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L. HALPERN
B. JOUVET

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On the stimulated photon-graviton conversion by an electromagnetic field

by

L. HALPERN
(Institut H. Poincaré)

and

B. JOUVET
(Laboratoire de Physique Atomique, Collège de France).

ABSTRACT. — We study by exact calculation of convenient propagators the stimulated photon-graviton conversion in a photon field. We resolve thereby the difficulty of infrared divergences which occur in the perturbation expansion in the limit of low energy stimulating photons.

The solutions for various frequencies of the stimulating photons are compared with the results of the perturbation expansion.

I. — INTRODUCTION

The linearized equations of the gravitational field of the general theory of relativity [5], which are equivalent to those of a massless field of spin two [6] have been used extensively to obtain estimates of the rate of emission of gravitational radiation. The validity of their solutions as an approximation is today little disputed except for the case where the forces that determine the motion of the radiating bodies are themselves gravitational. The universality of the gravitational interaction, which is manifested in the principle of equivalence, implies that massless particles also interact with the gravitational field and thus in a way possess a gravitational charge.

Some of the problems which arise in connection with the emission of gravitational radiation by massless particles have found their solution. The emission of a photon by a massless electron for example gives rise to difficulties which were however shown not to exist in case of the emission of gravitational radiation [*7*]. The infrared divergence associated with the emission of soft gravitons was later treated by Weinberg [*8*] who showed that the same kind of procedure as in quantum electrodynamics can eliminate it. The interaction of two massless particles can in principle give rise to an infrared problem if either is emitted at very low energy. The case where not the graviton but one of the massless particles with which it interacts is emitted at very low energy has been examined in reference [*4*]. It is shown there that there occurs no infrared divergence if the massless particle is produced by a spontaneous transition. The small phase space available for a soft particle in this case makes also the transition probability small.

The transition can however be stimulated by the presence of other particles of the same kind if one deals with bosons (for example photons). This stimulation of the emission is the inverse of the process of absorption and gives rise to the same cross section as the latter. The transition probability increases therefore with the flux of the stimulating particles. The flux of massless particles like photons can be however very large without its energy density being considerable if the energy of each photon is low enough. There results in the case of the photon-graviton interaction contemplated, a divergence of the transition amplitude in the limit when the energy of the stimulating photons tends to zero [*4*]. We may call this an infrared divergence of the second kind.

The emission or absorption of a very soft photon by a massive electron field has no physical observable effect. The problem however is also in this case of formal interest [*9*]. The photon-graviton interaction as deduced from general relativity implies however that each emission or absorption of an infrared photon transmutes the massless particle from photon to graviton or vice versa. The cross section of this process is, in the perturbation expansion of the first order, proportional to $\kappa^2 n/q_0$. The constant κ^2 is 8π times Newton's gravitational constant, n is the density of stimulating photons per cm^3 and q_0 is the energy of the stimulating photon in units for which $\hbar = c = 1$.

The possibility of stimulating the process is of considerable interest for example to enhance the emission of gravitational bremsstrahlung which is associated with any photon emission or absorption, the decomposition of a graviton into two photons can also be enhanced by stimulation [*4*]. The removal of the divergences of the infrared cathastrophy of the second kind appears even more imperative as it leaves the question open whether photons and gravitons can at all be distinguished experimentally; indeed if the transition probability of the process of graviton-photon conversion is large an uncharged massive source could be found to interact with electromagnetic radiation as if it were gravitational radiation of the same helicity [*4*]. Such effects would in any case be very hard to detect so that they are not excluded by experiment.

We show in the present work that the divergences disappear in the propagators that include the interaction of photon and graviton with the field of the stimulating photons in all orders. Because of the transmutation by the external field from virtual bare photon to virtual bare graviton at each vertex one has to deal with propagators that are coupled to currents or to momentum densities or mixed propagators rather then with photon or graviton propagators. We also compare the results obtained using these propagators with those of the perturbation expansion. The agreement is found to depend on the frequency of the stimulating photons and also on the frequency width of the source. The perturbation expansion yields transition amplitudes that are clearly unphysical in the case of soft stimulating photons whereas those obtained with the complete propagators appear to be physical everywhere.

The gravitational interaction is in our approach only considered to all orders in the case of the interaction of the external field with photon and graviton. This approximation proves however sufficient to remove the difficulties encountered in the perturbation expansion. A completely covariant treatment is still beyond our capability and one obtains by the present approach even better insight into the problem which part of the interaction is responsible for the difficulties of perturbation theory.

II. — THE FIELDS AND THEIR INTERACTIONS

The interaction of the electromagnetic field with the gravitational field is deduced from the form of the covariant Lagrangian of these fields in the general theory of relativity. This Lagrangian is of the form :

$$\mathscr{L} = -\frac{1}{2\kappa^2}(-g)^{\frac{1}{2}} g^{rs}(\Gamma^a_{rb}\Gamma^b_{sa} - \Gamma^a_{rs}\Gamma^b_{ab}) - \frac{1}{4}(-g)^{\frac{1}{2}} g^{ru}g^{sv}F_{rs}F_{uv} \qquad (1)$$

with

$$\Gamma^c_{ab} = \frac{1}{2} g^{bn}(g_{na,c} + g_{nc,a} - g_{ac,n}) \qquad F_{ik} = A_{k,i} - A_{i,k}$$

$$\kappa = (8\pi G)^{\frac{1}{2}} = 8{,}1 . 10^{-33} \text{ cm}$$

(G is Newton's gravitational constant in units for which $\hbar = c = 1$).

