C is free and proper. The reduced phase space $\check{P}$ coincides with the set $C/GS(P)_0$ of the orbits of the $GS(P)_0$-action in C,*

$$\check{P} = C/GS(P)_0.$$

It is a quotient manifold of C endowed with a weak Riemannian metric induced by the L^2 scalar product in P, and with a 1-form $\check{\theta}$ such that

$$\rho^*\check{\theta} = \theta_C,$$

where θ_C is the pull-back of θ to C. Moreover, $\check{\omega} = d\check{\theta}$ is weakly symplectic and

$$\rho^*\check{\omega} = \omega_C.$$

The constraint manifold C is a principal fibre bundle over $\check{P}$ with structure group $GS(P)_0$.

The existence of a symplectic structure in the reduced phase space is a folk theorem based on the assumption that the reduced phase space is a manifold. For the symplectic strata in the bag model it was proved in [13]. The existence of a weak Riemannian structure in the space of orbits of a subgroup of the gauge group acting on the space of connections in a compact manifold was shown in [14].

2. RESULTS OF EARDLEY AND MONCRIEF

The following results can be found in the Appendix to Reference [4]. They will be needed in the sequel.

Let

$$K(x) = \frac{1}{4\pi|x|}$$

be the Green's function for the Laplace equation in $\mathbb{R}^3$. For every $\rho \in H^1(\mathbb{R}^3, \mathfrak{g}) \cap L^{6/5}(\mathbb{R}^3, \mathfrak{g})$, the convolution

$$\phi = K * \rho$$

is a solution of the equation

$$\Delta\phi = \rho.$$

For $1 < p < \infty$, and $s = 3p/(3-2p)$, the following inequalities hold for an appropriate choice of constants c:

$$||K * \rho||_{L^s(\mathbb{R}^3, \mathfrak{g}\otimes\mathbb{R}^3)} \leq c||K||_{L^3_w(\mathbb{R}^3,\mathbb{R}^3)}||\rho||_{L^p(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)},$$

$$||grad(K * \rho)||_{L^s(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)} \leq c||grad\ K||_{L^{3/2}_w(\mathbb{R}^3,\mathbb{R}^3\otimes\mathbb{R}^3)}||\rho||_{L^p(\mathbb{R}^3,\mathfrak{g})},$$

where, for every normed vector space W, the space $L^p_w(\mathbb{R}^3, W)$ is the weak L^p space, see Ref. [15]. Let μ denote the Euclidean measure in $\mathbb{R}^3$ defined by the Euclidean metric. A map $u : \mathbb{R}^3 \to W$ is said to be in $L^p_w(\mathbb{R}^3, W)$ if there exists a constant $C < \infty$ such that

$$\mu\{x \mid ||u(x)||_W > t\} \leq Ct^{-p} \quad \forall t > 0.$$

For $u \in L^p_w(\mathbb{R}^3, W)$,

$$||u||_{L^p_w(\mathbb{R}^3,W)} = \sup_{t>0}(t^p\mu\{x \mid ||u(x)||_W > t\})^{1/p}.$$

This implies that

$$\rho \in L^{6/5}(\mathbb{R}^3, \mathfrak{g}) \Rightarrow K * \rho \in L^2(\mathbb{R}^3, \mathfrak{g}) \text{ and } grad(K * \mathfrak{g}) \in L^2(\mathbb{R}^3, \mathfrak{g}).$$

Moreover,

$$||grad(K * \rho)||_{H^1(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)} \leq ||\rho||_{L^2(\mathbb{R}^3,\mathfrak{g})} + c||\rho||_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} \tag{8}$$

and

$$||grad(K * \rho)||_{H^2(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)} \leq ||\rho||_{H^1(\mathbb{R}^3,\mathfrak{g})} + c||\rho||_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})}. \tag{9}$$

Hence, $H^1(\mathbb{R}^3, \mathfrak{g}) \cap L^{6/5}(\mathbb{R}^3, \mathfrak{g})$ is in the range of the Laplace operator Δ acting on $H^3(\mathbb{R}^3, \mathfrak{g})$. Moreover, the kernel of the Laplace operator restricted to $H^3(\mathbb{R}^3, \mathfrak{g})$ vanishes.

