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Foundations of quantum mechanics versus the electric Aharanov-Bohm effect

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ABSTRACT. — The significance of the complete wave function of a closed quantum mechanical system of many particles is discussed. Using basic properties of quantum mechanics, it is proved that the electric Aharonov-Bohm effect does not exist.

RÉSUMÉ. — La signification d'une fonction d'onde complète d'un système fermé de plusieurs particules est discutée. Employant les propriétés de base de la mécanique quantique il est démontré que l'effet électrique Aharonov-Bohm n'existe pas.

I. INTRODUCTION

Unlike many other quantum mechanical predictions, the Aharonov-Bohm (AB) effects [*1*] [*2*] were considered a controversial issue for a very long time [*3*]. The effects are derived from the solution of the quantum mechanical equation of motion of an electron in a nonsimply connected field-free region [*1*] [*2*]. Recently, it has been shown that the original derivation of the *electric* AB effects [*1*] [*2*] is based upon assumptions that violate energy conservation [*4*]. This new aspect of the problem calls for an investigation of the effect itself.

Classical equations of motion depend upon fields whereas the standard formulation of quantum mechanics is written in terms of potentials. Hence, relying upon the field-free requirement, the presently accepted interpretation of the AB effects claims that these effects have no classical ana-

logue [1] [2]. This is probably the underlying reason that led some people to deny the effects [5]-[7], to seek nonzero fields involved [8] or to look for conditions where they are eliminated [9].

There are two kinds of AB effects [1] [2]. The electric one discusses an electronic wave packet which is split into two coherent sub-packets, each of which moves in a field-free region at which the electric potential takes different values. At a later time the sub-packets are united and make an interference pattern. In the second kind of the AB effects, an electronic beam is split coherently into two parts, each of which moves at either side of an infinitely long and thin magnet (or at the inner and at the outer sides of a toroidal magnet) before being united on the interference screen.

This work discusses only the electric type of the AB effect. Unfortunately, this version of the AB effect has not been tested experimentally [4] [7]. However, it is shown in this work that basic properties of quantum mechanics predict the nonexistence of the electric AB effect. The magnetic AB effect, as well as a discussion of general physical implications of the two kinds of the AB effects, are beyond the scope of the present paper.

The cornerstone of this work is the significance of the complete wave function of a quantum mechanical system of more than one particle. The second section of this work discusses an atomic system from this point of view. Quantum mechanical properties associated with the Einstein-Podolski-Rosen (EPR) paradox [10] are discussed in the third section. The fourth section is devoted to the calculation of the interference pattern of the electric AB effect. The last section contains some concluding remarks.

II. WAVE FUNCTION OF AN ATOMIC SYSTEM OF MORE THAN ONE ELECTRON

It is well known that the quantum mechanical description of an atomic system uses a wave function Ψ which depends upon the coordinates of all particles of the system. The significance of this property is crucial for the main discussion of this work and the whole section is devoted to its presentation.

Atomic physics is a field where quantum mechanics has proved itself as a very successful theory. The following description of a solution of an atomic problem is presented and points of importance for the rest of this work are discussed. A detailed discussion of the subject can be found in textbooks [11].

The atomic wave function of n electrons is written as a linear combination of elements of a basis of a Hilbert space

$$\Psi = \sum_{\mathrm{I}} a_{\mathrm{I}} \mathrm{A} \psi_{i_1}(r_1) \ldots \psi_{i_n}(r_n) \tag{1}$$

where $\psi_{i_j}(r_j)$ denotes a single particle wave function which belongs to an orthonormal set, a_I denotes a (yet undetermined) coefficient and A denotes an antisymmetrization operator which guarantees the admissibility of Ψ as a wave function of identical fermions. The symbol I denotes a multiple index $I = (i_1, i_2, \ldots, i_n)$. (For the purpose of this work it is not required that the particles are identical.)