The gravitational field is thus a symmetric tensor field $\phi^{ik}(x)$. Following Gupta [2] ϕ^{ik} is defined by:

$$(-g)^{\frac{1}{2}} g^{ik} = \eta_{ik} - \kappa\phi^{ik}$$

η_{ik} are the components of the Minkowskian metric tensor of the signature $(1, -1, -1, -1)$. $\mathscr{L}$ can be expanded in powers of κ by expressing the g_{ik} in terms of ϕ^{ik} and η_{ik}. A convenient method of performing such an expansion has been developed in reference [3]. We are in the present work mainly interested in the case of emission of one of the photons which participate in the reaction into a state in which a large number n of other photons are present. The matrix element of such a transition is proportional to $(n + 1)^{\frac{1}{2}}$ in the case of emission and to $(n)^{\frac{1}{2}}$ in the case of absorption. We shall in section IV consider all vertices of this kind; we shall however neglect all powers of κ higher than the first if κ is not multiplied by the large factor $(n + 1)^{\frac{1}{2}}$ or $n^{\frac{1}{2}}$. All graphs with vertices of the form in Fig. 1*a* are considered whereas graphs of the form of Fig. 1*b* and Fig. 1*c* with

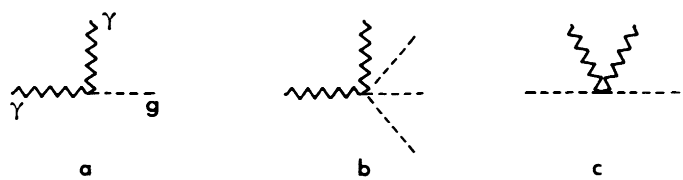

FIG. 1.

several gravitons are neglected. Wavy lines in the graphs denote photons and dotted lines gravitons. A vertical wavy line denotes one of the n photons which stimulate the transition. One may approximate the effect of these n photons by that of an external field $f_{ik}(x)$.

The Lagrangian of the interaction of the gravitational field with any boson field is in the linear approximation in κ of the general form:

$$\mathscr{L}_I = -\frac{1}{2}\kappa\phi^{ik}(x)\left[T^{ik}(x) - \frac{1}{2}\eta^{ik}T^n{}_n(x)\right] \qquad (1a)$$

T_{ik} denotes the symmetrized momentum tensor derived from the Lagrangian $\mathscr{L}_M$ of the boson field by variation with respect to the metric tensor:

$$T^{ik} = - \lim_{g_{mn} \to \eta_{mn}} \left(\frac{\delta \mathscr{L}_M}{\delta g_{ik}} + \frac{\delta \mathscr{L}_M}{\delta g_{ki}} \right) \tag{2}$$

in the case of the electromagnetic field:

$$T_{ik} = F_{in} F^{n}{}_{k} - \eta_{ik} \frac{1}{4} F_{mn} F^{nm}. \tag{2a}$$

The Lagrangian of the gravitational field becomes in the approximation of lowest order:

$$\mathscr{L}_G = \frac{1}{8} \left(\phi^{rs,t} \phi_{rs,t} - 2\phi^{rs,t} \phi_{rt,s} - \frac{1}{2} \phi^{,t} \phi_{,t} \right), \ (\phi = \phi^{n}{}_{n}). \tag{1b}$$

This is the Lagrangian of a symmetric tensor field corresponding to a particle of spin two and vanishing rest mass [6]. Indices are raised and lowered by the η_{ik} so that one can work in this approach like with a Lorentz covariant theory in flat space. The effect of the curvature of space is introduced by the interaction.

The Lagrangian $\mathscr{L}_G$ admits gravitational gauge transformations of the form:

$$\phi^{rs}(x) \to \phi^{rs}(x) + \Lambda^{r,s} + \Lambda^{s,r} - \Lambda^{n}{}_{,n} \eta^{rs} \tag{1c}$$

where $\Lambda^n(x)$ is a gauge vector field in analogy to the scalar gauge field of electrodynamics. A g-gauge is suitably chosen for which everywhere $\phi^{ik}{}_{,k} = 0$. This corresponds to the De Donder condition in the complete covariant theory.

The commutation relations for the free gravitational fields are obtained by imposing the additional subsidiary condition: $\phi^{n}{}_{n} = \phi$ and considering ϕ as an independent variable [1]. One finds:

$$[\phi^{rs}(x), \phi^{uv}(x')] = - i(\eta^{ru}\eta^{sv} + \eta^{rv}\eta^{su}) \, D(x - x') \tag{1d}$$

The admissible states have to fulfill the conditions:

$$\phi^{ik}{}_{,k}^{+} |\rangle = 0 \quad \text{and} \quad (\phi^{n}{}_{n}{}^{+} - \phi^{+}) |\rangle = 0 \tag{1e}$$

The difficulties which arise from g-gauge invariance and the definition of the vacuum were overcome by Gupta [1] in analogy to quantum electrodynamics with the help of an indefinite metric.