The last of the estimates from [4] needed here is

$$||[A; E]||_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} \leq c||A||_{H^2(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)}||E||_{H^1(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)}. \tag{10}$$

Similarly, we can prove

$$||\Psi^\dagger(I \otimes T^a)\Psi T_a||_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} \leq c||\Psi||^2_{H^2(\mathbb{R}^3,\mathbb{C}\otimes V)}. \tag{11}$$

3. PROOF OF THEOREM 1

For $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, $E \in H^1(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, and $\Psi \in H^2(\mathbb{R}^3, \mathbb{C}^4 \otimes V)$, all the bilinear terms in the constraint equation

$$div\ E + [A; E] - \Psi^\dagger (I \otimes T^a) \Psi T_a = 0$$

are in $H^1(\mathbb{R}^3, \mathfrak{g})$. Moreover, estimates (10) and (11) imply that they are also contained in $L^{6/5}(\mathbb{R}^3, \mathfrak{g})$. Hence, if (A, E, Ψ) is in the constraint set C, then $div\ E$ is contained in

$$B = H^1(\mathbb{R}^3, \mathfrak{g}) \cap L^{6/5}(\mathbb{R}^3, \mathfrak{g}). \tag{12}$$

It follows that the constraint set C is the zero level of the smooth map

$$F : P^0 \to B : (A, E, \Psi) \mapsto div\ E + [A; E] - \Psi^\dagger (I \otimes T^a) \Psi T_a,$$

where P^0 is given by Eq. (3). For each $(A, E, \Psi) \in C$ and $(a, e, \psi) \in P^0$,

$$DF_{(A,E,\Psi)}(a, e, \psi) = div\ e + [A; e] + [E; a] - \psi^\dagger (I \otimes T^a) \Psi T_a - \Psi^\dagger (I \otimes T^a) \psi T_a.$$

According to the Implicit Function Theorem $C = F^{-1}(0)$ is a submanifold of P^0 if, $DF_{(A,E,\Psi)}$ maps P^0 onto B for every $(A, E, \Psi) \in C$ and its kernel splits, [10].

Theorem 2 asserts that, for every $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, the covariant divergence Div_A, with respect to the connection A, given by

$$Div_A e = div\ e + [A; e],$$

maps

$$D = \{e \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3) | div\ e \in L^{6/5}(\mathbb{R}^3, \mathfrak{g})\} \tag{13}$$

onto B. Since

$$D \subset \tilde{D} = \{e \in H^1(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3) \mid div\ e \in B\},$$

it follows that Div_A maps $\tilde{D}$ onto B. This implies that $DF_{(A,E,\Psi)}$ maps P^0 onto B.

Moreover, if $(a, e, \psi) \in P$ and

$$div\ e + [A; e] + [E; a] - \psi^\dagger (I \otimes T^a) \Psi T_a - \Psi^\dagger (I \otimes T^a) \psi T_a = 0,$$

then $div\ e \in L^{6/5}(\mathbb{R}^3, \mathfrak{g})$ and $(a, e, \psi) \in P^0$. Hence, $\ker DF_{(A,E,\Psi)}$ is a closed subspace of P^0. Since the L^2-scalar product is continuous and non-degenerate in P^0, it follows that the L^2-orthogonal complement of $\ker DF_{(A,E,\Psi)}$ is closed in P^0 and its intersection with $\ker DF_{(A,E,\Psi)}$ is zero. Thus, the assumptions of the Implicit Function Theorem are satisfied and $C = F^{-1}(0)$ is a smooth submanifold of P^0.

Clearly, the same results hold in absence of the Dirac fields. This completes the proof of Theorem 1.

Since P^0 is a dense subspace of P, it may appear that the manifold topology in C is finer than the topology in C induced by the embedding $C \subset P$. This is not the case.

