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Horizons in five-dimensional theory

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RÉSUMÉ. — Nous étudions les conditions d'existence des horizons, donc des trous noirs, dans la théorie proposée. Celle-ci, proposée par M. A. Tonnelat, est fonction d'un paramètre et peut être considérée comme une extension de la Relativité Générale. Le résultat important de ce modèle est : la solution de Schwarzschild est la seule solution qui ait un horizon.

SUMMARY. — We will study the conditions of existence of the horizons, therefore the existence of black holes, according to the proposed model. This one, proposed by M. A. Tonnelat, is a function of one parameter and can be considered as an extension of the General Relativity. The important result, in this model, is the following: Schwarzschild's solution is the only solution which has a horizon.

I. PRINCIPLE OF THE THEORY

The theory is five-dimensional because we use a five-dimensional Riemannian space V_5, naturally the physical interpretation is made in the space-time V_4.

In the space V_5, the coordinates are x^M, the metric form is:

$$d\sigma^2 = \gamma_{MN}dx^M dx^N \qquad (M, N = 1, 2, 3, 4, 5) \tag{1}$$

and the components of the fundamental tensor γ_{MN} are independent of x^5.

We interpret, as the four-dimensional space-time, the Riemannian space V_4 with coordinates x^μ, with the metric form:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \qquad (\mu, \nu = 1, 2, 3, 4) \tag{2}$$

and the components φ_μ of the electromagnetic potential where:

(3) $$g_{\mu\nu} = \gamma_{\mu\nu} - \frac{\gamma_{\mu 5}\gamma_{\nu 5}}{\gamma_{55}}$$ (4) $$\gamma_{\mu 5} = \alpha\varphi_\mu\gamma_{55}$$

γ_{55} is a scalar field in V_4 and α is a constant.

The field equations come from a variational principle, expressed in V_5, proposed by M. A. Tonnelat [1]:

(5) $$\delta\int_{V_5} f(\gamma_{55})(R_{MN}\gamma^{MN} + \mathcal{M})\sqrt{\gamma}d^5x = 0$$

the variations $\delta\gamma^{MN}$ are arbitrary.

We assume that:

$$\gamma = \det(\gamma_{MN})$$

$\mathcal{M}$ is an invariant density which characterizes matter. R_{MN} are the covariant components of the Ricci tensor in V_5. f is an arbitrary real function of a real variable which permits to select the theory.

a) Using $f(\gamma_{55}) = 1$, we obtain Y. Thiry's theory [2], [3], [4]. We know that this theory does not give the correct value for the advance of the perihelion of planets [5], [6].

b) Using $f(\gamma_{55}) = (-\gamma_{55})^{-1/2}$, we obtain H. Leutwyller's variational principle [7].

c) In the case where:

(6) $$f(\gamma_{55}) = \beta(-\gamma_{55})^{p/2}$$

β et p are real constants, we shall study some consequences of this theory with respect to the parameter p. From the variational principle (5), we get the field equations [1]:

(7) $$P_{MN} = \chi T_{MN}$$

where:

(8) $$P_{MN} = R_{MN} - \frac{1}{2}\gamma_{MN}R - R\gamma_{5M}\gamma_{5N}\frac{df}{fd\gamma_{55}} - \frac{1}{f}(\nabla_M\nabla_N f - \gamma_{MN}\square_5 f)$$
$$R = R_M{}^M$$

χ is a scalar in V_4. Thus it can be a function of γ_{55} as f.

(9) $$T_{MN} = -\frac{1}{f\chi\sqrt{\gamma}}\frac{\partial(f\sqrt{\gamma}\mathcal{M})}{\partial\gamma^{MN}}$$

are the covariant components of the energy tensor in V_5. In the case of an incoherent fluid, we have in particular:

(10) $$T^{MN} = \rho c^2 u^M u^N$$

with:

(11) $$u^M = \frac{dx^M}{d\sigma}$$

M. A. Tonnelat has proved [1] that:

$$\nabla_M(f\chi T^{MN}) = 0 \tag{12}$$

then, in an incoherent fluid, the stream lines are geodesics of V_5.

In particular, the trajectory of a mass point is a geodesic of V_5. It verifies:

$$u^M \nabla_M u_N = 0 \tag{13}$$

If we consider equation (13) for $N = 5$, u_5 is a constant k on the trajectory.

