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A limitation on Bell's Inequality

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ABSTRACT. — It is shown that Bell's Inequality does not characterize all local hidden variable explanations of the polarization correlation experiments. If one considers theories in which a single polarization measurement is not independent of previous particle-polarizer interactions then it is possible to manufacture local hidden variable theories which agree with quantum mechanics for any of the experiments performed to date.

A relevant property here is ergodicity, and we can say that Bell's Inequality characterizes all ergodic local hidden variable theories (i. e. all local theories that give the same time and ensemble average) but not all non-ergodic local hidden variable theories. It is further shown that the most physically reasonable class of non-ergodic local hidden variable theories must also satisfy Bell's Inequality.

It might be concluded from this article that if one insists on believing in both local hidden variable theories and the polarization correlation experiments supporting quantum mechanics then one must also believe in the existence of a field, medium or ether that permeates space and has relatively stable states (memory).

I. INTRODUCTION

A. In 1964 J. S. Bell [*1*], building on some work of Einstein, Podolsky and Rosen [*2*] and Bohm and Aharonov [*3*], proposed to show that any local hidden variable theory must necessarily be inconsistent with the quantum mechanical predictions for certain types of experiments that measure the polarization correlations of two separated particles which are originally together in some state. Several experiments [*4-8*] have since been performed and the overall results support quantum mechanics [*9*] (accepting Bell's Inequality). A review of the hidden variable question and most of the references can be found in Reference [*12*].

In this article it is shown that Bell's Inequality does not characterize all local hidden variable explanations of the polarizarion correlation experiments (1). The existing demonstrations of Bell's Inequality assume that the experimental average is an ensemble average. That is, each measurement is absolutely independent of all other measurements. However for pratical reasons the experimental averages obtained in the laboratory are obtained by taking a number of experimental runs with a number of measurements in each run. It is implicitly assumed in the experiments that as long as there is a reasonable time interval between the measurements in any run it is a re-preparation of state and consequently the time average can be considered equivalent to an ensemble average.

The logical possibility that once a measuring system enters some state in the act of measurement it remains in that state indefinitely (i. e. is stable) is ignored as being physically unlikely. This indeed may be the case but it should be emphasized—especially in view of the not entirely consistent experimental results—that this is an assumption and is to the best of our knowledge not based on any experimental evidence.

Thus if one wants to characterize all local hidden variable explanations of the polarization correlation experiments as they are actually performed in the laboratory, one must consider possible interactions in time. That is, one must consider local hidden variable theories which are non-ergodic, i. e. do not give the same time and ensemble average. For example, the most simple type of non-ergodic theory that we can imagine is one in which the state of a measuring apparatus after a measurement is a function of its state before the measurement and the state of the particle it measured, with the crucial condition that the measuring apparatus remains in this state until the next measurement. A simple example of such a theory is given in the appendix. In such theories the states of two distant measuring apparatus can become correlated over time in a strictly local manner if they are measuring particles which are themselves in correlated states.

In this article it will be seen that *i*) any ergodic local hidden variable theory must satisfy Bell's Inquality (and consequently disagree with quantum mechanics), *ii*) the simple class of non-ergodic theories discussed above must also satisfy Bell's Inequality and *iii*) there exist non-ergodic theories

(1) It is well-known that Bell's Inequality does not characterize certain local hidden variable theories. Those are theories in which the distant measuring apparatus « communicate » with each other directly and consequently know each others states. Since such theories are experimentally testable by randonly changing the apparatus parameters between measurements (See end of Section IV.B), they are frequently excluded from discussion. They will be excluded here as well.

which agree with quantum mechanics and consequently do not satisfy Bell's Inequality but they involve controversal properties of space (Section I. C).

In the remainder of this section some terminology is developed (I.B) and the claims of the article are then expressed more precisely (I.C). In Section II a review of the relevant experiments is given along with a discussion of the mathematical representation of a hidden variable theory as iniciated by J. S. Bell. Section III contains the demonstration. The experimental evidence is discussed in Section IV, where an experimental proposal is also made. In Section V an observation on the Copenhagen and the Statistical Interpretations of quantum mechanics is presented.