We introduce also external c-number currents $j_n(x)$ and momentum densities $\tau_{ik}(x)$ that interact with the photon and graviton fields. The gravitational interaction is universal so that the momentum density of every field involved in a process is coupled to the gravitational field. To be consistent one has thus to take into account that the charged fields which give rise to the c-number current j_n have a momentum density τ_M^{ik} by themselves. The interaction of the current j with the photon field contributes also with a momentum density τ_I^{ik} the latter is in the case of the minimal electromagnetic interaction of the form: $\tau_{Iik} = (j_i A_k + j_k A_i)$. The emission of a photon is thus always associated with gravitational radiation even then if the transition causing the emission of the photon is of the dipole type with $\Delta J = \pm 1$. The source of the gravitational radiation is the momentum tensor of the complete system: photon + interaction + atom (atom = external charged source) $T^{ik}_{\text{total}} = T^{ik}_{(\text{photon})} + \tau_I^{ik} + \tau_M^{ik}$. Only the sum of these tensor densities is in general conserved: $T^{ik}_{\text{total},k} = 0$. The conservation of the momentum tensor is however essential for the invariance of the result with respect to the g-gauge transformation (1 c). This is in analogy to quantum electrodynamics where the conservation of the current $j^k_{,k} = 0$ guarantees e-gauge invariance. The term $\tau_I + \tau_M$ cannot be separated if one assumes the minimal electromagnetic interaction which is introduced by replacing: $i\partial_k \to i\partial_k + eA_k$ because they are related by electromagnetic gauge transformations. For example in case of an electron:

$$\tau_{Mik} + \tau_{Iik} = \overline{\Psi}_1 \left(\frac{i}{4}(\gamma_k \overleftrightarrow{\nabla}_i + \gamma_i \overleftrightarrow{\nabla}_k) + \frac{1}{2}\gamma_k A_i + \frac{1}{2}\gamma_i A_k \right) \Psi_2$$

with

$$\overline{\Psi} \overleftrightarrow{\nabla} \Psi = \overline{\Psi} \Psi_{,i} - \overline{\Psi}_{,i} \Psi$$

($\Psi_{1,2}$ are the wavefunctions of the states between which the transition occurs).

The introduction of the Coulomb gauge for which $A_0 = 0$ may eliminate the contribution to the matrix element of the emission of gravitational radiation of the term τ_I because only the multipole moments of τ_{00} are significant here. There is however not a single case where we have a precise knowledge of τ_M. We may assume that it is small in case of an energetic spontaneous transition of $\Delta J = 1$ in case of large energy difference between the levels. The transition probability of a process where two quanta are emitted depends on the separation of the energy levels which occur virtually in the matrix element. The perturbation of the system by an external field may however alter the states in such a way that even if the above assumptions are true the smallness of the contribution of τ_M is questionable.

These difficulties are avoided in a model where the electromagnetic transition is caused by a gauge invariant interaction of the type $\sigma_{\mu\nu}F^{\mu\nu}$ between the electromagnetic field and neutral particles as the ϕ_0-meson and the π_0-meson. This interaction is of the form:

$$\mathcal{L}_I = \mu\pi_0\varepsilon^{abcd}(B_{a,b} - B_{b,}{}^{a})F_{cd} \tag{3}$$

where $\pi_0(x)$ is the neutral pseudo-scalar field and $B_n(x)$ is the neutral vector field. Such an interaction gives even no contribution to τ_1 because ε^{abcd} is a constant tensor density also in the curved space of general relativity so that its variation with respect to g_{ik} vanishes. The contribution of τ_M becomes in this case small if the emitted gravitational quantum is energetic because one of the two neutral particles is either before or after the emission of the photon not on the mass shell (Fig. 2). The process considered can

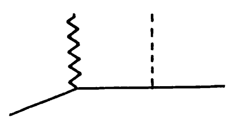

FIG. 2.

only be separated from the production mechanism if the life time of the vector meson is large compared to the inverse of the energy of the gravitational quantum; otherwise the gravitational quantum may be emitted by the producing particles of the ϕ-meson which is created in a virtual state and decays to a real π-meson and a photon.

We have seen that every electromagnetic transition even a dipole transition is associated with the emission of gravitational radiation as a kind of Bremsstrahlung. For example the exchange of a $J = 1$ quantum between two sources may not only occur via the exchange of one photon but even

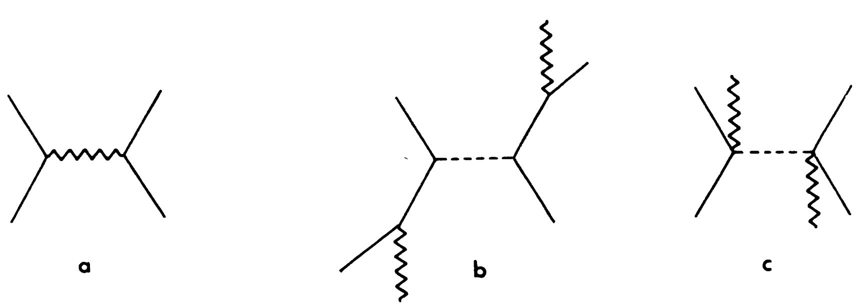

FIG. 3.

via a graviton if each source emmits or absorbs one photon of the external field. Some of these processes are represented graphically in Fig. 3 *a*, *b*. In case of the minimal electromagnetic interaction also the graph of the type of Fig. 3 *c* has to be considered. The matrix elements of all these graphs have to be added.