LEMMA 1. – *The topology of C as a subset of P coincides with its manifold topology.*

Proof. – The topology of C defined by the norm (4) is finer than the topology induced by the embedding $C \subset P$. On the other hand, if $p_n = (A_n, E_n, \Psi_n)$ is a sequence in C convergent in P to $p = (A, E, \Psi)$. Since C is closed in P, as the zero level of a continuous function then, then $p \in C$. The constraint equation (2) yields

$$\begin{aligned} div(E - E_n) &= [A_n; E_n] - [A; E] + \Psi^\dagger (I \otimes T^a)\Psi T_a - \Psi_n^\dagger (I \otimes T^a)\Psi_n T_a \\ &= [(A_n - A); E] + [A_n; (E_n - E)] + (\Psi - \Psi_n)^\dagger (I \otimes T^a)\Psi T_a \\ &\quad + \Psi_n^\dagger (I \otimes T^a)(\Psi - \Psi_n)T_a. \end{aligned}$$

Taking into account the estimates (10) and (11) we get

$$\begin{aligned} \|div(E - E_n)\|_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} \leq\ & c\|A_n - A\|_{H^2(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)}\|E\|_{H^1(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)} \\ &+ c\|A_n\|_{H^2(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)}\|E_n - E\|_{H^1(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)} \\ &+ c\|\Psi - \Psi_n\|_{H^2(\mathbb{R}^3,\mathbb{C}\otimes V)}\|\Psi\|_{H^2(\mathbb{R}^3,\mathbb{C}\otimes V)} \\ &+ c\|\Psi_n\|_{H^2(\mathbb{R}^3,\mathbb{C}\otimes V)}\|\Psi - \Psi_n\|_{H^2(\mathbb{R}^3,\mathbb{C}\otimes V)}. \end{aligned}$$

Hence $\|div(E - E_n)\|_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} \to 0$ as $n \to \infty$, which implies that the sequence p_n converges to p in the topology induced by the norm (4). Therefore the manifold topology in C coincides with its topology induced by the embedding $C \subset P$.

4. PROOF OF THEOREM 2

LEMMA 2. – *For $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, the map*

$$H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3) \to L^{6/5}(\mathbb{R}^3, \mathfrak{g}) : e \mapsto [A; e]$$

is compact.

Proof. – It follows from (10) that $A, e \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$ implies that $[A; e] \in L^{6/5}(\mathbb{R}^3, \mathfrak{g})$. Let $\{e_n\}$ be a bounded sequence in $H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$ with a bound M. Then, for every $m, n \in \mathbb{N}$,

$$||e_m - e_n||_{H^2(\mathbb{R}^3, \mathfrak{g}\otimes\mathbb{R}^3)} \leq 2M.$$

Let U be a bounded domain in $\mathbb{R}^3$ with complement V, and χ_U and χ_V the characteristic functions of U and V, respectively. Using the triangle inequality and (10) applied to functions in domains U and V, we get

$$\begin{aligned}
&||[A; e_k] - [A; e_m]||_{L^{6/5}(\mathbb{R}^3, \mathfrak{g})} = ||[A; e_k - e_m]||_{L^{6/5}(\mathbb{R}^3, \mathfrak{g})} \\
&= ||\chi_U([A; e_k - e_m]) + \chi_V([A; e_k - e_m])||_{L^{6/5}(\mathbb{R}^3, \mathfrak{g})} \\
&\leq ||[A; e_k - e_m]||_{L^{6/5}(U, \mathfrak{g})} + ||[A; e_k - e_m]||_{L^{6/5}(V, \mathfrak{g})} \\
&\leq c||A||_{H^2(U, \mathfrak{g}\otimes\mathbb{R}^3)} ||e_k - e_m||_{H^1(U, \mathfrak{g}\otimes\mathbb{R}^3)} \\
&\quad + c||A||_{H^2(V, \mathfrak{g}\otimes\mathbb{R}^3)} ||e_k - e_m||_{H^1(V, \mathfrak{g}\otimes\mathbb{R}^3)} \\
&\leq c||A||_{H^2(\mathbb{R}^3, \mathfrak{g}\otimes\mathbb{R}^3)} ||e_k - e_m||_{H^1(U, \mathfrak{g}\otimes\mathbb{R}^3)} \\
&\quad + c||A||_{H^2(V, \mathfrak{g}\otimes\mathbb{R}^3)} ||e_k - e_m||_{H^1(V, \mathfrak{g}\otimes\mathbb{R}^3)}.
\end{aligned}$$