II. FIELD EQUATIONS AND JORDAN'S THEORY

In order to study the horizons as a function of p, we shall give the exact external solution of the field equations, in the static case with spherical symmetry in V_4, without electromagnetic field [8]. In this article, we will not study the electromagnetic part of the theory, thus there is no electromagnetic field: $\gamma_{\mu 5} = 0\,(\mu = 1, 2, 3, 4)$. We will now only consider non-charged mass-points ($u_5 = 0$) thus their trajectories are also geodesics of V_4.

In this case, we are going to compare the field equations (7) with these of P. Jordan's theory [9].

We put:

$$\gamma_{55} = -\xi^2 \tag{14}$$

and we assume that:

$$f(\gamma_{55}) = \beta\xi^p \tag{15}$$

Then we can write if $p \neq -1$:

$$P_{\mu\nu} = \widehat{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\widehat{R} - \frac{\widehat{\nabla}_\mu\widehat{\nabla}_\nu\xi^{1+p}}{\xi^{1+p}} + g_{\mu\nu}\frac{\widehat{\Delta}\xi^{1+p}}{\xi^{1+p}} + \frac{2p}{(1+p)^2}\left(\frac{\widehat{\nabla}_\mu\xi^{1+p}\widehat{\nabla}_\nu\xi^{1+p}}{\xi^{2(1+p)}} - \frac{1}{2}g_{\mu\nu}\frac{\widehat{\nabla}_\rho\xi^{1+p}\widehat{\nabla}^\rho\xi^{1+p}}{\xi^{2(1+p)}}\right) \tag{16}$$

$$P_{55} = \frac{\xi^2}{2}(1+p)\left[\widehat{R} - \frac{2p}{(1+p)^2}\left(\frac{2\widehat{\Delta}\xi^{1+p}}{\xi^{1+p}} - \frac{\widehat{\nabla}_\mu\xi^{1+p}\widehat{\nabla}^\mu\xi^{1+p}}{\xi^{2(1+p)}}\right)\right] \tag{17}$$

We denote with $\frown$ an expression which is related to the metric form of V_4.

If we decided to put:

$$\chi = \tau\xi^{1+p} \tag{18}$$, $$\zeta = \frac{2p}{(1+p)^2} \tag{19}$$

where τ is a constant, we should get the first terms of the equations of P. Jordan's theory [9] which, in the case without electromagnetic field, gives the equations:

$$P_{\mu\nu} = \chi\widehat{T}_{\mu\nu} \qquad P_{55} = 0$$

It is important to note that:

a) χ is also in equations (7), thus, in general, we cannot put relation (18). The value of χ, in function of ξ, depends in particular on the electromagnetic study of the theory [*2*], [*3*], [*4*]. Thus the two theories are in general irreducible.

b) Relation (19) limits the values of ζ to the interval $\left]-\infty, \frac{1}{2}\right]$, contrarily to P. Jordan's theory.

c) In the important case $p = -1$, relations (16) and (17) are not valid, thus we cannot study this case in P. Jordan's theory where we have $\zeta = -\infty$.

d) If $T_{MN} = 0$, we have the external case and the field equations (7) are independent of χ. Thus we get the solution of the external case when we put:

$$\chi = \tau\xi^{1+p} \quad , \quad \zeta = \frac{2p}{(1+p)^2} \qquad (p \neq -1)$$

in the corresponding solution of P. Jordan's theory, and we limit the values of ζ to the interval $\left]-\infty, \frac{1}{2}\right]$. This excludes some forms of the solution. The values of the components of the fundamental tensor in V_4: $g_{\mu\nu}(x^\rho)$ are the same.

III. THE EXACT EXTERNAL SOLUTIONS OF THE STATIC CASE, WITH SPHERICAL SYMMETRY IN V_4, WITHOUT ELECTROMAGNETIC FIELD

Thus, we may consider the field equations in the case:

$$\begin{aligned} ds^2 &= -e^{2l}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) + e^{2n}dct^2 \\ \gamma_{5\mu} &= 0, \ \gamma_{55} = -\xi^2, \qquad T_{MN} = 0, \qquad f(\gamma_{55}) = \beta\xi^p \end{aligned} \tag{20}$$

l, n and ξ are functions of r only.

In P. Jordan's theory, the corresponding solutions are O. Heckmann's solutions [*10*]. All these solutions are not a solution of the problem because ζ is limited to the interval $\left]-\infty, \frac{1}{2}\right]$.