A gravitational quantum can decompose by the interaction (1 *a*) into two photons (Fig. 4 *a*). Also this process may be stimulated by the presence of other photons. The exchange of a J = 2 quantum between sources in an external field can occur via a graviton ; it may also occur via a photon (Fig. 4 *b*) if the sources have electromagnetic properties.

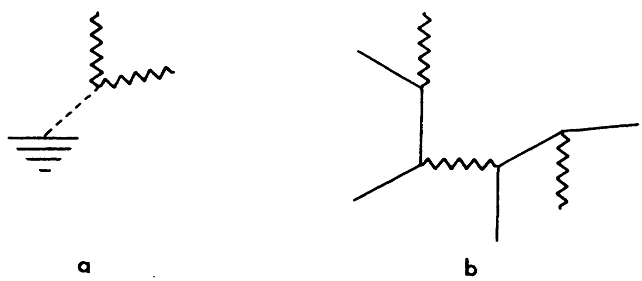

FIG. 4.

III. — THE DIVERGENCE OF THE PERTURBATION EXPANSION

The virtual emission by a current j_n of a photon and the subsequent absorption by a momentum density T_{ik} of the converted graviton is in the first order in κ of the perturbation expansion:

$$M = \frac{i(2\pi)^{-4}\kappa}{k^2(k \pm q)^2}\{ 2j^n(k)T^*_{nu}(-k \mp q)f^{um}k_m \mp 2j^n f_n{}^u T^*_{uv} q^v - j_n f^{nm} k_m T^{*s}{}_s \} \qquad (1)$$

$$f_{ik} = (a_k q_i - a_i q_k)(e^{iqx} + e^{-iqx})$$

The conservation of the current: $j^n(k)k_n$ and of the momentum density:

$$T^{*mn}(-k \mp q)(k_n \pm q_n) = 0$$

have been taken into account in deriving this expression. The virtual photon is quasi real near the pole $k^2 = 0$. The product of the matrix element of free photon emission and of absorption of the converted graviton × current × momentum density is $i\pi$ times the residuum at this pole (see Fig. 5 *a*).

Due to the decomposition:

$$D^f(x) = \lim_{\varepsilon \to 0} - (2\pi)^{-4} \int \frac{e^{-ikx}}{k^2 + i\varepsilon} d^4k = (2\pi)^{-4} \int \left[i\pi\delta(k^2) - \mathscr{P}\left(\frac{1}{k^2}\right) \right] e^{-ikx} d^4k$$

one obtains:

$$\underset{(k^2=0)}{M} = - \frac{(2\pi)^{-3}\kappa}{8k_0(qk)} \{ 2j^n T^*_{nu} f^{um} k_m \mp 2j_n f^{nu} T^*_{uv} q^v - j_n f^{nm} k_m T^{*s}{}_s \} \qquad (1\,a)$$

This expression diverges in the limit of vanishing q if f_{ik} remains different from zero. The results of section V of reference [4] shows that such a

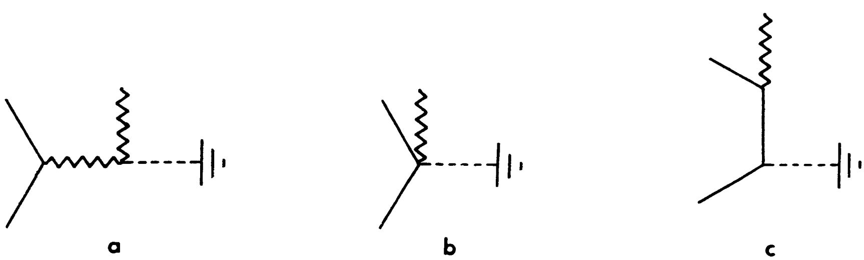

FIG. 5.

divergence does not occur if the photon of momentum q is emitted spontaneously instead of being stimulated by the photon field f_{ik}. The square of the matrix element is there multiplied by the phase space factor d^3q so that the transition probability tends to zero in the limit $q \to 0$. Reference [4] also shows that in the case of the stimulated emission of the photon of momentum q the factor in the numerator is $n(q)d^3q$. $n(q)$ denotes the spatial density of photons of momentum q per frequency interval. The numerator does not in this case tend to zero for small q if the stimulating photons are within this narrow frequency interval. The transition probability diverges therefore because of the denominator (qk). One arrives at the equs (1 and 1 *a*) if one approximates the stimulating photons by an external field f_{ik}.

The analysis of the interaction in section II shows that the matrix elements of Fig. 5 *b*, *c* have to be added to those of Fig. 5 *a* in case of the minimal electromagnetic interaction. The sum of the contributions from $\tau_M + \tau_I$ is *e*-gauge invariant but not that from each term. τ_M is however not known in detail so that the complete expression cannot be calculated ;

it depends strongly on the particular choice of the source. The cancelation of the divergence of eq. (1 *a*) due to the effect of a weak external field on the photon-graviton transition of the source appears unlikely.