B. In order to make the claim of this paper more precise it is necessary to review some more or less standard terminology to clearly distinguish between the various ways of obtaining an experimental average of some measurable quantity.

First, there is the *ensemble average* which ideally one would obtain by using a large number of identical apparatus, taking exactly one measurement with each apparatus. In practice one would make a series of measurements (with one apparatus) following each other in time with the condition that the states of the apparatus were somehow randomized (re-prepared) prior to each measurement. We call this type of experiment an *ensemble experiment*.

Secondly, we call a *single time average* an average which is obtained by making a series of measurements (with one apparatus) following each other in time, taking no action to randomize the states (re-prepare) prior to each measurement. In all the polarization correlation experiments it is implicitly assumed that the states of the measuring apparatus automatically revert to random states (i. e. are re-prepared) if there is at least some small interval of time between each measurement, and therefore these experiments are implicitly assumed to give an ensemble average.

Third, we call an *overall time average* an average obtained by using a large number of single time averages. That is, an average obtained over a large number of distinct experimental runs, with each of these runs having a large number of measurements. We call the experiment necessary to obtain a single time average or an overall time average a *time series experiment*.

In the case of the polarization correlation experiments quantum mechanics predicts that these three averages must agree, but because of experimental error and drift effects, an overall time average was used in all the previously performed experiments. In general one can imagine theories in which these three averages may or may not agree. When they all agree then we call the theory *ergodic* [*13*, *14*].

C. It will be shown that Bell's Inequality applies only to ergodic local hidden variable theories in general and therefore in general only to ensemble experiments where ergodic and non-ergodic theories must agree. In addition, the type of simple non-ergodic theory discussed in Section I.A can disagree with Bell's Inequality for single time averages but must agree for overall time averages. That is, this type of theory cannot agree with quantum mechanics for an overall time average. One can find more complicated non-ergodic local theories which agree with quantum mechanics for either a single time average or an overall time average.

Physically speaking these previously mentioned theories assign controversal properties to space. These theories assume that the states of the apparatus can affect the states of the particle-pairs over time, which in turn affect the states of the other apparatus and *vice versa*. To accept the proceeding and preserve locality (1) it must be imagined that a field, medium or ether with relatively stable states or memory exists. The example given in Section III.C should clarify this conclusion. Therefore it could be concluded from this article that the insistence on the belief in both local hidden variable theories and the correlation experiments supporting quantum mechanics necessitates a belief in the existence of a field, medium or ether with relatively stable states.

References [*15*] and [*16*] contain some work related to ergodicity.

II

A. We first review the relevant experiment and terminology. We follow Clauser and Horne [*17*] and refer to figure 1. « A source of coincident two particle emissions is viewed by two analyzer-detector essemblies 1 and 2.

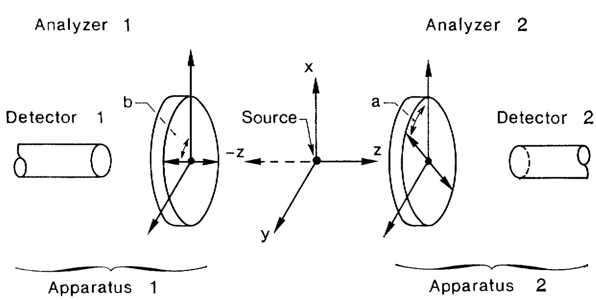

FIG. 1. — (Following Clauser and Horne [*17*]). Scheme considered for a discussion of local hidden variable theories. A source emitting particle pairs is viewed by two apparatus. Each apparatus consists of an analyzer and an associated detector. The analyzers have parameters, *a* and *b* respectively, which are externally adjustable. In the figure, *a* and *b* represent the angles between the analyzer axes and a fixed reference axes.

(1) Voir note p. 380.