Repeating the argument in [12] we see that, for every $n \in \mathbb{N}$, there exists U_n such that $||A||_{H^2(V_n, \mathfrak{g}\otimes\mathbb{R}^3)} < 1/4cMn$ which implies that

$$c||A||_{H^2(V_n, \mathfrak{g}\otimes\mathbb{R}^3)} ||e_k - e_m||_{H^1(V_n, \mathfrak{g}\otimes\mathbb{R}^3)} < 1/2n.$$

Moreover, we may choose the sequence of domains U_n such that $U_k \subseteq U_m$ for $k \leq m$.

By the Rellich-Kondrachov Theorem, see [16], the embedding of $H^2(U_n, \mathfrak{g} \otimes \mathbb{R}^3)$ into $H^1(U_n, \mathfrak{g} \otimes \mathbb{R}^3)$ is compact. This implies that, for every n, the sequence $\{\chi_{U_n} e_k\}$ has a subsequence $\{e_k^n\}$ convergent in $H^1(U_n, \mathfrak{g} \otimes \mathbb{R}^3)$. We can choose this subsequence in such a way that, for $m < n$, $\{e_k^n\} \subseteq \{e_k^m\}$.

If n is chosen so that $c||A||_{H^2(V_n, \mathfrak{g}\otimes\mathbb{R}^3)} ||e_k - e_m||_{H^1(V_n, \mathfrak{g}\otimes\mathbb{R}^3)} < 1/2n$, let N_n be the integer such that, for all $k, m \geq N_n$, $||e_k^n - e_m^n||_{H^1(U_n, \mathfrak{g}\otimes\mathbb{R}^3)} < 1/2nc||A||_{H^2(\mathbb{R}^3, \mathfrak{g}\otimes\mathbb{R}^3)}$. Then,

$$||[A; e_k^n] - [A; e_m^n]||_{L^{6/5}(\mathbb{R}^3, \mathfrak{g})} < 1/n \text{ for all } \quad k, m \geq N_n.$$

The subsequence $\{[A; e_{N_n}^n]\}$ of $\{[A; e_n]\}$ is convergent in $L^{6/5}(\mathbb{R}^3, \mathfrak{g})$. This implies that the map $H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3) \to L^{6/5}(\mathbb{R}^3, \mathfrak{g}) : e \mapsto [A; e]$ is compact. ■

LEMMA 3. – *For* $k \geq 1$, *and* $A \in H^k(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, *the map*

$$H^k(\mathbb{R}^3, \mathfrak{g}) \to H^{k-1}(\mathbb{R}^3, \mathfrak{g}) : e \mapsto [A; e]$$

is compact.

Proof. – For $k = 1$, we follow the argument given in [12]. Let $\{e_n\}$ be a bounded sequence in $H^1(\mathbb{R}^3, \mathfrak{g})$. By the Sobolev Imbedding Theorem, this sequence is bounded in $L^4(\mathbb{R}^3, \mathfrak{g})$, and there exists $M > 0$ such that

$$||e_n - e_m||_{L^4(\mathbb{R}^3, \mathfrak{g})} < M.$$

For any bounded domain $U \subset \mathbb{R}^3$ with complement $V = \mathbb{R}^3 - U$,