The solution of the problem is an eigenfunction of the Hamiltonian H

$$H\Psi = E\Psi. \tag{2}$$

This Hamiltonian can be written as follows

$$H = H_1 + H_2 \tag{3}$$

where H_1 is a sum of single particle operators and H_2 is a sum of two particle operators. Due to practical reasons, the expansion (1) is truncated and the solution is sought in a finite dimensional space. Substituting the truncated form of (1) into (2), one reduces the problem to the eigenvalue problem of the Hamiltonian matrix.

The first step is the calculation of the matrix elements of the Hamiltonian, each of which is a sum of matrix elements of one or two particles operators. A single-particle operator takes the following form

$$f = \sum_j f_j \tag{4}$$

where f_j operates on the coordinates of the j'th electron. Hence, the required matrix element of H_1 is a sum of quantities of the following form

$$\langle \psi_{i_1}(r_1) \ldots \psi_{i_n}(r_n) \mid f_j \mid \psi_{i'_1}(r_1) \ldots \psi_{i'_n}(r_n) \rangle . \tag{5}$$

Consider two cases of (5). In the first case $i_1 = i'_1, \ldots, i_n = i'_n$ while the second one is the same except that $i_n \neq i'_n$ and $n \neq j$. Using the orthonormality of the ψ_k, one finds that in the first case the integration on the coordinates of all r_i except r_j yields unity and (5) reduces to the form

$$\langle \psi_{i_j}(r_j) \mid f_j \mid \psi_{i_j}(r_j) \rangle . \tag{6}$$

On the other hand, in the second case the integration on the coordinates of the n'th particle vanishes. Indeed

$$\langle \psi_{i_n}(r_n) \mid \psi_{i'_n}(r_n) \rangle = \delta_{i_n, i'_n} \tag{7}$$

which vanishes for $i_n \neq i'_n$. It follows that the matrix element (5) vanishes in the second case. A related general law says that a matrix element of a single particle operator vanishes between two states which differ in more than one individual set of quantum numbers [*12*]. An analogous law holds for two-particles operators.

This example shows that matrix elements of a single-particle operator

depend not only on the coordinates of the particular particle upon which it operates but also on the coordinates of the other particles of the system as well.

This result (henceforth called the completeness property) is the characteristic feature required for the following discussion. Therefore, no further analysis of atomic wave functions is carried out here. It is appropriate to mention that the completeness property is a part of a theory that achieves an amazing accuracy when compared with atomic data [*13*]. Physics is a natural science and, besides logical selfconsistency, it should fit to experiment. Hence, it can be concluded that the completeness property, which passes these tests, is a part of what can be stated as a good description of Nature.

III. WAVE FUNCTION OF TWO PARTICLES SEPARATED IN SPACE

The completeness property was shown to hold for an atomic system where the single particle wave functions of all the electrons are nonzero at the same atomic volume. Nevertheless, it is shown in this section that this result is valid for a wave function of two electrons even when they are very far apart. This property is proved experimentally in a test of the EPR paradox [*10*]. The following is a description of a Gedanken experiment realizing Bohm's version of this subject [*14*]. (This part of Bohm's work is irrelevant to the AB effect.)

Consider a pair of isoenergetic electrons that move along the Z-axis in opposite directions and undergo a collision after which they are scattered at a right angle along the x-axis. After traveling a certain distance, each of the electrons enters a Stern-Gerlach analyzer (see fig. 1). Let us write a quantum mechanical expression for this process.

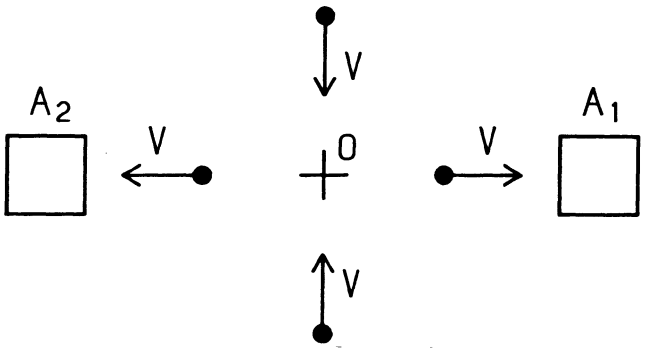

FIG. 1. — Two isoenergetic electronic beams move along the z-axis in opposite directions. Two electrons interact at the origin O and are scattered at right angles along the x-axis. Each of the electrons enters a Stern-Gerlach analyzer, A_1 and A_2, respectively. The analyzers are located at $r = (\pm x_0, 0, 0)$, respectively.