Each apparatus has an adjustable parameter; let a denote the value of the parameter at apparatus 1 and b that at apparatus 2. In figure 1, a and b are taken to be angles specifying the orientation of analyzers, e. g. the axes of linear polarizers, or the directions of the field gradients of Stern-Gerlach magnets for spin 1/2 particles ».

Let $A_n(a)$ (resp. $B_n(b)$) be the measured quantities which by definition assume the value 1 or -1 depending on whether or not one of the particles of the n^{th} particle-pair was detected at detector 1 (resp. detector 2). Here n could denote the time t_n of a measurement in a time series experiment or the name of a particular apparatus in an ensemble experiment. Define $Z_n(a, b)$ by

$$Z_n(a, b) = A_n(a)B_n(b)$$

which represents the correlation of the measurements. It has the value 1 if both or neither of the particles of the n^{th} particle-pair pass the polarizers. And it has the value -1 if only one particle passes a polarizer.

From these measured quantities one calculates the experimental averages

$$P(a) = \frac{1}{N} \sum_{n=1}^{N} A_n(a)$$

$$P(b) = \frac{1}{N} \sum_{n=1}^{N} B_n(b)$$

$$P(a, b) = \frac{1}{N} \sum_{n=1}^{N} Z_n(a, b) = \frac{1}{N} \sum_{n=1}^{N} A_n(a)B_n(b)$$

where N is the total number of particle pairs and is assumed to be large.

Depending on the type of particle-pair and analyzer, quantum mechanics gives a certain value of $P(a, b)$. For spin 1/2 particles this value is (with the two particle system in a singlet state)

$$P_q(a, b) = -\cos(a - b)$$

with a and b angles. Bell proposed to show that no local hidden variable theory could agree with this value for all values of a and b. This result is expressible as an inequality having the form

$$| P(a, b) - P(a, b') | + | P(a', b') + P(a' b) | \leq 2 \tag{1}$$

This form is the result of several workers [*18*].

There are a number of variations of this inequality [*19-23*] and they are all referred to as Bell's Inequality. Judiciously choosing the angles a, b, a' and b' and using the appropriate quantum mechanical expression $P_q(a, b)$ for $P(a, b)$, etc., the inequality can be violated [*19*].

B. We can now discuss Bell's characterization of a hidden variable theory. It is instructive to make explicit several things which are implicity assumed.

Classically speaking, in a measurement process there is an interaction between the measuring apparatus and the thing (i. e. particle) being measured, which gives as a result a certain number (a meter reading). We can characterize the state of the apparatus by the variable s (in general a set of variables) and the state of the object by the variable λ (in general a set of variables) with domains S and Γ respectively. In the completely ideal classical apparatus s would be considered to be a constant. Then the result (the numerical value) of some measurement A_n should be expressible as some function of s and λ. That is

$$A_n = f(s, \lambda)$$

for some $s \in S$, $\lambda \in \Gamma$. In our case $f(s, \lambda) \in \{ -1, 1 \}$ but this is not relevant to the discussion. Then we can express the experimental average P for many measurements as

$$P = \frac{1}{N} \sum_{n=1}^{N} A_n = \int_S \int_\Gamma f(s, \lambda) \rho(s, \lambda) ds d\lambda \tag{2}$$

where $\rho(s, \lambda)$ is the density function for s and λ which intuitively expresses the relative or normalized frequency of occurrence of the pair (s, λ). By its meaning ρ must be assumed to satisfy

$$\int_S \int_\Gamma \rho(s, \lambda) ds d\lambda = 1.$$

Almost by definition in any good experiment s and λ are considered to be independent variables since we are thinking of the apparatus as distinct from the object being measured. This being the case we can write

$$\rho(s, \lambda) = \rho_s(s) \rho_\lambda(\lambda).$$

So Eq. 2 becomes

$$P = \int_S \int_\Gamma f(s, \lambda) \rho(s) \rho(\lambda) ds d\lambda.$$

Here we have dropped the subscripts on ρ and will continue to do so. It will be clear from the variable that, for example, $\rho(s)$ and $\rho(\lambda)$ refer to different functions.