$$\begin{aligned}
&||[A, e_n] - [A, e_m]||_{L^2(\mathbb{R}^3, \mathfrak{g})} \\
&= \int_{\mathbb{R}^3} ||[A, (e_n - e_m)]||^2 d_3x \\
&= \int_U ||[A, (e_n - e_m)]||^2 d_3x + \int_V ||[A, (e_n - e_m)]||^2 d_3x \\
&\leq ||A||^2_{L^4(U, \mathfrak{g})} ||e_n - e_m||^2_{L^4(U, \mathfrak{g})} + ||A||^2_{L^4(V, \mathfrak{g})} ||e_n - e_m||^2_{L^4(V, \mathfrak{g})} \\
&\leq ||A||^2_{L^4(\mathbb{R}^3, \mathfrak{g})} ||e_n - e_m||^2_{L^4(U, \mathfrak{g})} + ||A||^2_{L^4(V, \mathfrak{g})} ||e_n - e_m||^2_{L^4(\mathbb{R}^3, \mathfrak{g})} \\
&\leq ||A||^2_{L^4(\mathbb{R}^3, \mathfrak{g})} ||e_n - e_m||^2_{L^4(U, \mathfrak{g})} + M^2 ||A||^2_{L^4(V, \mathfrak{g})}.
\end{aligned}$$

Since $||A||^2_{L^4(\mathbb{R}^3, \mathfrak{g})}$ is the limit of $||A||^2_{L^4(U, \mathfrak{g})}$ as U increases to $\mathbb{R}^3$, for every $k > 0$, there exists an open bounded set $U_k \subset \mathbb{R}^3$ such that

$$M^2 ||A||^2_{L^4(V_k, \mathfrak{g})} < 1/2k,$$

where $V_k = \mathbb{R}^3 - U_k$.

For a bounded domain U_k in $\mathbb{R}^3$, the Rellich-Kondrachov Theorem implies that the embedding $H^1(R^3, \mathfrak{g}) \to L^4(U_k, \mathfrak{g})$ is compact, [16]. Hence the sequence $\{\chi_{U_k} e_n\}$, consisting of the restrictions of e_n to U_k, has a subsequence convergent in $L^4(U_k, g)$. Following the argument in the proof of Lemma 2 we construct a subsequence $\{e^n_{N_n}\}$ of $\{e_n\}$ such that $\{[A; e^n_{N_n}]\}$ converges in $L^2(\mathbb{R}^3, \mathfrak{g})$.

For $k > 1$, we can proceed as follows. Let $\{e_n\}$ be a bounded sequence in $H^k(\mathbb{R}^3, \mathfrak{g})$. We have

$$||[A, e_n] - [A, e_m]||_{H^{k-1}(\mathbb{R}^3)} \leq \sum_{i=0}^{k-1} ||D^i[A, e_n] - D^i[A, e_m]||_{L^2(\mathbb{R}^3)} \leq$$

$$c \sum_{i=0}^{k-1} \sum_{j=0}^{i} ||[D^j A, D^{i-j} e_n] - [D^j A, D^{i-j} e_m]||_{L^2(\mathbb{R}^3)}.$$

In order to avoid a cumbersome notation, in the description of the norms we have omitted the target space. Since $A \in H^k(\mathbb{R}^3)$, it follows that $D^j A \in H^1(\mathbb{R}^3)$ for $0 \leq j \leq k-1$. Similarly, $D^{i-j} e_n \in H^1(\mathbb{R}^3)$. We can apply the results for $k = 1$ to each of the operators $[D^j A, D^{i-j} e]$. Hence, there exists a subsequence $\{e^n_{N_n}\}$ of $\{e_n\}$ such that $\{[A, e^n_{N_n}]\}$ is convergent in $H^k(\mathbb{R}^3, \mathfrak{g})$. This implies that the map $H^k(\mathbb{R}^3, \mathfrak{g}) \to H^{k-1}(\mathbb{R}^3, \mathfrak{g})$ given by $e \mapsto [A; e]$ is compact. ■

It follows from Lemmas 2 and 3 that, for $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, the operator $Div_A : D \to B : e \mapsto div\ e + [A; e]$ is a compact perturbation of the divergence operator div, where B and D are given by Eq. (12) and Eq. (13), respectively. Since the results of Eardley and Moncrief discussed in Section 2 imply that the divergence maps D onto B, it follows that Div_A is semi-Fredholm. Therefore, the range of Div_A is closed, [11]. It remains to show that the annihilator of the range of Div_A in the dual B' of B vanishes.