On the other hand, in the important case $p = -1$, there is no O. Heckmann's solution.

We are going to express our solutions under a suitable form to study them with respect to p.

Then we get the following solution, which depends on three constants: m, a and ξ_0 as a function of n, n varying from $-\infty$ to 0.

$$r = r_0 \frac{(y_1 - y_2)e^{-y_2 n}}{1 - e^{(y_1 - y_2)n}}, \qquad e^{2l} = \frac{(y_1 - y_2)^2 e^{(y_1 - y_2)n}}{\left(y_2 - y_1 e^{(y_1 - y_2)n}\right)^2}, \qquad \xi = \xi_0 e^{an}$$

with $r_0 = Gm/c^2$, y_1 and y_2 are the roots of:

(21) $\quad \varphi(y) \equiv y^2 - 2(1 + a + ap)y + a(1 + p + ap) = 0 \ (y_1 > y_2)$

this equation has always two real and distinct roots.

Naturally, we can write O. Heckmann's solution in the same form, but the roots y_1 and y_2 are not always real and distinct.

IV. STUDY OF THE SOLUTIONS

We may class the results with respect to a and p (fig. 1).

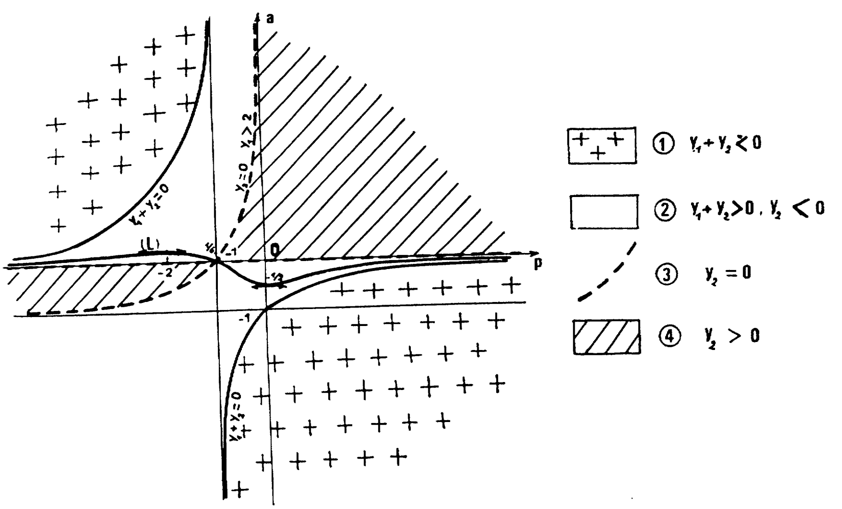

FIG. 1.

We get the type ①, ②, ③, ④ of variations of r and e^{2l} with respect to n (fig. 1, 2, 3). O. Heckmann's solutions, where y_1 and y_2 are real and distinct, may be also classed according to these four types.

The horizon does not exist in cases ① and ②. In case ④, we have not $e^{2n} = 0$ when e^{2l} is infinite as in Schwarzschild's case. Case ④ is difficult to interpret physically, since we cannot extend the solution for $r < r_\beta$, even if we changed the signature of $d\sigma^2$. On the other hand, case ③ gives

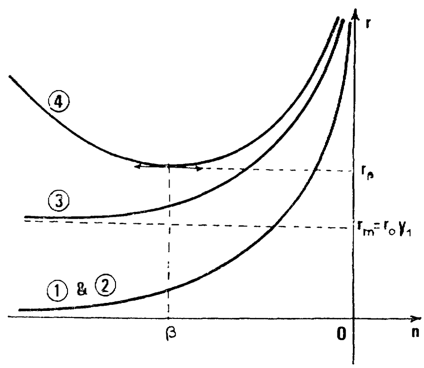

FIG. 2.

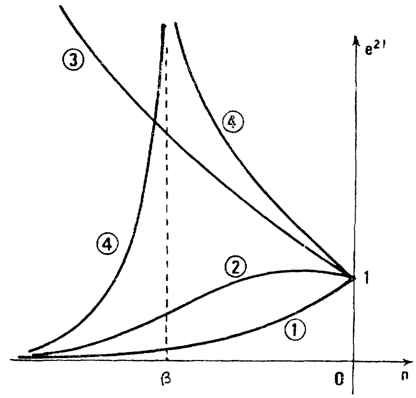

FIG. 3.

a similar solution to Schwarzschild's and coincides with it for $a = 0\,(y_1 = 2, y_2 = 0)$.