The previous, which is implicit in Bell's Inequality, certainly expresses a good part of what we intuitively mean by « theories and measurements following deterministic or causal laws ». We accept this as the mathematical definition of a deterministic or causal process and consequently as a valid way of expressing any hidden variable theory. Any questions of the existence and behavior of integrals are ignored (2).

(2) In particular, we assume the relevant integrals necessary to express any hidden variable theory always exist and that they satisfy the conditions of Fubini's theorem.

The expression (2) is quite general and can be used in the case where s and λ may not, for some reason, be independent. In general s and λ could be directly related (this would be a bad experiment) or statistically related (this may be unavoidable for certain experiments) because of some underlying causal process which effects both s and λ. $\rho(s, \lambda)$ is this case would represent an average relative or normalized frequency of occurrence of the pair (s, λ) so to speak.

When this is the case $\rho(s, \lambda)$ is not factorable, that is

$$\rho(s, \lambda) \neq \rho(s)\rho(\lambda)$$

as is easy to see.

Also it can happen that s and λ may be independent for ensemble experiments but not in time series experiments (see example in appendix), which is the possibility that is of interest to us. For instance, if we think of a time series experiment with measurements made at times t_n then we can write

$$A_n = f(s_n, \lambda_n)$$

where we let s_n represent the state of the apparatus at time t_n and λ_n be the state of the n^{th} particle. It is possible in general that the state s_n is not statistically independent of the states $\lambda_{n-1}, \lambda_{n-2}$, etc. and *vice versa* as was discussed in the introduction.

When one considers such theories one cannot factor $\rho(s, \lambda)$ as above. This unfactorability of ρ is the mathematical reason why Bell's Inequality fails to characterize all local theories as will be seen.

III

A. If the polarization correlation measurements in the experiment of figure 1 are to be described by some hidden variables, then by the previous section there exist functions f and g such that

$$\begin{aligned} P(a) &= \frac{1}{N}\sum_{n=1}^{N} A_n(a) = \int_s\int_\Gamma f(a, s, \lambda)\rho(a, s, \lambda)dsd\lambda \\ P(b) &= \frac{1}{N}\sum_{n=1}^{N} B_n(b) = \int_{s'}\int_\Gamma g(b, s', \lambda)\rho(b, s', \lambda)ds'd\lambda \end{aligned} \tag{3}$$

with

$$\begin{aligned} &\int_s\int_\Gamma \rho(a, s, \lambda)dsd\lambda = 1 \\ &\int_{s'}\int_\Gamma \rho(b, s', \lambda)ds'd\lambda = 1 \end{aligned} \tag{4}$$

where s and s' represent the states of the analyzers 1 and 2 respectively and a and b the parameter values of the apparatus.

Then the average of the product of the measurements is

$$P(a, b) = \frac{1}{N}\sum_{n=1}^{N} Z_n(a, b) = \frac{1}{N}\sum_{n=1}^{N} A_n(a)B_n(b)$$

$$= \int_S\int_{S'}\int_\Gamma f(a, s, \lambda)g(b, s', \lambda)dsds'd\lambda\rho(a, b, s, s', \lambda) \quad (5)$$

with the condition

$$\int_S\int_{S'}\int_\Gamma \rho(a, b, s, s', \lambda)dsds'd\lambda = 1 \quad (6)$$

Also the various density functions must be taken to satisfy the following because of their significance.

$$\begin{aligned} \rho(a, s, \lambda) &= \int_{s'} \rho(a, b, s, s', \lambda)ds' \\ \rho(b, s', \lambda) &= \int_{s} \rho(a, b, s, s', \lambda)ds \end{aligned} \quad (7)$$