Since B contains the Schwarz space S of rapidly decaying smooth functions on $\mathbb{R}^3$, it follows that elements of the dual B' of B are tempered distributions. For each $\mu \in B'$, and $\phi \in D$,

$$\langle \mu | Div_A e \rangle = \langle \mu | div\ u + [A, e] \rangle = -\langle grad \mu + [A, \mu] | e \rangle,$$

where the last equality follows from the ad-invariance of the metric in $\mathfrak{g}$. Hence μ annihilates the range of Div_A if and only if

$$grad \mu + [A, \mu] = 0$$

in the sense of distributions.

The space Let $\bar{B} = L^2(\mathbb{R}^3, \mathfrak{g}) \cap L^{6/5}(\mathbb{R}^3, \mathfrak{g})$ be endowed with the norm $||f||_{\bar{B}} = ||f||_{L^2(\mathbb{R}^3, \mathfrak{g})} + ||f||_{L^{6/5}(\mathbb{R}^3, \mathfrak{g})}$. It is the completion of B with respect to $||f||_{\bar{B}}$.

LEMMA 4. – *For $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, let $\mu \in B'$ satisfy the distribution equation $grad \mu + [A, \mu] = 0$. Then μ extends to a continuous linear functional on $\bar{B}$.*

Proof. – Since Δ is onto B, it follows that every $f \in B$ can be expressed in the form $f = \Delta \phi$, where $\phi = K * f \in H^3(\mathbf{R}^3, \mathfrak{g})$. Hence,

$$\langle \mu | f \rangle = \langle \mu | \Delta \phi \rangle = -\langle grad \mu | grad \phi \rangle = \langle [A, \mu] | grad \phi \rangle =$$

$$= \langle [A, \mu], grad(K * f) \rangle = -\langle \mu, [A; grad(K * f)] \rangle,$$

where the last equality follows from the ad-invariance of the metric in $\mathfrak{g}$. Moreover,

$$||[A; grad(K * f)]||_{H^1(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)} \leq ||A||_{H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)} ||grad(K * f)||_{H^1(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)}.$$

Similarly, (10) yields

$$||[A; grad(K * f)]||_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} \leq c||A||_{H^2(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)}||grad(K * f)||_{H^1(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)}.$$

Since the norm in B is given by

$$||f||_B = ||f||_{H^1(\mathbb{R}^3,\mathfrak{g})} + ||f||_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})},$$

it follows that

$$|\langle\mu|f\rangle| \leq ||\mu||_{B'}||[A; grad(K * f)]||_B =$$

$$= ||\mu||_{B'}\{||[A; grad(K * f)]||_{H^1(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)} + ||[A; grad(K * f)]||_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})}\} \leq$$
$$\leq ||\mu||_{B'}||A||_{H^2(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)}||grad(K * f)||_{H^1(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)}\{1 + c\}.$$

Taking into account (8) and the definition of the norm in $\bar{B}$ we get

$$||grad(K * f)||_{H^1(\mathbb{R}^3,\mathfrak{g}\otimes\mathbb{R}^3)} \leq ||f||_{L^2(\mathbb{R}^3,\mathfrak{g})} + c'||f||_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} \leq (1 + c')||f||_{\bar{B}}.$$

Therefore,

$$|\langle\mu|f\rangle| \leq c''||A||_{H^2(\mathbf{R}^3,\mathfrak{g}\otimes\mathbf{R}^3)}||\mu||_{B'}||f||_{\bar{B}},$$

which implies that μ extends to a continuous linear form on $\bar{B}$. ■

LEMMA 5. – *For $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$, let $\mu \in \bar{B}'$ satisfy the distribution equation $grad\mu + [A, \mu] = 0$. Then $\mu = 0$.*

Proof. – The Yang-Mills potential A defines parallel transports in $\bar{B}$ and $\bar{B}'$. The distribution equation $grad\mu + [A, \mu] = 0$ implies that μ is invariant under the parallel transport in $\bar{B}'$.