The z-component of the linear momentum of each of the incoming electrons is the sole nonvanishing component of their linear momentum. Hence, the z-component of the overall orbital angular momentum L vanishes. Assume that spin-orbit interaction can be ignored. It follows that the eigenvalue $M_L = 0$ is a constant of the motion. Hence, the angular-spin part of the wave function can be written as follows

$$\Psi = \sum_L a_L P_L(\cos\theta)\chi_L \tag{8}$$

where $P_L(\cos\theta)$ are the Legendre polynomials and χ_L denotes the overall spin part of the L'th term of this expansion.

The scattered particles are emitted at a right angle. Therefore, for an odd L the factor $P_L(0)$ vanishes. It follows that, in (8), the summation runs on even values of L and the angular part of the wave function is symmetric. Using Fermi-Dirac statistics, one finds that χ_L must be antisymmetric. Therefore, the overall spin part of the wave function of the scattered electrons is

$$\chi = (s_{1+}s_{2-} - s_{1-}s_{2+})/2^{\frac{1}{2}} \tag{9}$$

where $s_{i\pm}$ denotes that the z-component of the spin of the i'th electron is $\pm$ 1/2, respectively. This discussion shows that the scattered electrons have a well defined overall spin: $S = M_S = 0$.

The scattered electrons move freely until each of them enters the respective analyzer. Each analyzer measures the s_z component of one of the electrons and m_{s_i}, which is the m quantum number of the i'th spin, becomes a good quantum number. Therefore, after the electrons emerge from the analyzers, the overall spin, S, is not a good quantum number and the wave function (9) is projected onto either of the following wave functions

$$\chi_a = s_{1+}s_{2-} \tag{10}$$

$$\chi_b = s_{1-}s_{2+}\,. \tag{11}$$

It follows that quantum mechanics predicts that after the analysis is carried out for the two *space-like separated* events, one finds the following correlation

$$\langle m_{s_1} m_{s_2} \rangle = -1/4\,. \tag{12}$$

This discussion shows that results of spin measurements depend upon the overall spin wave function χ_a or χ_b of (10) and (11). The wave function (9) shows that each electron has an equal probability to yield $m_{s_i} = \frac{1}{2}$ or $m_{s_i} = -\frac{1}{2}$. If this result depends just upon the spin part of the *single particle wave function* then there will be no correlation between the measurements of the two z-components of the spins and the quantity (12) should vanish.

Experiments related to the one described above have been carried out and prove the above property of quantum mechaniscs [*15*]. Other implications of experiments of this kind are discussed in the literature [*16*]. Considering the subject of this work, the significance of the experiment described in this section is that it shows that the completeness property of quantum mechanics holds even if the constituents are not located at the same spatial volume.

IV. THE ELECTRIC AHARONOV-BOHM EFFECT

The foregoing considerations provide a good basis for the analysis of the electric AB effect. Consider an electronic wave packet which is split into two coherent sub-packets [*1*] (see fig. 2). Each sub-packet enters a very long hollow cylinder whose electric potential vanishes at the entrance time. When the sub-packets are well inside the cylinders, the electric potential of one of them changes in time and vanishes again while the sub-packets are still far from the cylindrical end points. The sub-packets emerge from the cylinders and interfere on a screen. This interference pattern is compared with the one obtained from a null experiment where the potentials of the two cylinders vanish identically.