If we assume λ is independent of both s and s', i. e. the states of the apparatus are completely unrelated to the states of the particles, then

$$\rho(a, b, s, s', \lambda) = \rho(a, b, s, s')\rho(\lambda)$$

Further if we assume the states of the measuring apparatus are independent of each other, then

$$\rho(a, b, s, s') = \rho(a, s)\rho(b, s').$$

This is the mathematical expression of the famous locality conditions which says that there is no coupling between the two measuring apparatus. Thus given

$$\rho(a, b, s, s', \lambda) = \rho(a, s)\rho(b, s')\rho(\lambda)$$

we can write Equations (3) and (5) as

$$\begin{aligned} P(a) &= \int_\Gamma A(a, \lambda)\rho(\lambda)d\lambda \\ P(b) &= \int_\Gamma B(b, \lambda)\rho(\lambda)d\lambda \\ P(a, b) &= \int_\Gamma A(a, \lambda)B(b, \lambda)\rho(\lambda)d\lambda \end{aligned} \quad (8)$$

where

$$\begin{aligned} A(a, \lambda) &= \int_s f(a, s, \lambda)\rho(a, s)ds \\ B(b, \lambda) &= \int_{s'} g(b, s', \lambda)\rho(b, s')ds' \end{aligned} \quad (9)$$

Equations (8) are Bell's expressions in the form [*18*]

$$-1 \leq A(a, \lambda)(B(b, \lambda)) \leq 1$$

From these expressions one derives Bell's Inequality in the usual manner [*18*].

Equations (8) are not valid in general when $\rho(a, b, s, s', \lambda)$ is not factorable. In particular, in the case of a time series experiment what the states s, s' and λ are not in general independent, Equation (5) must be used. Observe that mathematically this (the unfactorability of $\rho(a, b, s, s', \lambda)$) can be equivalent to a situation in which the states of the measuring apparatus were physically coupled.

When we use Equation (5) the demonstration of Bell's Inequality fails. This is easier to see if we write Equation (5) in the special for (and keeping in mind the usual proof).

$$\mathrm{P}(a, b) = \int_{\Gamma} \mathrm{A}(a, \lambda)\mathrm{B}(b, \lambda)\rho(a, b, \lambda)d\lambda \tag{10}$$

where $\mathrm{A}(a, \lambda)$ and $\mathrm{B}(b, \lambda)$ are either 1 or -1. λ in this form must then include the states of the apparatus. Therefore the density function depends on both parameters a and b in an unfactorable way, in general, for time series experiments.

One shows in general that the form (5) can give any quantum mechanical prediction, $\mathrm{P}_q(a, b)$, by manufacturing an appropriate example like the one given in Section (III.C) below.

B. In Section (I.B) we commented on the physical reasonableness of a certain type of non-ergodic theory, i. e. theories in which $s_n = s_n(s_{n-1}, \lambda_n)$ (in the below notation) and the states of the measuring apparatus are stable. We now show that such theories must satisfy Bell's Inequality for an overall time average but can agree with quantum mechanics for single time averages.

For any given experimental run we can write λ_i, s_i, s_i', $i = 1, 2, \ldots, \mathrm{N}$ to represent the states of the particle-pair, and the two measuring apparatus respectively for the measurements made at the times t_i. Observe that for such theories we can write

$$\begin{aligned} s_1 &= s_1(s_0, \lambda_1) \\ s_2 &= s_2(s_1, \lambda_2) \\ s_3 &= s_3(s_2, \lambda_3) = s_3(s_2(s_1, \lambda_2), \lambda_3) \\ &\equiv s_3(s_0, \lambda_1, \lambda_2, \lambda_3) \\ &\vdots \\ s_n &= s_n(s_0, \lambda_1, \lambda_2, \ldots, \lambda_n) \\ &\vdots \end{aligned}$$

where s_0 represents the initial state of the apparatus. It is more convenient to write s_n as

$$s_n = s_n(s_0, n, \bar{\lambda}), \qquad \bar{\lambda} = (\lambda_1, \lambda_2, \ldots \lambda_{\mathrm{N}})$$