Here we found a result proved by S. W. Hawking [*17*] in Brans-Dicke Theory: the horizons are Schwarzschild's horizons.

V. DEFLECTION OF LIGHT RAYS AND THE ADVANCE OF THE PERIHELION OF PLANETS

We are going to calculate these values in function of m, a and p, and compare them with their experimental value. Thus, we can limit the permit solutions in the plane (p, a).

We know that the trajectory of a neutral mass-point is a geodesic of V_4. A geodesic of V_4 where $ds = 0$ is the trajectory of a light ray. The expansion of e^{2n} and e^{-2l} in inverse powers of r gives:

$$\left\{\begin{aligned} e^{2n} &= 1 - \frac{2Gm}{c^2 r} - 2a(1+p)\left(\frac{Gm}{c^2 r}\right)^2 + \dots \\ e^{-2l} &= 1 - 2(1 + a + ap)\frac{Gm}{c^2 r} - a(1 + p + ap)\left(\frac{Gm}{c^2 r}\right)^2 + \dots \end{aligned}\right.$$

Then the deflection of light rays is, after a classical calculation:

$$\delta = \frac{4Gm}{c^2 R_\odot}\left[1 + \frac{a(1+p)}{2}\right]$$

$R_\odot$ is the radius of the sun

and the advance of the perihelion of planets is:

$$\Delta\varpi = \frac{2\pi}{\mathrm{P}} \frac{\mathrm{G}m}{c^2} [3 + 2a(1 + p)]$$

P is the latus rectum of the orbit.

We get the experimental values if $a\,(1 + p)$ is small.

This condition excludes case ①.

VI. RADIAL GEODESICS IN V_4

Here we can see the very peculiar aspect of Schwarzschild's solution which is the only one to have an horizon.

θ and φ are constant on the trajectory of the mass-point. We have:

$$ds^2 = \frac{r_0^2(y_1 - y_2)^4 e^{(y_1 - 3y_2)n}}{\left(1 - e^{(y_1 - y_2)n}\right)^4\left(e^{-2n}\mathrm{C}^2 - 1\right)} dn^2 \qquad dct = \mathrm{C}e^{-2n} ds$$

where C is a constant.

Then

$$c(t_1 - t_0) = \int_{n_0}^{n_1} \frac{\mathrm{C}r_0(y_1 - y_2)^2 e^{(y_1 - 3y_2)\frac{n}{2}} e^{-2n}}{\left(1 - e^{(y_1 - y_2)n}\right)^2\sqrt{e^{-2n}\mathrm{C}^2 - 1}} dn \tag{22}$$

For example, in the case of a radial fall, we have $n_1 < n_0 < 0$ (n_0: initial value of n). For finite values of n_1, the integral (22) has always a finite value. When $n_1 \to -\infty$, (22) has a finite value if $y_1 - 3y_2 - 2 > 0$. We can write this condition:

$$(y_1 + y_2 - 2)/4 > y_2$$

Using (21), we get:

$$\varphi\left(\frac{y_1 + y_2 - 2}{4}\right) = -\frac{a^2}{4} [2(1 + p)^2 + (1 - p)^2] < 0 \qquad \text{if} \qquad a \neq 0$$

then we have:

$$y_2 < \frac{y_1 + y_2 - 2}{4} < y_1 \qquad \text{if} \qquad a \neq 0$$

Thus, if $a \neq 0$, all the values of r with $n < n_0$ are reached in a finite time-coordinate (consequently in a finite proper time). This is true even in cases ③ and ④ which have a minimum value for r.

For a light ray, we have the same result. Thus, for $a \neq 0$ there never is an horizon.

But if $a = 0$, we have Schwarzschild's solution ($y_1 = 2$, $y_2 = 0$) and we know that there is an horizon when $n \to -\infty$ [*11*], [*12*].

VII. MOTION OF A NEUTRAL MASS POINT WHEN $| n |$ BECOMES LARGE ON THE TRAJECTORY [13]

The peculiar aspect of Schwarzschild's solution is even more evident in this study.