The following realization of this effect is discussed in this section. One of the cylinders is removed. The other cylinder consists of two cylindrical layers made of insulating materials. The outer layer is rigid and is covered uniformly with negative charges. The same amount of positive charges

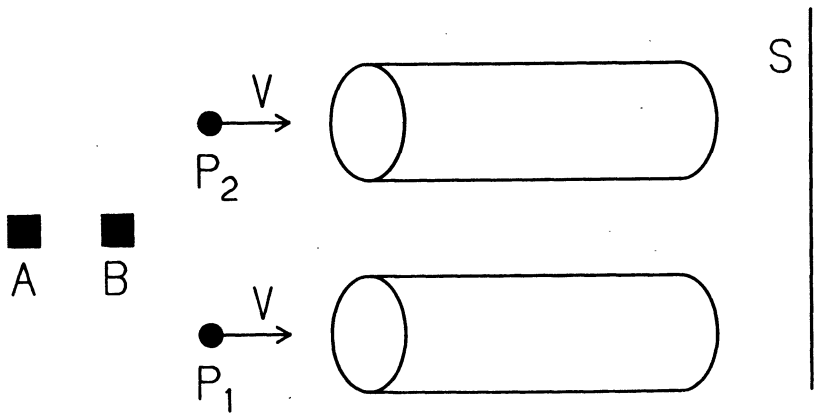

FIG. 2. — An isonergetic electronic beam moves from left to right. The beam is chopped at A into rather short packets, each of which is split coherently at B into two subpackets. Each subpacket moves inside a long hollow cylinder. After emerging from the cylinders, the subpackets interfere on the screen S.

covers uniformly the inner flexible layer. When the electronic sub-packet approaches the cylinder, the distance between its two layers is infinitesimal and the electric potential at its inner part, as well as at its outer one, vanishes. Later, when the electronic sub-packet is deep inside the cylinder, a special device releases a fixed amount of mechanical energy. This mechanical energy pushes adiabatically the inner flexible layer along the radial direction in a way that conserves its cylindrical shape (see fig. 3). After a while the flexible layer expands adiabatically back to its original size and the potential vanishes again before the sub-packet approaches the other end of the cylinder. The term « moving electron » denotes the electron whose interference pattern is discussed in this section.

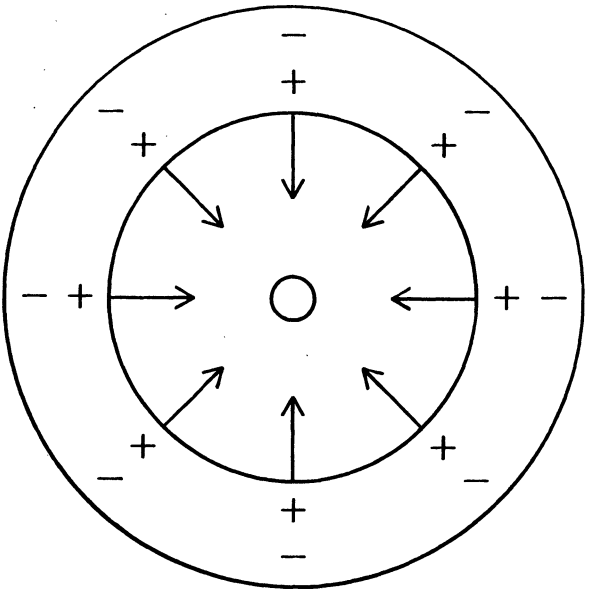

FIG. 3. — A cross section of the cylinder whose potential at its inner part varies in time. The outer layer contains motionless negative charges. The inner layer contains positive charges which, at the instant depicted, move towards the cylindrical axis. The small circle at the center denotes the inner subpacket of the moving electron while traveling along the cylindrical axis.

Outside the cylinder the electric potential vanishes identically. Here, like in the original presentation of the electric AB effect [*1*] [*2*], edge phenomena are ignored. This approximation relies upon the fact that the cylinder is very long and that the moving electron is far from the cylindrical ends when the distance between the two layers is nonzero. At the cylinder's inner part the potential does not vanish during a part of the time. At this

time the electric field is nonzero only between the two layers and near the edges. Hence, the two electronic sub-packets move in a nonsimply connected field-free region. The interference pattern depends upon the relative phases of the two parts of the wave function. These phases are calculated below. It is assumed that the velocities of the charges are small and that the non-relativistic limit holds.

