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Gravitational radiation and the arrow of time in cosmology

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ABSTRACT. — The propagation of type N gravitational radiation in steady state and Einstein-de Sitter cosmologies is considered. It is shown that full retarded solutions are consistent in steady state cosmology and full advanced solutions in Einstein-de Sitter cosmology. Full advanced solutions are not consistent in the former and full retarded solutions in the latter.

1. — INTRODUCTION

All known basic laws of physics are time symmetrical. This means that corresponding to the existing physical systems one can also obtain others by making the time run backwards. Yet inspite of this symmetry there are certain phenomena indicating the lack of reversibility of time. For instance, though, Maxwell's equations for electrodynamics admit retarded and advanced solutions in perfect symmetry, in practice only the former are considered since they correspond to reality while the latter are rejected on the grounds that they affect causality. In thermodynamics the entropy of a physical system increases along the positive direction of time. Therefore, the question arises: if the physical theories are insensitive to the direction of time, how does it come about that Nature favours the retarded fields and thereby defines an arrow of time ?

Gold [2] argued on general grounds that in a universe in which all physical theories are time symmetrical, the arrow of time must ultimately be associated with the large scale properties of the Universe. The expansion of the Universe provides an arrow of time. It is then interesting to find how this arrow of time is related to the local sense of time determined by the irreversible processes.

In a nonstatic universe the future and past absorbers (the particles in the Universe lying in the future light cone of the source are called by the collective name of « future absorber » while the particles on the past light cone of the source constitute the « past absorber ») are entirely different and also their interaction with radiation. Hogarth [3] found a necessary and sufficient condition for Maxwell's time-symmetric electrodynamics to be compatible with observational electrodynamics and he has shown that an electromagnetic fully retarded solution is self-consistent in steady state cosmology but not in Einstein-de Sitter's while the situation is reversed with advanced solutions. Very recently Hoyle and Narlikar [5] in a more subtle analysis of the same problem arrived at similar conclusions.

In the present paper the propagation of type N gravitational radiation through Einstein-de Sitter and steady state cosmological models is studied. To define gravitational radiation in nonempty space-time, the statement of Pirani [10], « ... a space-time with matter and a type N or type III Weyl tensor is pervaded by gravitational radiation » is used. It has been found that the propagation of full retarded radiation is self-consistent in steady state model but not in Einstein-de Sitter's and with full advanced radiation the situation is reversed.

2. — CONSTRUCTION OF THE METRIC FORM AND FIELD EQUATIONS

In a previous paper (Krishna Rao [6]), a class of exact wave solutions of the Einstein-Rosen [1] cylindrically symmetric space-time corresponding to the field equations of Lichnerowicz's [8] « total radiation » ($R_{ij} = \sigma w_i w_j$, $w_i w^i = 0$, σ is a scaler $\neq 0$) is given. A particular solution (Krishna Rao [7]) which can be expressed in cylindrical polar coordinates r, Φ, z and time t as

$$ds^2 = e^{2f}(dt^2 - dr^2) - r^2 d\Phi^2 - dz^2, \tag{2.1}$$

where f is a function of $(t \pm r)$, is interpreted as representing a nonempty space-time pervaded by gravitational radiation. The Weyl tensor of the space-time given by (2.1) is of type N according to Petrov's [9] classification. The congruence of null rays associated with the field (2.1) is geodetic, hypersurface-orthogonal, with nonvanishing shear and divergence.

Now, if we consider the conformal forms of (2.1), their Weyl tensors also belong to the same type and therefore can be interpreted as material distributions pervaded by gravitational radiation (Pirani [*10*]). Any such metric form can be expressed as

$$ds^2 = e^{2h}[e^{2f}(dt^2 - dr^2) - r^2 d\Phi^2 - dz^2], \tag{2.2}$$

where h is a function of the coordinate variables. However, if we take h as a function of time only, (2.2) represents for certain values of h, the Einstein-de Sitter and steady state cosmological models pervaded by type N gravitational radiation, the radiation being retarded or advanced according as f is a function of the argument $(t - r)$ or $(t + r)$ respectively.

