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Pure radiation fields in general relativity

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A METHOD FOR GENERATING PURE RADIATION FIELDS FROM EMPTY GRAVITATIONAL FIELDS IS GIVEN

The object of this investigation is to obtain a method for generating pure radiation fields from empty gravitational fields. Such methods were obtained earlier by Ehlers [*1*], Bonnor [*2*] and the present author [*3*] in other cases.

For this purpose we consider the three metrics

$$ds^2 = e^{2\sigma} d{*}s^2 - \bar{e}^{2\sigma}(dx^3)^2, \tag{1}$$

$$d\bar{s}^2 = e^{2W} d{*}s^2 - \bar{e}^{2W}(dx^3)^2, \tag{2}$$

$$d{*}s^2 = {*}g_{ij}dx^i dx^j \tag{3}$$

$$(i, j, \ldots = 1, 2, 4;\ \alpha, \beta, \ldots = 1, 2, 3, 4)$$

where σ, W and ${*}g_{ij}$ are functions of x^i only.

Hereafter quantities defined with respect to (2) and (3) will be denoted by an overhead bar or an arterisk respectively.

The Ricci tensor for the metric (1) is given by

$$R_{ij} = {*}R_{ij} + 2\sigma_i\sigma_j + {*}g_{ij}{*}\Delta_2\sigma \tag{4}$$

$$R_{33} = \bar{e}^{4\sigma}{*}\Delta_2\sigma, \tag{5}$$

$$R_{3i} = 0, \tag{6}$$

where

(7) $$\sigma_i = \frac{\partial \sigma}{\partial x^i},$$

and $^*\Delta_2$ is the Beltrami differential parameter of the second kind.

The empty space-time field equations for the metric (1) are therefore

(8) $$^*R_{ij} + 2\sigma_i\sigma_j = 0,$$

(9) $$^*\Delta_2\sigma = 0,$$

and since (9) follows from (8) in view of the contracted Bianchi identities, we are left with only equations (8) for determining σ and $^*g_{ij}$.

Now the Ricci tensor for the metric (2) is given by

(10) $$\bar{R}_{ij} = {}^*R_{ij} + 2W_iW_j + {}^*g_{ij}{}^*\Delta_2 W,$$

(11) $$\bar{R}_{33} = \bar{e}^{4W*}\Delta_2 W,$$

(12) $$\bar{R}_{3i} = 0,$$

where

(13) $$W_i = \frac{\partial W}{\partial x_i}.$$

If we assume that W is a function of σ so that the level surfaces of W and σ coincide, then

(14) $$W_i = W'\sigma_i, \qquad W_{i;j} = W'\sigma_{i;j} + W''\sigma_i\sigma_j, \ \ldots$$

where an overhead dash denotes ordinary differentiation with respect to σ and a semicolon followed by a lower index denotes covariant differentiation with respect to the metric (3).

From equations (11) and (14) we get

(15) $$\bar{R}_{33} = \bar{e}^{4W}W''{}^*g^{ij}\sigma_i\sigma_j,$$

in view of (9). And if we assume that σ_i is a null vector then

(16) $$\bar{R}_{33} = 0.$$

Also from (10), (14), (8) and (9) and the fact that σ_i is a null vector we get

(17) $$\bar{R}_{ij} = 2(W'^2 - 1)\sigma_i\sigma_j.$$

Thus the Ricci tensor for the metric (2), in veiw of (12), (16) and (17), can be written as

(18) $$\bar{R}_{\alpha\beta} = \theta\sigma_\alpha\sigma_\beta,$$

where

(19) $$\theta = 2(W^{12} - 1),$$

and $\sigma\beta$ is a null vector.

Now equations (18) are precisely the field equations for the unidirectional flow of pure radiation for the metric (2). Hence we have arrived at the following reseult:

For every solution of the empty space-time field equations corresponding to the metric (1) a solution of the field equations for the unidirectional flow of pure radiation, i. e. (18), is given by the metric (2) where W is an arbitrary function of σ and the gradient of σ is null vector.

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