The interaction $\sigma_{\mu\nu}F^{\mu\nu}$ considered in section II (eq. II.3) admits no graphs like Fig. 5 *b*. The graph in Fig. 5 *c* where the graviton is not emitted by the photon but by the neutral particles, gives small contributions for large transition energies k_0 and can then be neglected. The divergence of the graph of Fig. 5 *a* occurs also for this interaction. The derivatives of the field B_n and of the propagator introduce factors p and k in the numerator which do not compensate for the smallness of (qk) in the denominator.

1. The energy density of a field of a given average spatial density of n photons, decreases with the wavelength of these photons. A wavepacket composed of frequencies of the order of q_0 must cover a volume of at least q_0^{-3} cm^3. The total energy of such a wavepacket of average spatial density of n quanta is therefore proportional to q_0^{-2} and diverges in the limit $q \to 0$.

The transition probability of the process in Fig. 5 *a* is determined by the magnitude of $\kappa^2 n/q_0$. This can be infered from eq. (1 *a*). It has been derived in detail in reference [*4*]. This quantity reaches unity for $n = q_0/\kappa^2$. A wave packet of the dimensions of one wavelength attains the Schwarzschild radius due to its own mass at such a spatial density n. The transition probability of the process becomes thus important if the total mass of the external photon field produces a considerable curvature of space. This curvature need not to be observable locally; it may however affect the propagators of the fields so that eq. (1 *a*) is no longer valid in the case of a closed universe whose energy exists mainly in the form of soft electromagnetic radiation.

Arguments of such a nature were also presented in the discussions of the classical infrared problem before it was discovered that its solution emerged from the structure of the theory itself. We try to show in the following that even the present divergence can be removed within the framework of a simplified approach where only the vertices of the graphs in Fig. 1 *a*, are considered to all orders and vertices which have not as many external field lines as graviton lines (Fig. 1 *b*) are considered only to the first order in κ. This appears satisfactory because the process considered can cause the conversion of photons to gravitons even if the mass of the wavepacket is not too large, if only a sufficiently large number of photon emissions and absorptions by the current j occur [*4*].

2. The cancelation of the divergence of eq. (1 *a*) in the limit $q \to 0$ due to other graphs as it occurs in quantum electrodynamics is excluded in

the present case. The infrared divergence of quantum electrodynamics is removed by the sumation of graphs which are of even order in the coupling constant and of graphs which are of odd order in the coupling constant. In the present model every vertex with a line of the external photon field contributes one power of κ and converts a photon to a graviton. The square of the matrix elements which are of even powers in κ cannot be added to the square of those which are of odd powers in κ if the two types of particles are distinguishable experimentally by their interaction with the sources.

IV. — THE COMPLETE PROPAGATORS

The conversion of photon to graviton by the stimulating photon field results in relations between the propagators of the photon-graviton system in the external field. Some of these relations are shown graphically in Fig. 6 *a*, *b*, *c*. We call the complete propagators which include all the

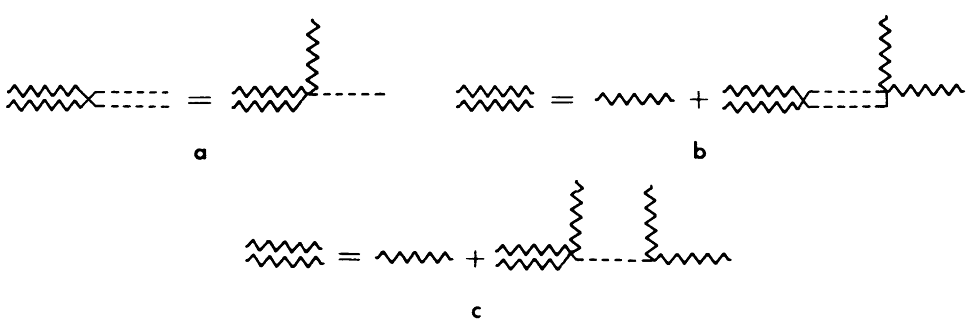

FIG. 6.

vertices to which the stimulating field contributes *v-v*, *v-t*, *t-v*, *t-t*-propagators depending on their vector or tensor character. They are drawn in Fig. 6 as double lines. Fig. 6 *a* illustrates the relation of the *v-v*-propagator to the *v-t*-propagator. The latter represents the emission of a particle coupled to a current and the absorption of a particle coupled to the momentum tensor. Fig. 6 *b* illustrates the relation between the bare photon propagator the *v-t*-propagator and the *v-v*-propagator. Fig. 6 *c* illustrates how the graphs of Fig. 6 *a*, *b* can be combined to obtain a relation (a system of integral equations) for the *v-v*-propagator $G_i(x, y)_k$.