Let f_0 be any smooth compactly supported function with values in $\mathfrak{g}$, and a the diameter of the support of f_0. For $n \geq 1$, let $P_{na}f_0$ be the parallel transport of f_0 by distance na in the direction of the x-axis. Define

$$f_n = \frac{1}{n}P_{na}f_0 \text{ and } f = \sum f_n.$$

Then

$$||f_n||^2_{L^2(\mathbb{R}^3,\mathfrak{g})} = \frac{1}{n^2}||f_0||^2_{L^2(\mathbb{R}^3,\mathfrak{g})}$$

and

$$||f||^2_{L^2(\mathbb{R}^3,\mathfrak{g})} = \sum \frac{1}{n^2}||f_0||^2_{L^2(\mathbb{R}^3,\mathfrak{g})} < \infty.$$

Similarly,

$$||f_n||^{6/5}_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} = \frac{1}{n^{6/5}}||f_0||^{6/5}_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})},$$

and

$$\|f\|^{6/5}_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} = \sum \frac{1}{n^{6/5}} \|f_0\|^{6/5}_{L^{6/5}(\mathbb{R}^3,\mathfrak{g})} < \infty.$$

Hence, $f \in \bar{B}$.

On the other hand, since μ is a continuous linear functional on $\bar{B}$,

$$\langle \mu, f \rangle = \left\langle \mu, \sum \frac{1}{n} P_{na} f_0 \right\rangle = \sum \frac{1}{n} \langle \mu, P_{na} f_0 \rangle.$$

The invariance of μ under the parallel transport implies that

$$\langle \mu, P_{na} f_0 \rangle = \langle \mu, f_0 \rangle.$$

Hence,

$$\langle \mu, f \rangle = \sum \frac{1}{n} \langle \mu, f_0 \rangle.$$

Since the series $\sum \frac{1}{n}$ diverges, it follows that $\langle \mu, f_0 \rangle = 0$. Hence, the distribution μ vanishes on the space of all compactly supported functions in S. Since this space is dense in S, it follows that $\mu = 0$.

This completes the proof of Theorem 2.

5. PROOF OF THEOREM 3

LEMMA 6. – *The action of $GS(P)_0$ in P is free.*

Proof. – In [9] we claimed that the action of $GS(P)_0$ in P is free, however our argument proved only that it is locally free. Let $\Phi \in GS(P)_0$ be in the stability group of $p = (A, E, \Psi) \in P$. It follows from (5) that

$$grad \Phi + [A, \Phi] = 0,$$

that is Φ is covariantly constant with respect to the connection A. Hence, for every $x \neq 0$ in $\mathbb{R}^3$, and $t \neq 0$,

$$\frac{d}{dt} \Phi(tx) = [A(tx) \cdot x, \Phi(tx)]. \tag{14}$$

For $A \in H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3)$ we can define a gauge transformation Γ such that the transformed gauge potential

$$\tilde{A} = \Gamma A \Gamma^{-1} + \Gamma grad\, \Gamma^{-1}$$

satisfies the Cronström gauge condition

$$\tilde{A}(x) \cdot x = 0,$$

see [5] and [17]. Under this gauge transformation $\Phi \mapsto \tilde{\Phi} = \Gamma\Phi\Gamma^{-1}$ and Eq. (14) implies that $\tilde{\Phi}$ is constant along the rays emanating from the origin. Since at infinity $\Phi \to identity \in G$, it follows that $\tilde{\Phi}(x) = identity$ for all $x \in \mathbb{R}^3$. Hence, $\Phi(x) = identity$ for all $x \in \mathbb{R}^3$ which implies that Φ is the identity in $GS(P)_0$. Therefore, the action of $GS(P)_0$ in P is free. ■

Since the action of $GS(P)_0$ in P is free and proper, and the manifold topology of C coincides with the topology induced by the embedding of C into P, we obtain

COROLLARY 1. – *The action of $GS(P)_0$ in C is free and proper.*

Theorem 3 is a consequence of smoothness of the constraint set, the properness and freeness of the action of $GS(P)_0$ in C and several results which can be found in the literature. For each $p \in C$, we denote by O_p the orbit of $GS(P)_0$ through p. Since the action of $GS(P)_0$ is free and proper, the isotropy group of p is trivial.