The single particle wave function of the moving electron is written as a sum of two quantities

$$\psi = \psi_{\mathrm{I}} + \psi_{\mathrm{O}} \tag{13}$$

where the subscripts I and O denote the inner and the outer sub-packets, respectively. Let ϕ denote the wave function of the rest of the system. The overall wave function is tentatively written in the following form

$$\Psi = (\phi_{\mathrm{I}}\psi_{\mathrm{I}} + \phi_{\mathrm{O}}\psi_{\mathrm{O}})/2^{\frac{1}{2}}. \tag{14}$$

The significance of the distinction between ϕ_{I} and ϕ_{O} will be discussed later. Its origin pertains to the different responses of the cylindrical charges to the positions of the moving electron associated with ψ_{I} and ψ_{O}, respectively. Like in the original derivation of the electric AB effect [*1*] [*2*], it is assumed here that the cylindrical parts of the wave function $\phi_{\mathrm{I}}(t)$ and $\phi_{\mathrm{O}}(t)$ vary adiabatically and that, at the interference time T, $\phi_{\mathrm{I}}(\mathrm{T}) = \phi_{\mathrm{O}}(\mathrm{T})$ [*2*].

Notice also that the form of (14) is analogous to that of (8). Indeed, using (9), one can write (8) as follows

$$\Psi = \sum_{\mathrm{L}} a_{\mathrm{L}} \mathrm{P}_{\mathrm{L}} (\cos\theta)\, s_{1+} s_{2-} - \sum_{\mathrm{L}} a_{\mathrm{L}} \mathrm{P}_{\mathrm{L}} (\cos\theta)\, s_{1-} s_{2+}. \tag{8'}$$

Both (8') and (14) are written as a sum of two parts, each of which consists of a multiplication of factors depending on appropriate coordinates. Introducing the radial coordinate and the center of mass ones, (8') can be written explicitly in terms of the coordinates of the two electrons. Evidently, the spatial location of the first electron in (8') is macroscopically far from that of the second one. Considering (14), one finds that the same is true for the moving electron with respect to the cylindrical charges. Thus, the analogy between (8') and (14) is established.

Henceforth, r and $-e$ denote the coordinates of the moving electron and its charge, respectively. r_i and e_i denote the coordinates of the i'th particle at the cylinder and its charge, respectively. The index i runs on n charges. For simplicity, it is assumed that the charged particles at the cylinder are not electrons. Therefore, no antisymmetrization is required for ψ. This assumption simplifies notation and does not affect the results of this section.

The Hamiltonian is written as follows

$$\mathrm{H} = \mathrm{H}_c + \mathrm{H}_e + \mathrm{V} \tag{15}$$

where H_c denotes the part of the Hamiltonian that operates only on the cylindrical constituents, H_e denotes the part that represents the kinetic energy of the moving electron and V denotes the interaction between the moving electron and the charges at the cylinder.

The interference patterns of two experiments are compared. In the first experiment the electric potential associated with the cylindrical charges vanishes identically while in the second one the inner cylindrical layer moves as described above. Omitting V from (15), one finds for the first experiment

$$i\hbar \frac{\partial \Psi}{\partial t} = (H_c + H_e)\Psi . \tag{16}$$

The substitution of (14) into (16) yields

$$i\hbar \frac{\partial \Psi}{\partial t} = [(H_c \phi_I)\psi_I + (H_c \phi_O)\psi_O + \phi_I(H_e \psi_I) + \phi_O(H_e \psi_O)]/2^{\frac{1}{2}} . \tag{17}$$