The equations which correspond to the graph of Fig. 6 *a* are

$$G_i(x,z)^{rs} = -\kappa \int \left\{ [G_i(x,x')_{n,a} - G_i(x,x')_{a,n}] f^n{}_b(x') - \frac{1}{2}\eta_{ab} G_i(x,x')_{n,m} f^{nm}(x') \right\} \cdot P^{arbs} D^f(x'-z) dx' \quad (1)$$

those which correspond to Fig. 6 *b* are

$$G_i(x,y)_k = \eta_{ik} D^f(x-y) - \kappa \int G_i(x,z)^{rs} \left\{ f_r{}^n(z)[D^f{}_{,n}(z-y)\eta_{sk} - D^f{}_{,s}(z-y)\eta_{nk}] - \frac{1}{2}\eta_{rs} D^f{}_{,n}(z-y) f_k{}^n(z) \right\} dz \quad (2)$$

The bare graviton propagator is assumed to be:

$$P^{arbs} D^f(x-y) = -(\eta^{ar}\eta^{bs} + \eta^{as}\eta^{br}) D^f(x-y)$$

The combination of equs. (1) and (2) results in an integral equation for the v-v-propagator

$$G_i(x,y)_k = \eta_{ik} D^f(x-y) + \kappa^2 \int \left\{ [G_i(x,x')_{n,a} - G_i(x,x')_{a,n}] f^n{}_b(x') - \frac{1}{2}\eta_{ab} G_i(x,x')_{n,m} f^{nm}(x') \right\} D^f(x'-z)(\eta^{ar}\eta^{bs} + \eta^{as}\eta^{br}) \left\{ f_r{}^n(z) \cdot \left[-D^f{}_{,n}(z-y)\eta_{sk} + D^f{}_{,s}(z-y)\eta_{nk} \right] + \frac{1}{2}\eta_{rs} D^f{}_{,n}(z-y)\eta_{km} f^{mn}(z) \right\} dx'dz \quad (3)$$

This equation corresponds to Fig. 6 *c*. It is solved here only for the special case of the field of a monochromatic plane wave

$$f_{ik}(x) = f_{ik}(e^{-iqx} + e^{iqx}) \qquad f_{ik} = f(a_k q_i - a_i q_k) \quad \text{with} \quad a_n a^n = -1$$

Propagators exist for all possible numbers n, m of photons of the stimulating field in the initial and the final state. For example:

$$G_i(x,y)_k = \frac{(m|T(A_i(x)A_k(y)S|n)}{(m|S|n)}$$

Propagators of equal n and different m have to be distinguished in the matrix elements (but not necessarily in the transition probability if the energy resolution of the experiment performed is not sharp enough). The propagators that leave the number of photons unaltered ($n = m$) we call free propagators. Equ. (3) expressed in momentum space becomes in this case

$$G_i(k,k)_k = -\eta_{ik}/k^2 - \frac{2\kappa^2}{(k^2)^2 - 4(qk)^2} G_i(k,k)_c$$

$$\{f^{cn}Q_n k_k - k^c Q^n f_{nk} + \delta^c_k Q^2 - k^2 f^{cn} f_{nk}\} \quad (3\,a)$$

$$Q_n = f_{nm} k^m$$

The solution of equ. (3 *a*) is found with the help of the relations

$$G_i(k,k)_n(k^n, q^n, f^{nm}Q_m) = -\frac{1}{k^2}(k_i, q_i, f_{in}Q^n)$$

and (4)

$$f_{in}f^n{}_k = f^2 q_i q_k, \qquad f_{in}Q^n = f^2 q_i(qk), \qquad Q^2 = -f^2(qk)^2$$

The free *v*-*v*-propaga'or is

$$G_i(k,k)_k = \frac{1}{k^2\left[k^4 - 4(qk)^2\left(1+\frac{1}{2}f^2\kappa^2\right)\right]}$$

$$\{-\eta_{ik}[k^4 - 4(qk)^2] + 2f^2\kappa^2[-k^2 q_i q_k + (qk)(q_i k_k + q_k k_i)]\} \quad (5)$$

with

$$k^4 = (k^2)^2$$

The last two terms in the second bracket give no contribution because of current conservation.

Expressed in momentum space eq. (1) for the *v*-*t*-propagator with

$m = n \pm 1$ is

$$G_i(k, k \pm q)^{rs} = -\frac{i\kappa}{(k \pm q)^2}$$

$$\{G_i(k,k)_n(f^{nr}k^s + f^{ns}k^r) + G_i(k,k)^r Q^s + G_i(k,k)^s Q^r - G_i(k,k)_n Q^n \eta^{rs}\} \quad (1\,a)$$

With the help of the relations

$$G_i(k,k)_n Q^n = -Q_i \frac{k^4 - 4(qk)^2}{k^2\left[k^4 - 4(qk)^2\left(1+\frac{1}{2}f^2\kappa^2\right)\right]}$$

(4 *a*)

$$G_i(k,k)_n f^{na} = \frac{-f_i^a[k^4 - 4(qk)^2] - 2f^2\kappa^2(qk)q_i Q^a}{k^2\left[k^4 - 4(qk)^2\left(1+\frac{1}{2}f^2\kappa^2\right)\right]}$$

one finds the v-t-propagator

$$G_i(k, k \pm q)^{rs} = \frac{i\kappa}{k^2\left[k^4 - 4(qk)^2\left(1 + \frac{1}{2}f^2\kappa^2\right)\right]}\Big\{(k \mp q)^2[f_i{}^r k^s + f_i{}^s k^r - \eta^{rs}Q_i$$

$$+ \delta_i^r Q^s + \delta_i^s Q^r] + \frac{2f^2\kappa^2}{(k \pm q)^2}[-(qk)k_i(q^r Q^s + q^s Q^r) + k^2 q_i(q^r Q^s + q^s Q^r)]\Big\}. \quad (6)$$

The t-t-propagator is found from $G_i(k \mp q, k)^{rs}$ by joining another vertex. The evaluation is much more complicated in this case and we do not need it here.