and similarly

$$s_n' = s_n'(s_0', n, \bar{\lambda})$$

Then for an overall time average we have

$$P(a, b) = \int_X A(s_0, n, \bar{\lambda})B(s'_0, n, \bar{\lambda})\rho(a, b, s_0, s'_0, n, \bar{\lambda})ds_0 ds'_0 dn d\bar{\lambda}$$

where dn is the counting measure, $\rho(n) = 1/N$ and $X = S \times S' \times [0, N] \times \bar{\Gamma}$. It is clear that here ρ is factorable since the initial states s_0 and s'_0 of the apparatus for an experimental run and $\bar{\lambda}$ the sequence of states of the particle-pairs for an experimental run are all independent of each other. So we can write

$$\rho(a, b, s_0, s'_0, n, \bar{\lambda}) = \rho(a, s_0)\rho(b, s'_0)\rho(n)\rho(\bar{\lambda})$$

and therefore Bell's Inequality follows in the usual manner.

It is not difficult to modify the example given in the appendix to construct a non-ergodic theory in which one can violate Bell's Inequality for single time averages. That is, for an experiment in which one makes exactly four experimental runs at each of the four possible parameter combinations. To construct the example it is crucial to choose different initial states for the apparatus at each of the parameter settings. Such an example must agree with Bell's Inequality when one takes an average over many experimental runs by the above proof.

C. We now give a manufactured example of a non-ergodic local hidden variable theory which agrees with quantum mechanics for an overall time average. Assume that there are a sufficient number, N, of measurements in each run so that the first M measurements affect the average in no significant way (i. e. $N \gg M$). Imagine a theory which can be simulated by assigning a lattice structure to space with each lattice element having stable states. Also assume that when a particle passes a lattice element the state of that element after the particle passes is a function of its prior state, the state of the particle, and the states of the bordering lattice elements. In addition assume that the state of the particle can be affected by the state of the lattice element and that the state of the measuring apparatus affects the states of bordering lattice elements in the process of a measurement.

Therefore after the first measurement of a run the state of the lattice element (called the first element) closest to the left-hand measuring apparatus is a function of its prior state, the state of the particle, and the state of the left-hand measuring apparatus and *vice versa.* After the second measurement the state of the second lattice element is a function of the states of the first lattice element (which in turn is a function of the states of the left-hand measuring apparatus and the first particle) and the second particle and *vice versa.* After the i^{th} measurement the $i^{th} + 1$ lattice element « knows » the state of (and consequently the parameter value) of the left-hand measuring apparatus as well as the states of all i particles and *vice versa.*

Now if we take the distance between the two measuring apparatus to be M lattice elements, then after M measurements the state of the right-hand measuring apparatus is not independent of some prior states of the left hand measuring apparatus and *vice versa.* Consequently after sufficient time the result of a measurement at each measuring apparatus depends on, among other things, the parameter values of both measuring apparatus. That is, after a sufficient number of measurements a non-ergodic local theory can behave like a non-local theory. Now since it is assumed that the first M measurements do not affect the average it is not necessary to explicitly give this development in time for the first M measurements. Therefore after M measurements it is justifiable to give the relationships below. It should be emphasized that these relationships taken by themselves define a non-local theory. They give a valid example of a non-ergodic local theory which agrees with quantum mechanics for a time series experiment only with the above conditions (3). It would, of course, be better to explicitly show the evolution in time but this has not been done here. No difficulty is anticipated in modifying the example in the appendix with a lattice structure to construct an example which violates Bell's Inequality for a time series experiment (but does not agree with quantum mechanics) while explicitly showing the evolution in time.