We will not study case ④, $y_2 > 0$. Indeed $r(n)$ attains a minimum and becomes infinite when $| n |$ becomes infinite; then the physical interpretation seems difficult.

We have:

$$\varphi = \varphi_0, \qquad r^2 \frac{d\theta}{ds} = \mathrm{D}, \qquad e^{2n} \frac{dct}{ds} = \mathrm{C}$$

$$- e^{2l}\left(\frac{dr}{ds}\right)^2 - r^2\left(\frac{d\theta}{ds}\right)^2 + e^{2n}\left(\frac{dct}{ds}\right)^2 = 1$$

We suppose that $\mathrm{D} \neq 0$ (we have studied $\mathrm{D} = 0$ in § VI).

1° If $y_2 < 0$, $r \to 0$ when $n \to -\infty$, we get an approximate differential equation of the trajectory:

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{\mathrm{A}^2\mathrm{C}^2}{\mathrm{D}^2} r^{(y_1+3y_2+2)/y_2} - \mathrm{B}^2 r^{(y_1+y_2)/y_2}$$

A and B are constants $\neq 0$ which depend on r_0, y_1 and y_2. We can discuss the different kinds of trajectories:

a) $-1 < y_2 < 0$

$$\left(\frac{dr}{d\theta}\right)^2 \sim \frac{\mathrm{A}^2\mathrm{C}^2}{\mathrm{D}^2} r^{(y_1+3y_2+2)/y_2}$$

In the plane (p, a), we can prove that, if $-1 < y_2 < 0$, then $(y_1 + 3y_2 + 2)/y_2 < 2$; the trajectory reaches the central mass (fig. 4).

b) $y_2 = -1$ the approximations are only valid if $\mathrm{A}^2\mathrm{C}^2/\mathrm{D}^2 > \mathrm{B}^2$, then we have the same result as in *a*).

c) $y_2 < -1$ when $r \to 0$ we have:

$$\frac{\mathrm{A}^2\mathrm{C}^2}{\mathrm{D}^2} r^{(y_1+3y_2+2)/y_2} - \mathrm{B}^2 r^{(y_1+y_2)/y_2} \leqslant 0$$

then r has a minimum value $\neq 0$ on the trajectory (fig. 5).

2° If $y_2 = 0$ (then $y_1 \geqslant 2$) $r \to r_0 y_1$ when $n \to -\infty$.

The approximate differential equation of the trajectory is:

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{\mathrm{C}^2}{\mathrm{D}^2} (r_0 y_1)^{(2/y_1)+3} (r - r_0 y_1)^{1-(2/y_1)} - \left(1 + \frac{r_0^2 y_1^2}{\mathrm{D}^2}\right) r_0 y_1 (r - r_0 y_1)$$

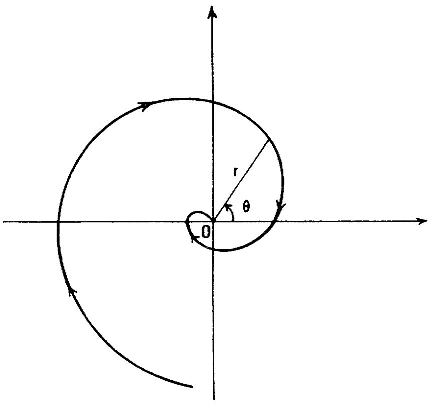

FIG. 4.

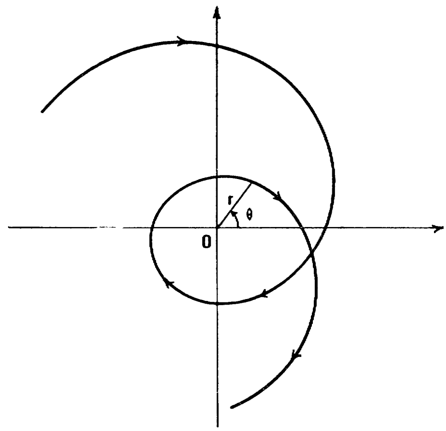

FIG. 5.

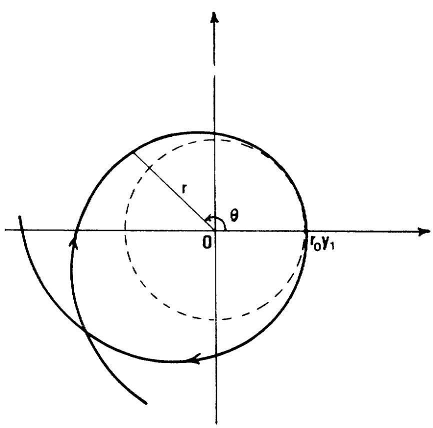

FIG. 6.