V. — THE PROPAGATION OF THE REAL PARTICLES

The plane wave solutions that correspond to free particles are in general associated to poles of the propagators. The poles of the free v-v-propagator $G_i(k, k)_k$ are determined by the equations $k^2 = 0$ and

$$\left[k \pm q\left(1 + \frac{1}{2}f^2\kappa^2\right)^{\frac{1}{2}}\right]^2 = 0$$

The pole due to k^{-2} is situated at the points: $\pm \bar{k} = k_0$ where $\bar{k}$ is the absolute value of the spacelike part of k. The residue of the pole for which $k_0 = \bar{k}$ is

$$\operatorname*{Res}_{k^2=0} G_i(k, k)_k = \frac{-\eta_{ik}}{2k_0 b^2} \qquad b = \left(1 + \frac{1}{2}f^2\kappa^2\right)^{\frac{1}{2}} \quad (1)$$

The displaced poles due to $(k \pm qb)^{-2}$ are situated at

$$k_0 = \mp \frac{1}{2}bq_0(\pm)(b^2q_0^2/4 + \bar{k}^2 \pm 2q_0 b|\bar{k}| \cos\alpha)^{\frac{1}{2}} \qquad \alpha = \sphericalangle(\bar{k}\bar{q})$$

The signs in the brackets are those of the square root whereas the free signs correspond to the point $k \pm qb$ chosen. The residues of these poles are

$$\operatorname*{Res}_{(k \pm qb)^2=0} G_i(k, k)_k = -\frac{\eta_{ik}f^2\kappa^2}{2k_0 4b^2} \pm \frac{\kappa^2 f^2 q_i q_k}{2k_0 4(qk)b} \quad (1\ a)$$

k_0 is here the root of the equation $(k \pm qb)^2 = 0$. $E = f^2q_0^2$ is the energy density of the inducing photon field. With today's lasers it may reach values up to $f^2q_0^2 \approx 10^{28}\ \mathrm{cm}^{-4}$. The quantity $f^2q_0^2$ is therefore in empirical cases extremely small and consequently the second term of eq. (1 *a*) is negligible. There is also no contribution if $\bar{q} \parallel \bar{k}$ because then also $(qj) = 0$. The quantity $\kappa^2 f^2$ may however become large in the limit $q \to 0$ if E/q_0^2 is large enough. The contribution of the pole $k^2 = 0$ eq. (1) vanishes in such a case whereas the first term of eq. (1 *a*) for each pole has the value $-\eta_{ik}/(4k_0b^2)$. If the energy density E is different from zero then κf and $b = \left(1 + \frac{1}{2}\kappa^2 f^2\right)^{\frac{1}{2}}$ diverge in the limit $q \to 0$ yet $qb \to q\kappa\mathrm{E}^{\frac{1}{2}}/q_0$ which remains small but different from for empirical values of $\mathrm{E} \neq 0$. The displacement of the poles of $(k \pm qb)^{-2}$ with respect to those of k^{-2} is thus small but nonvanishing in this case. Apart from this tiny displacement and apart from the very small second term of eq. (1 *a*), the propagator which results from the sum of the two displaced poles is in the limit equal to that of the free photon.

The *v-t*-propagator which serves to calculate the emission of a photon followed by the absorption of a graviton has an additional pole at $(k \pm q)^2 = 0$. Only one of these two particles can be on the mass shell unless the transition amplitude vanishes as in the case of $\bar{k} \parallel \bar{q}$. The pole at k^{-2} is chosen if the photon is on the mass shell and the pole of $(k \pm q)^{-2} = 0$ if the graviton is on the mass shell.

The residue of the pole of k^{-2} is

$$\operatorname*{Res}_{k^2=0} \mathrm{G}_i(k, k+q)^{rs} = \frac{i\kappa}{2k_0 2(qk)b^2}\left(-f_i{}^r q^s - f_i{}^s q^r - \eta^{rs}\mathrm{Q}_i + \delta_i^r \mathrm{Q}^s + \delta_i^s \mathrm{Q}^r\right) \tag{2}$$

The terms which vanish because of current and momentum conservation have again been omitted. The residues of the poles of $(k \pm qb)^{-2}$ are

$$\begin{aligned} \operatorname*{Res}_{(k \pm qb)^2=0} \mathrm{G}_i(k, k+q)^{rs} &= \frac{i\kappa}{2k_0(qk)4b^2}\Big\{-(1 \pm b)\left(-f_i{}^r q^s - f_i{}^s q^r - \eta^{rs}\mathrm{Q}_i + \delta_i^r \mathrm{Q}^s + \delta_i^s \mathrm{Q}^r\right) \\ &\mp \frac{2f^2\kappa^2 b}{(1 \mp b)(qk)} q_i(q^r\mathrm{Q}^s + q^s\mathrm{Q}^r)\Big\}. \end{aligned} \tag{2 a}$$

