ANNALES DE L'I. H. P., SECTION A

L. B. RÉDEI

Model hamiltonian for low energy baryon pseudo-scalar meson scattering

Annales de l'I. H. P., section A, tome 13, nº 1 (1970), p. 99-102
<http://www.numdam.org/item?id=AIHPA_1970__13_1_99_0>

Ann. Inst. Henri Poincaré,
Vol. XIII, n° 1, 1970, p. 99-102

Section A :
Physique théorique.

Model hamiltonian for low energy baryon pseudo-scalar meson scattering

by

L. B. RÉDEI
Institute of Physics, Department of Theoretical Physics,
Umea University, Umea, Sweden.

In a recent paper [*1*] by the author, subsequently referred to as I, a model Hamiltonian was derived from pseudo-scalar γ_5 coupling for low energy pion-nucleon scattering. The Hamiltonian was derived by neglecting virtual nucleon-antinucleon pair creation terms. The correct relativistic expression for the nucleon was used, enabling one to obtain an approximate expression for the pion-nucleon scattering amplitude which after coupling constant renormalization converged in the infinite cut-off limit (In the infinite nucleon mass limit one recovers the results of the static model [*2*]). The approximate unitary S-matrix was obtained in I by summing to infinite order those terms in the perturbation expansion which do not contain higher intermediate states than one nucleon and two mesons. By fitting the value of the coupling constant $\frac{g^2}{4\pi}$ to reproduce the experimental value of the 3/2 3/2 scattering length one obtained $\frac{g^2}{4\pi} = 23{,}25$ and for the mass of $N^*_{3/2}$ (1238) 1368 MeV.

The purpose of this short note is to compare these results with some recent calculations [*3*] by Katz and Wagner. In this work a broken SU_3, relativistic Schrödinger equation is solved to obtain the masses of the baryon resonances. The potential is obtained by making an off energy
recent calculations [*3*] by Katz and Wagner. In this work a broken SU_3, relativistic Schrodinger equation is solved to obtain the masses of the baryon resonances. The potential is obtained by making an off energy
shell extrapolation of the baryon exchange Born term derived from an SU_3

symmetric Lagrangian. The breaking of SU_3 symmetry is introduced by using the physical masses for the baryons and the mesons. The value of the pion-nucleon coupling constant is determined by requiring the model to reproduce the mass of the $N^*_{3/2}$ (1238) resonance. This gives $\frac{g^2}{4\pi} = 38$. The dependence of the mass was found approximately linear in the inverse power of the coupling constant. For the 3/2 3/2 channel this agrees very well with the results of I. In fact the linear dependence of the mass of $N^*_{3/2}$ on the inverse coupling constant follows directly from equation (37) in reference [1]. Also, if one instead of fitting $\frac{g^2}{4\pi}$ to the scattering length determines its value by fitting it to the experimental mass of the $B^*_{3/2}$ one obtains in I $\frac{g^2}{4\pi} = 36$ in good agreement with the figure quoted in reference [3]. If thus seems that these two models are in good agreement at least in the 3/2 3/2 pion-nucleon channel. Encouraged by this we shall briefly discuss the SU_3 extension of the Hamiltonian in I (The model discussed in I has the advantage over the baryon exchange model [3] that it does not involve an arbitrary off energy shell extrapolation).

We start with the SU_3 symmetric pseudo-scalar Hamiltonian for the baryon pseudo-scalar meson system

$$H_{\text{int.}} = ig_0\colon \int d^3x((1 - 2f)\bar{B}(x)^i_j\gamma_5 B(x)^j_k P(x)^k_i + \bar{B}(x)^i_j\gamma_5 B(x)^k_i P(x)^j_k)\colon \quad (1)$$

where $B(x)^i_j$ is the 3×3 SU_3 matrix made out of the eight baryon field operators and $P(x)^k_i$ the meson matrix constructed from pseudo-scalar meson fields [4]. Neglecting virtual baryon-antibaryon pair creation terms one has

$$H = H_0 + V_+ + V_- = H_0 + V \quad (2)$$

where H_0 is the free Hamiltonian and $V_\pm$ raises (lowers) the meson number by one while leaving the number of baryons unchanged. Explicity

$$V_+ = (V_-)^+ = ig_0\colon \int d^3x((1 - 2f)\bar{B}^{(-)}(x)^i_j\gamma_5 B^{(+)}(x)^j_k P^{(-)}(x)^k_i + \bar{B}^{(-)}(x)^i_j\gamma_5 B^{(+)}(x)^k_i P^{(-)}(x)^j_k)\colon \quad (3)$$