The quantity relevant to the interference pattern is the phase difference between $\phi_I \psi_I$ and $\phi_O \psi_O$. In the first experiment, the static charges at the cylindrical layers do not gain energy. Therefore, in this case, the eigenvalue of $H_c \phi_I$ equals that of $H_c \phi_O$ at every instant. (As a matter of fact, in the first experiment $\phi_I(t) = \phi_O(t)$ for all values of the time t and the distinction between these functions is redundant.) It follows that the contribution of the first term of (17) to the phase difference cancels that of the second one. The resultant phase difference is

$$\begin{aligned}
\delta\alpha &= -\frac{1}{\hbar}\Big[\int \phi^\dagger{}_I \psi^\dagger{}_I H \phi_I \psi_I d^3r d^3r_1 \ldots d^3r_n dt \\
&\qquad - \int \phi^\dagger{}_O \psi^\dagger{}_O H \phi_O \psi_O d^3r d^3r_1 \ldots d^3r_n dt\Big] \\
&= -\frac{1}{\hbar}\Big[\int (\phi^\dagger{}_I H_c \phi_I)\psi^\dagger{}_I \psi_I d^3r d^3r_1 \ldots d^3r_n dt \\
&\qquad - \int (\phi^\dagger{}_O H_c \phi_O)\psi^\dagger{}_O \psi_O d^3r d^3r_1 \ldots d^3r_n dt \\
&\qquad + \int \phi^\dagger{}_I \phi_I (\psi^\dagger{}_I H_e \psi_I) d^3r d^3r_1 \ldots d^3r_n dt \\
&\qquad - \int \phi^\dagger{}_O \phi_O (\psi^\dagger{}_O H_e \psi_O) d^3r d^3r_1 \ldots d^3r_n dt\Big] \\
&= -\frac{1}{\hbar}\Big[\int \psi^\dagger{}_I H_e \psi_I d^3r dt - \int \psi^\dagger{}_O H_e \psi_O d^3r dt\Big] .
\end{aligned} \tag{18}$$

The expression on the first right hand side of (18) is obtained from the Schrödinger equation. In the second expression, the integration is written separately for the various terms. The last expression is obtained after eliminating the first two terms of the preceding expression. Cross terms which

contain $\psi_I\psi_O$ in the integrand are omitted because at least one of these single particle wave functions of the moving electron vanishes at every volume element.

The Schrödinger equation for the second experiment is

$$ih\frac{\partial\Psi}{\partial t} = (H_c + H_e + V)\Psi$$
$$= [(H_c\phi_I)\psi_I + H_c\phi_O)\psi_O + \phi_I(H_e\psi_I) + \phi_O(H_e\psi_O)$$
$$+ V\phi_I\psi_I + V\phi_O\psi_O]/2^{\frac{1}{2}}. \quad (19)$$

As stated above, the moving electron travels in a field-free region and is macroscopically far from the rest of the charges. In this case the classical limit holds. Ehrenfest's theorem shows that the kinetic energy of the moving electron changes due to the external Lorentz force. This force vanishes in the field-free region where the moving electron travels. It follows that the third and the fourth terms of (19) make the same contribution to the phase difference as the corresponding terms of (17).

The potential V at the location of the moving electron takes the following form

$$V(r) = \sum_{i=1}^{n}\int \phi^\dagger \frac{e_i}{|r - r_i|}\phi d^3r_1 \ldots d^3r_n \quad (20)$$

where ϕ denotes either ϕ_I or ϕ_O and the summation runs on the n particles of the cylindrical layers. Evidently, classical electrodynamic considerations prove that the contribution of this quantity to the phase shift is

$$\delta\alpha = \frac{e}{\hbar}\left[\int \psi_I^\dagger V(r)\psi_I d^3r dt - \int \psi_O^\dagger V(r)\psi_O d^3r dt\right]$$
$$= \frac{e}{\hbar}\int \psi^\dagger{}_I\left\{\sum_{i=1}^{n}\int \phi^\dagger{}_I \frac{e_i}{|r - r_i|}\phi_I d^3r_1 \ldots d^3r_n\right\}\psi_I d^3r dt \quad (21)$$

where the second term of the first line vanishes since $V(r) = 0$ where $\psi_O(r)$ is nonzero.

Up to this point, the calculations agree with the corresponding ones carried out in section III of reference 2. Let us turn to the first and the second terms of (19). The charges at the cylinder move in a finite external field. Their energy changes due to the field of the moving electron. The change of the energy of these charges can be calculated, like that of the moving electron, from Ehrenfest's theorem. Using again simple classical calculations, one finds that the electric field associated with ψ_O makes no contribution to the overall self energy of the cylindrical layers. On the other hand, the field associated with ψ_I *does make a change* of the self energy of the charges at the cylinder.