We have shown before that if $\mathrm{E} \neq 0$ the quantity b diverges in the limit $q \to 0$ whereas qb remains finite. The term $(qk)b^2$ in the denominator of eq. (2) then diverges so that the pole of k^{-2} gives no contribution in this case.

$$\lim_{q \to 0} \operatorname*{Res}_{k_2=0} \mathrm{G}_i(k, k+q)^{rs} = 0 \tag{2'}$$

The first term in the paranthesis of eq. (2 *a*) has a factor $(1 \pm b)$ so that it remains finite in the limit

$$\lim_{q\to 0} \operatorname*{Res}_{(k\pm qb)^2=0} G_i(k, k+q)^{rs} = \frac{\mp i\kappa}{2k_0 2(qk)b}\frac{1}{2}(-f_i{}^r q^s - f_i{}^s q^r - \eta^{rs}Q_i + \delta_i^r Q^s + \delta_i^s Q^r) \quad (2'\,a)$$

The residues of these two poles are in this limit however of opposite sign so that their sum will give no contribution unless the resolution of the source is sharp enough to distinguish the two frequencies $(k \pm qb)$. We have shown that qb can remain finite in the limit.

The contribution of eq. (2) increases with q if E remains unchanged.

The values of κf of the electromagnetic radiation that can be produced at present in terrestrial laboratories is very small. One is thus near the limit $b \to 1$. The residue of the pole of k^{-2} is in this limit

$$\lim_{b\to 1} \operatorname*{Res}_{k^2=0} G_i(k, k+q)^{rs} = \frac{i\kappa}{2k_0 2(qk)}(-f_i{}^r q^s - f_i{}^s q^r - \eta^{rs}Q_i + \delta_i^r Q^s + \delta_i^s Q^r). \quad (2'')$$

The residue of the pole of $(k + qb)^{-2}$ is

$$\lim_{b\to 1} \operatorname*{Res}_{(k+qb)^2=0} G_i(k, k+q)^{rs} = \frac{i\kappa}{2k_0(qk)}\left\{-\frac{1}{2}(-f_i{}^r q^s - f_i{}^s q^r - \eta^{rs}Q_i + \delta_i^r Q^s + \delta_i^s Q^r) + \frac{2}{(qk)}q_i(q^r Q^s + q^s Q^r)\right\} \quad (2''\,a)$$

The first term in the paranthesis of eq. (2″ *a*) is of equal value and of opposite sign to that of eq. (2″). These terms give thus only a contribution to the transition amplitude if the frequency width of the source is smaller than q_0. The second term of eq. (2″ *a*) is only by a factor of the order of q_0/k_0 smaller than the first term. There is however no term of opposite sign to compensate it at the other poles.

The residue of the pole of $(k - qb)^{-2}$ is small in the limit $b \to 1$ for the propagator $G_i(k, k+q)^{rs}$. The contributions from the two poles of $(k \pm qb)^{-2}$ are exchanged if $k + q$ is replaced by $k - q$. The transition probability vanishes in the limit of $\bar{k} \parallel \bar{q}$ because of momentum conservation and the opposite sign of the equal terms of the poles in equation (2) and (2 *a*).

The properties discussed in this section remain valid independent of whether the v-propagator is coupled to the source by the minimal interaction or by an interaction of the form $\sigma_{ik}F^{ik}$. The derivatives of the propagator in the latter case introduce only an additional factor k in the numerator which does not alter appreciably the results in the limits discussed.

VI. — CONCLUSIONS

Propagators have been obtained which include the interaction of photon and graviton with an external electromagnetic field in all orders. The application of these propagators modifies the results that are obtained by the perturbation expansion of the first order in the external field. The perturbation expansion gives rise to multiple poles of the photon-graviton transition amplitude in the limit of vanishing momentum q of the external photon field and results, consequently, in divergent amplitudes. The poles of the complete propagators which we obtained are displaced so that no divergences occur. The displacement of the poles in the limit considered is of the order of $\kappa E^{\frac{1}{2}}$ where E is the energy density of the external field and $\kappa = 8{,}1 . 10^{-33}$ cm.

The comparison of the results with the perturbation expansion is instructive. The magnitude of the transition amplitudes become comparable if $q_0 \gg \kappa E^{\frac{1}{2}}$. The residua of some of the poles which contribute nearly equally in this region are however of opposite sign. They nearly cancel each other in the transition amplitude if the frequency width of the source is not smaller than the displacement of the poles. The results of perturbation theory are of the correct order of magnitude if the frequency resolution of the source is narrower than q_0.

The external electromagnetic field enhances the emission of gravitational bremsstrahlung by photons and modifies slightly the propagation of electromagnetic radiation. Such effects are however below the threshold of observability in all empirically known cases.

The exchange of a particle of spin 1 by sources with electromagnetic properties involves not only the photon propagator but also the graviton propagators. The contributions of the latter cannot be determined in case of the minimal electromagnetic interaction without a detailed knowledge of the internal energy distribution of the radiating system. A simplified model has for this reason been introduced, where the electromagnetic interaction is of the form $\sigma_{ik}F^{ik}$ and where the contribution of the gravitational term is shown to be small.

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