where $B^{(+)}$ describes baryon annihilation, $\bar{B}^{(-)}$ baryon creation and $P^{(-)}$

meson annihilation. One can now proceed as in I to obtain an effective potential for baryon-meson scattering. Using the expression

$$\langle b | S | a \rangle = \langle b | a \rangle - 2\Pi i \delta(W_a - W_b) \lim_{\substack{z \to W_a \\ \mathrm{Im}\, z > 0}} \langle b | T(z) | a \rangle, \tag{4}$$

where $|a\rangle$, $|b\rangle$ are one baryon, one meson states and T(z) given by

$$T(z) = V \frac{1}{1 + R_0(z)V},$$

$$R_0(z) = (H_0 - z)^{-1},$$

one gets an approximate expression for T(z) by using

$$\tilde{T}(z) = PVP \frac{1}{1 + R_0(z)PVP}, \tag{6}$$

where P is the projection operator which is one for states not containing more than two mesons and is zero otherwise. This approximation is clearly equivalent to keeping those terms in the perturbation expansion of the right hand side of equation [*4*] which do not contain more than two mesons in their intermediate states. By expanding in terms V and resumming the series one obtains that

$$\langle b | \tilde{T}(z) | a \rangle = \langle b | \tilde{K}(z) \frac{1}{1 + R_0(z)\tilde{K}(z)} | a \rangle, \tag{7}$$

where

$$-\tilde{K}(z) = V_+ R_0(z) V_- + V_- R_0(z) V_+. \tag{8}$$

The operator $\tilde{K}(z)$ does not change the meson number. In general it will have a diagonal part which describes a self-energy effect due to permenant meson cloud formation. The diagonal part can be eliminated by renormalization and it is to be dropped from the S-matrix [*5*]. Writing

$$\langle b | K(z) | a \rangle = \langle b | \tilde{K}(z) | a \rangle - K_{\mathrm{diag}}(z) \langle b | a \rangle \tag{9}$$

one has, in view of the proceeding discussion, the following approximate baryon-meson S-matrix

$$\langle b | S | a \rangle = \langle b | a \rangle - 2\Pi i \delta(W_a - W_b) \lim_{z \to W_0} \langle b | K(z) \frac{1}{1 + R_0(z)K(z)} | a \rangle. \tag{10}$$

Comparing this with the original expressions (4) and (5) it follows that K(z) can be looked upon as a generalized energy dependent meson-baryon potential. The operator K(z) becomes non-hermitian for Re z above production threshold corresponding to meson production.

To summarize, the potential K(z) is defined by equation (8) and (9), where the SU_3 symmetric interaction Hamiltonian $V = V_+ + (V_+)^+$ is given by equation (3). The breaking of SU_3 invariance can be introduced in the customary manner by using the physical masses for the particles.

The potential K(z) could in principle be used to calculate the masses of the baryon resonances.

Note added after receiving proofs. — The model described above has been used to calculate the mass of the Ω^- particle as a $K\Xi$ bound state. Using the same value $g^2/4\pi = 23{,}25$ as in I 1796 MeV was obtained for the Ω^- mass (L. B. Rédei, *Nuclear Physics*, B **17**, 1970, 38). The exotic K^+p scattering has also been treated (L. B. Rédei, Field theoretic model for low energy $J = 3/2^+K^+p$ scattering, to be published in *Phys. Rev.*).

REFERENCES

[1] L. B. RÉDEI, *Nuclear Physics*, B **10**, 1969, 419.
[2] G. F. CHEW and F. E. LOW, *Phys. Rev.*, **101**, 1956, 1570.
[3] J. KATZ, *Nuovo Cimento*, **58** A, 1968, 125 and J. KATZ and S. WAGNER, Desy report 69/16, April 1969.
[4] See reference (3) and also e. g. S. GASIOROWICZ, *Elementary particle physics*. John Wiley and Sons, Inc., New York, 1966, chapter 18.
[5] For a detailed justification of this, see L. VAN HOVE, N. M. HUGENHOLTZ and L. P. HOWLAND, *Quantum theory of Many particle systems*. W. A. Benjamin Inc., New York, 1961, p. 136.

(Manuscrit reçu le 26 janvier 1970).

Directeur de la publication : GUY DE DAMPIERRE.

IMPRIMÉ EN FRANCE

DÉPOT LÉGAL ÉD. N° 1710*a*. IMPRIMERIE BARNÉOUD S. A. LAVAL, N° 6111. 11-1970.