LEMMA 7. – *For each $p \in C$,*

$$T_pO_p = \{u \in T_pC \mid \omega(u, v) = 0 \quad \forall \quad v \in T_pC\}.$$

Proof. – It is a consequence of the equality of the constraint set and the zero level of the momentum map $\mathcal{J}$, Eq. (7), and the identity

$$T_pO_p = \{u \in T_pP \mid \omega(u, v) = 0 \quad \forall \quad v \in \ker d\mathcal{J}_p\}$$

established in [19] for momentum maps of equivariant actions of Hilbert-Lie groups. ■

LEMMA 8. – *For each $p \in C$, there exists a smooth submanifold S_p of C through p such that*

$$T_pC = T_pS_p \oplus T_pO_p,$$

$$T_qC = T_qS_p + T_qO_q \quad \forall \quad q \in S_p,$$

if $q \in S_p$ and $\Phi \in gs(P)_0$, then

$$\Phi q \in S_p \Rightarrow \Phi = identity$$

Proof. – This lemma is a special case of the Slice Theorem established in [18] for actions of non-compact Lie groups. This result was extended to Hilbert-Lie groups in [13]. In the case under consideration the result is trivial. The extended phase space $P = H^2(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3) \times H^1(\mathbb{R}^3, \mathfrak{g} \otimes \mathbb{R}^3) \times H^2(\mathbb{C}^4 \otimes V)$ can be considered as an affine space with a weak Riemannian structure defined by the L^2-scalar product in P. For each $p \in P$, and a pair of vectors (a, e, ψ) and $(a', e'\psi')$ in T_pP,

$$\langle (a, e, \psi) \mid (a', e', \psi') \rangle_{L^2} = \int_{\mathbb{R}^3} (a \cdot a' + e \cdot e' + \psi^\dagger \psi') d_3 x.$$

The action of $GS(P)_0$ in P is affine and it preserves this Riemannian structure. Hence, we can take S_p to be an open neighbourhood of p in the intersection with C of the affine subspace of P which is L^2-orthogonal to T_pO_p. ■

COROLLARY 2. – *It follows from Lemmas 5 and 6 that* $\ker \omega_C$ *is a distribution on* C *and its integral manifolds coincide with the orbits of* $GS(P)_0$ *in* C. *The reduced phase space* $\check{P}$ *is a quotient manifold of* C.

Since the action of $GS(P)_0$ in P is free, the fibres of $\rho : C \to \check{P}$ are diffeomorphic to $GS(P)_0$. A proof that a smooth, free and proper action of a Lie group G_0 on a manifold M gives rise to a G_0-principal bundle structure in M is given in [c-b]. It extends without any changes to smooth, free and proper Banach-Lie group actions on infinite dimensional manifolds. Hence, $\rho : C \to \check{P}$ is a $GS(P)_0$-principal fibre bundle.

The weak Riemannian structure in P induces a weak Riemannian structure in C which is preserved by the action of $GS(P)_0$. Hence it gives rise to a weak Riemannian structure in the quotient space $\check{P} = C/GS(P)_0$, [14]. Similarly, the pull-back θ_C the 1-form θ to C is preserved by the action of $GS(P)_0$. The tangent bundle spaces of the orbits of $GS(P)_0$ are spanned by the vector fields ξ_P for $\xi \in gs(P)_0$. Eqs. (6) and (7) imply that θ_C annihilates these vector fields. Hence θ_C projects to a 1-form $\check{\theta}$ in the reduced phase space $\check{P}$. Clearly, $\check{\omega} = d\check{\theta}$ pulls-backs under the projection map ρ to the pull-back of ω to C, and Lemma 5 implies that $\check{\omega}$ is non-degenerate, [13]. Hence, $\check{\omega}$ is weakly symplectic. This completes the proof of Theorem 3.

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