The law of energy conservation, which holds in classical electrodynamics, can be used in this case. It shows that the change of the self energy of the charges at the cylinder balances the work done on these charges by the external field. In the case discussed here, the sole external field is the one associated with the moving electron. The corresponding potential at the location of the i'th charge is

$$V(r_I) = -e\int \psi^{\dagger}_I \frac{1}{|r - r_i|}\psi_I d^3r\,. \tag{22}$$

The difference between the self energy of ϕ_I and that of ϕ_O is

$$\delta\varepsilon = \sum_{i=1}^{n}\int \phi^{\dagger}_I\left(\int \psi^{\dagger}_I \frac{ee_i}{|r - r_i|}\psi_I d^3r\right)\phi_I d^3r_1 \ldots d^3r_n\,. \tag{23}$$

Multiplying this quantity by $-1/\hbar$ and integrating on the time, one obtains the contribution of H_c to the phase shift. It is evident thas this quantity and (21) cancel each other.

It is interesting to compare the different origins of (21) and (23). In (21) the interaction energy is written as a quantity associated with the moving electron. On the other hand, the sum (23) represents the contributions of an *astronomical number* of charges at the cylinder. As proved above, this quantity balances the energy difference derived in (21) for a *single* electron. Using this sum, one shows energy conservation where the potential energy associated with the interaction between the moving electron and the cylinder balances the change of the cylindrical self-energy. Evidently, the change of the self-energy of each of the charges at the cylinder follows its motion in the field of the moving electron. Due to the macroscopic distance between the moving electron and the cylinder, this quantity is very small. However, as shown above and in reference 4, the utilization of the sum of all these quantities (each of which is very small) is vital for the restoration of the energy balance of the system. The authors of reference 2 use unjustified approximations and neglect this quantity (see p. 1518).

The previous discussion shows that a nonvanishing contribution to the phase difference is obtained from the third and the fourth terms of (19). The discussion presented after (19) shows that the overall phase difference of (19) is precisely the same as that written in (18). The latter quantity was obtained in the first experiment where the cylindrical electric potential vanishes identically at all times. This result shows that the electric version of the AB effect does not exist.

The main point used in this section is that there is no separate single particle phase. If Ψ is written as a sum of terms, like $(\phi_I\psi_I + \phi_O\psi_O)/2^{\frac{1}{2}}$ in (14), then each term has its own phase. *It is impossible to split the phase between the various factors of each term.* This feature is a direct outcome

of the completeness property of quantum mechanical wave function. The calculations carried out above are based upon this property and prove the main result of this work.

V. CONCLUDING REMARKS

The dependence of a single particle matrix element upon the complete wave function is a vital element of this work. This dependence, called above the completeness property, is well known in atomic spectroscopy. It is also a property of quantum mechanical states used in experiments which test the EPR paradox. These fields of research show that quantum mechanical predictions are confirmed by experiment.

Using the completeness property, it is shown in the fourth section of this work that the electric AB effect does not exist. As a matter of fact, it is already stated in the introduction that this version of the AB effect has never been tested experimentally [4] [7]. Hence, the result of this paper is not incompatible with experiment.

The main conclusion of this work disagrees with the corresponding result of references 1 and 2. The discrepancies of these articles are as follows. Reference 1 uses a single particle wave function which is in disagreement with the completeness property of quantum mechanical wave functions. Reference 2 *neglects* the contribution of the wave function ϕ of the apparatus to the phase shift. It is shown in the previous section and in reference 4 that energy balance is achieved when the overall wave function is written as in (14). Doing it this way, one realizes that the electric AB effect disappears.

It is mentioned in the introduction that, unlike other quantum mechanical predictions, the AB effects were considered a controversial issue for a long time. It turns out that most of the debate refers to the magnetic AB effect. One may suppose that the results of this work are relevant to this controversy. However, a discussion of these implications is beyond the scope of the present article.

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