With the previously established notation, let S and S′ both be singleton sets $\{ s \}$ and $\{ s' \}$ respectively. That is each of the apparatus has only one state. For arbitrary $f : S \times \Gamma \to \{ -1, 1 \}$ and $g : S' \times \Gamma \to \{ -1, 1 \}$ define

$$S_+ \times \Gamma_+ = f^{-1}(1) \qquad S'_+ \times \Gamma'_+ = g^{-1}(1)$$
$$S_- \times \Gamma_- = f^{-1}(-1) \qquad S'_- \times \Gamma'_- = g^{-1}(-1)$$

where x is the set product. Also define

$$\Gamma_1 = \Gamma_+ \cap \Gamma'_+ \qquad \Gamma_3 = \Gamma_- \cap \Gamma'_+$$
$$\Gamma_2 = \Gamma_+ \cap \Gamma'_- \qquad \Gamma_4 = \Gamma_- \cap \Gamma_-$$

Since S and S′ are singleton sets $S = S_+ = S_-$ and $S' = S'_+ = S'_-$. Define $\rho(a, b, s, s', \lambda)$ as follows

$$\rho(a, b, s, s', \lambda) = \begin{cases} \alpha_1(1/2 + P_q(a, b)/4) & \text{on the set} \quad S \times \Gamma_1 \times S' \\ \alpha_2(1/4 - P_q(a, b)/4) & \text{on the set} \quad S \times \Gamma_2 \times S' \\ \alpha_3(1/4 - P_q(a, b)/4) & \text{on the set} \quad S \times \Gamma_3 \times S' \\ \alpha_4 P_q(a, b)/4 & \text{on the set} \quad S \times \Gamma_4 \times S' \end{cases}$$

(3) One would expect that, as with this example, the greater the distance between the two measuring apparatus the less the agreement with quantum mechanics (all other experimental conditions kept constant) to be a general property of those non-ergodic theories which agree with quantum mechanics for an overall time average. A distance relationship was reported in reference (5), one of the experiments which disagreed with quantum mechanics.

where

$$\frac{1}{\alpha_i} = \int_S \int_{S'} \int_{\Gamma_i} ds ds' d\lambda . \; i = 1, 4.$$

Then

$$\begin{aligned} P(a, b) &= \int_S \int_{S'} \int_{\Gamma} f(a, s, \lambda) g(b, s', \lambda) \rho(a, b, s', \lambda) ds ds' d\lambda \\ &= \int_S \int_{S'} \int_{\Gamma_1} \rho(a, b, s, s', \lambda) ds ds' d\lambda + \int_S \int_{S'} \int_{\Gamma_1} \rho(a, b, s, s', \lambda) ds ds' d\lambda \\ &- \int_S \int_{S'} \int_{\Gamma_2} \rho(a, b, s, s', \lambda) ds ds' d\lambda - \int_S \int_{S'} \int_{\Gamma_3} \rho(a, b, s, s', \lambda) ds ds' d\lambda \\ &= 1/2 + P_q(a, b)/4 + P_q(a, b)/4 \\ &- (1/4 - P_q(a, b)/4) - (1/4 - P_q(a, b)/4) \\ &= P_q(a, b). \end{aligned}$$

Similarly one obtains $P(a) = 1/2 = P(b)$ as well as the relationships of Equations (4) and (6).

IV

A. As stated in Section I the experimental averages obtained in the laboratory are overall time averages and are equivalent to an ensemble average only with the additional seemingly reasonable assumption:

i) Waiting a short period of time between consecutive measurements of a run constitutes a repreparation of the system.

Since quantum mechanics predicts an ensemble average, the experimental evidence can be interpreted as supporting quantum mechanics only to the extent that this assumption is accepted. For example, quantum mechanics predicts a strict correlation (both or neither of the particles pass the polarizers) for certain particle systems when both measurements are made along the same direction. The Experiments (4, 7 and 8) (reference numbers) can be interpreted as evidence that this is actually true for an ensemble average only with this physical assumption. This physical assumption is tantamount to excluding ceitain non-ergodic local hidden variable theories, mainly those where the measuring system has stable states or memory.