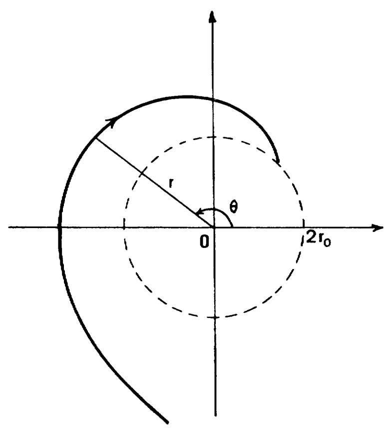

FIG. 7 (Schwarschild's solution).

a) $y_1 > 2$: the trajectory is tangent to the circle $r = r_0 y_1$ (fig. 6).

b) $y_1 = 2$ this is Schwarzschild's solution (fig. 7).

For all the values of y_1 and $y_2 \leqslant 0$, we still have $y_1 - 3y_2 - 2 > 0$ as in § VI and the time-coordinate on the trajectory is finite, except when $y_2 = 0$ and $y_1 = 2$ (Schwarzschild's solution). Thus we note that Schwarzschild's solution is very peculiar in this theory. The trajectories of neutral mass points in the field of Schwarzschild's solution do not appear as the limit of trajectories when the scalar field becomes nearly constant.

The studies of the § VI and VII are not valid in Jordan's theory because y_1 and y_2 do not satisfy the same relations. In particular, we no longer have $y_1 - 3y_2 - 2 > 0$ and we can no longer assert that the time-coordinate is finite on the trajectory when the solution is not Schwarzschild's.

VIII. NEWTONIAN APPROXIMATION OF THE FIELD EQUATIONS

With the internal solution, we could calculate χ and a with respect to p. But we have not solved this problem (yet we have an approximate solution which is also valid when p is equal to -1 [14]). Here, we use the Newtonian approximation of the field equations and we are going to prove that, in general, the value of χ must be different from $8\pi G/c^4$ (contrarily to P. Jordan's hypothesis [9]), the relation between a and p will be represented by a curve (L) (fig. 1). The solution is of the type ② and we have no horizon.

In equation (18), we can choose τ; thus this equation is always valid in Newtonian approximation as χ takes a constant value χ_0. Then the field equations are the same as in Jordan's theory. As in General Relativity we get the value of χ_0 using the Newtonian approximation of the field equations.

We use harmonic coordinates in $V_5(\square_5 x^M = 0)$ with $x^4 = ct$. Here:

$$T^{MN} = \rho c^2 u^M u^N \tag{10}$$

We use the following approximations:

$$u_1 = u_2 = u_3 = 0, \quad u_5 = 0 \text{ (no charges)}, \quad u_4 = 1, \quad \chi = \chi_0 + \dots$$

If we put

$$\begin{cases} \gamma_{ij} = -\delta_{ij} + \varepsilon^2\alpha_{ij} + 0(\varepsilon^4) \\ \gamma_{4i} = \varepsilon^3\alpha_{4i} + 0(\varepsilon^5) \\ \gamma_{44} = 1 + \varepsilon^2\alpha_{44} + 0(\varepsilon^4) \\ \gamma_{5i} = \varepsilon^3\alpha_{5i} + 0(\varepsilon^5) \\ \gamma_{54} = \varepsilon^2\alpha_{54} + 0(\varepsilon^4) \\ \gamma_{55} = \left(-1 + \varepsilon^2\alpha_{55} + 0(\varepsilon^4)\right)\xi_0^2 \end{cases} \qquad \begin{aligned} & i, j = 1, 2, 3 \\ & \delta_{ij} = \begin{cases} 0 & \text{if} \quad i \neq j \\ 1 & \text{if} \quad i = j \end{cases} \\ & \varepsilon \sim \frac{1}{c} \end{aligned}$$

by straightforward calculation, we find:

$$\begin{cases} \dfrac{\varepsilon^2}{4}\Delta[(1+p)\alpha_{44} - (1+p)\alpha_{ll} + (1-3p)\alpha_{55}] = 0 & \Delta\alpha_{4i} = 0 \\ \dfrac{\varepsilon^2}{4}\Delta[\alpha_{44} + \alpha_{ll} + (1+2p)\alpha_{55}] = \chi_0\rho c^2 & \Delta\alpha_{5i} = 0 \\ \dfrac{\varepsilon^2}{2}\Delta\left[\alpha_{ij} + \dfrac{1}{2}\delta_{ij}(\alpha_{44} - \alpha_{ll} - \alpha_{55}(1+2p))\right] = -p\dfrac{\varepsilon^2}{2}\partial_{ij}\alpha_{55} & \Delta\alpha_{54} = 0 \end{cases}$$

where:

$$a_{ll} = \alpha_{11} + \alpha_{22} + \alpha_{33}, \qquad \Delta = \partial_{11} + \partial_{22} + \partial_{33}$$

It follows that:

$$\alpha_{4i} = 0 \qquad \alpha_{5i} = 0 \qquad \alpha_{54} = 0$$

After a short calculation, we get α_{44}, α_{55}, α_{ll} and we obtain α_{ij}.

Thus, in the case of a mass point, whose coordinate are $x^i = 0$ and the mass: m, we get:

$$\begin{cases} \varepsilon^2\alpha_{ij} = -\dfrac{\chi_0 mc^2}{4\pi(3+2p+3p^2)}\left[p(1+p)\dfrac{x^i x^j}{r^3} + (2-p+p^2)\dfrac{\delta_{ij}}{r}\right] \\ \varepsilon^2\alpha_{44} = -\dfrac{1+p+p^2}{3+2p+3p^2}\dfrac{\chi_0 mc^2}{\pi r} \\ \varepsilon^2\alpha_{55} = -\dfrac{1+p}{3+2p+3p^2}\dfrac{\chi_0 mc^2}{2\pi r} \end{cases}$$

We can now compare our rigorous solution with this approximate solution. To do this, we must write our rigorous solution in the same rectangular and harmonic coordinates.

Then we get for the exact solution:

$$\begin{cases} \varepsilon^2\alpha_{ij} = apr_0\dfrac{x^i x^j}{r^3} - (2+3ap+2a)r_0\dfrac{\delta_{ij}}{r} \\ \varepsilon^2\alpha_{44} = -2\dfrac{r_0}{r} \\ \varepsilon^2\alpha_{55} = 2a\dfrac{r_0}{r} \end{cases}$$

In comparing these results, we have:

a) If $p \neq -1$ the same results if and only if:

$$(23)\quad \chi_0 = \frac{2\pi G(3p^2+2p+3)}{c^4(p^2+p+1)}, \qquad (24)\quad p+1+2a(p^2+p+1) = 0$$

Thus χ_0 must be a function of p, if $p \neq -1$, its value is not $8\pi G/c^4$. This contradicts P. Jordan's [*9*] and F. Hennequin [*6*] hypothesis and we get a different relation between a and p. Thus when p is different from -1, we do not get the same advance of the perihelion of planets and the same deflection of light rays, in particular in Y. Thiry's theory [*5*] ($p = 0$).

But we have the same conclusion as K. Just [*5*]: we do not obtain the experimental values if $p \neq -1 (\zeta \neq \infty)$.

The relation (24) between a and p is the equation of a curve (L) (fig. 1) wich is always in the region ② of the plane (p, a).

Now, we know that in the case ②, the physical interpretation is easy: we get a solution as a function of r, r varying from 0 to ∞.

b) If $p = -1$.

$\chi_0 = 8\pi G/c^4$, there is no definite relation between a and p. A second order calculation gives [*14*]:

$a = - Gm/5c^2R$

There R is the radius of the central corper.

We are in the case (2) without horizons.

These calculations prove that Schwarzschild's solution is not in general the solution of the problem. Yet, we cannot conclude by an approximate calculation in favour of the non-existence of particular Schwarzschild's horizons in this theory.

But we can prove by the study of the internal solution that we have no horizon if p belongs to the interval $[-1, -1/3]$ [*16*].

Thus, in the important case $p = -1$, we never have horizons. We can erase Schwarzschild's horizon with the introduction of the scalar field $\gamma_{55}(x^\mu)$.

Using very different considerations A. Janis, E. Newman and J. Winicour [*15*] have found similar results.

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