Interpreting the experimental evidence is further complicated by the fact that the experimental results are not without inconsistencies. For example, Experiments (4, 7 and 8) support quantum mechanics while Experiments (5 and 6) support local hidden variable theories. Experiment (8) is a replication of Experiment (6). In view of the results of this article it is natural to ask whether the conflicting results in the previous two experiments are a result of the way the overall time averages were obtained rather than due to experimental error. That is, do they differ significantly in the number of experimental runs and/or in the number of measurements made in each run ? We are unable to assess this possibility from the published data.

B. In principle one could distinguish between quantum mechanics and any local hidden variable theory, ergodic or not, by performing a time series experiment while in some way randomizing the states of the apparatus between particle passages so as to get an ensemble average. One way to do this might to be flash intense beams of white light from independent sources through both apparatus between photon passages so as to destroy any effect the object being measured might have on the measuring apparatus. See Reference [*16*] for some technical details where a similar idea is used in a different but related context.

One might also consider performing the same experiment with a greater time difference between particle passages on the very plausible thinking that if there is any effect in time it should decrease when the time difference is increased. We feel that such experiments would not test the perhaps most likely hidden variable candidates. If there is anything characteristic about quantum mechanics in contrast to classical mechanics, it is the existence of discrete relatively stable states that do not change continuously in time. Increasing the time interval between passages would not test hidden variable theories which have this property.

Finally, another possibility might be to change the parameter values in some random manner between photon passages. A French group (A. Aspect, *Physical Review*, D, *14* (8), 1944 (1976)) has plans for such an experiment although their intentions are to examine the local theories mentioned in Footnote (1). It is clear from this article that such an experiment would not necessarily be a test between quantum mechanics and such local hidden variable theories but could be looked at as a test between quantum mechanics and non-ergodic local hidden variable theories.

V

It is commonly thought that the Statistical and Copenhagen Interpretations of quantum mechanics are experimentally equivalent [*24*]. This being the case then both or neither interpretations are consistent with local hidden variable theories. This article sheds a different light on the previous. For example, if it were experimentally found that a true ensemble experiment agreed with Bell's Inequality and that a time series experiment agreed with quantum mechanics then this would be consistent with the Statistical Interpretation but not with the Copenhagen Interpretation. This result would be consistent with the Statistical Interpretation since within this interpretation one could take as an element of the ensemble (for which the probability amplitude, ψ, defines an average) not a single particle-pair, but many particle-pairs following each other in time (i. e. a single experimental run). In other words, interpret $\psi\psi^*$ as an overall time average. The consequences of treating $\psi\psi^*$ as an overall time average are examined in Reference [*15*].

APPENDIX

The following is a simple but descriptive example of the type of non-ergodic local « theory » discussed in Section (I.A). Referring to Figure 1, let our particle source produce billiard balls which are either red or blue, with the condition that they are produced under a « conservation of color » law. That is, both balls of the pair are either red or blue. Also assume that the colors are random with red and blue appearing each with a probability of one-half. Let Analyzer 1 be such that it has the following properties. It is a two state filtering device, b or r, which will pass a blue ball if it is in state b, and will pass a red ball if it is in state r. In addition assume that the analyser's state becomes r if it measured a red ball regardless if that ball passed or not (*i. e.* regardless of it's prior state). Likewise assume that the state of the analyzer becomes b if it measures a blue ball. Let Analyzer 2 be identical with Analyzer 1.

Then it is easy to see that the correlation average, P, is one and zero for a single time (or overall time) average and ensemble average respectively. Also observe (in the notation of the test) that the density function $\rho(s, s, \lambda)$ is not factorable in either of the forms $\rho(s)\rho(\lambda)\rho(s)$ or $\rho(s, \lambda)\rho(s, \lambda)$ for a time series experiment, and that there are no non-local interactions involved. The above types of theories can violate Bell's Inequality for single time averages but must agree with it for overall time averages as shown in Section (III.B).

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[13] *Almost any book on Stochastic Processes would discuss ergodicity.* Also we emphasize that we use the word, ergodic, as it is used in the study of Stochastic Processes and not as it is used in Statistical Mechanics. There are some language distinctions that could cause confusion in this article.

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