For the space-time given by (2.2), the nonvanishing components of the Einstein tensor G_{ij}, connected with the Ricci tensor R_{ij} by the relation

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R, \tag{2.3}$$

where

$$R = g^{ij} R_{ij}$$

are given by:

$$\begin{aligned} G_{11} &= 2\ddot{h} + \dot{h}^2 + f'(\bar{r}^1 \mp 2\dot{h}), \\ G_{22} &= G_{33} r^2 = \bar{e}^{2f} r^2 (2\ddot{h} + \dot{h}^2), \\ G_{44} &= -3\dot{h}^2 + f'(\bar{r}^1 \mp 2\dot{h}), \\ G_{14} &= -f'(\bar{r}^1 \mp 2\dot{h}), \end{aligned} \tag{2.4}$$

where $\dot{h} = dh/dt$, $\ddot{h} = d^2h/dt^2$, and $f' = df/d(t \mp r)$ as the case may be, Here and in what follows whenever $\mp$ (or $\pm$) occurs, always the upper sign corresponds to the retarded solution while the lower one stands for advanced solution.

3. — PROPAGATION OF RADIATION IN EINSTEIN-DE SITTER AND STEADY STATE COSMOLOGICAL MODELS

To study the propagation of radiation in Einstein-de Sitter and steady state models we assume the distribution to be a mixture of smoothed out fluid (without internal stresses) and radiation. The energy momentum tensor for such a distribution can be chosen as

$$T_{ij} = \rho u_i u_i + \sigma w_i w_j, \qquad u_i u^i = 1, \qquad w_i w^i = 0, \tag{3.1}$$

where ρ and σ are respectively the material and radiation densities.

i) Einstein-de Sitter model.

From (2.2), (2.4) and (3.1) through the field equations of general relativity

$$G_{ij} = -\, 8\pi T_{ij}, \tag{3.2}$$

we get

$$\begin{aligned}
e^h &= (at)^2, \\
8\pi\rho &= 12/e^{2f}a^4t^6, \\
8\pi\sigma &= \begin{cases} -f'(r^{-1} - 4a^{-1}t^{-1}), \ldots \text{ retarded} \\ f'(r^{-1} + 4a^{-1}t^{-1}), \ldots \text{ advanced} \end{cases} \\
u^i &= (0, 0, 0, e^{-f}a^{-2}t^{-2}), \\
w^i &= e^{-2f}a^{-4}t^{-4}(\pm 1, 0, 0, 1)
\end{aligned} \tag{3.3}$$

where a is a positive constant. The forms of e^h, ρ and σ show that the solution represents Einstein-de Sitter universe pervaded by type N gravitational radiation. It can be easily seen that if we consider the propagation of retarded radiation, for certain sets of values of r and t, the radiation density given in (3.3) becomes equal to either zero or a negative quantity and therefore the model ceases to be physical while in the case of advanced radiation the model continues to be physical until we reach spatial and or temporal infinity.

ii) Steady state model.

The field equations of steady state theory as given by Hoyle and Narlikar [3] are

$$G_{ij} = -\, 8\pi\left[T_{ij} - \varepsilon\left(c_i c_j - \frac{1}{2} g_{ij} c_l c^l\right)\right], \tag{3.4}$$

where the expression in round brackets on the right hand side of (3.4) represents the creation field and ε is a coupling constant. For the motion, of the smoothed out fluid to be geodetic we must have $c^i = u^i$.

From (2.2), (2.4), (3.1) and (3.4) we get

$$\begin{aligned}
e^h &= (bt^{-1}) \\
8\pi\rho &= 6e^{2f}b^2 \\
8\pi\sigma &= \begin{cases} -f'(r^{-1} + 2bt^{-1}), \ldots \text{ retarded} \\ f'(r^{-1} - 2bt^{-1}), \ldots \text{ advanced} \end{cases} \\
u^i &= c^i = (0, 0, 0, bte^{-f}), \\
w^i &= (bte^{-f})^2(\pm 1, 0, 0, 1)
\end{aligned} \tag{3.5}$$

where b is an arbitrary positive constant. It is easy to verify from the expression for radiation density given in (3.5) that for the propagation of retarded and advanced fields, what happens here is exactly the reverse of Einstein-de Sitter's case.

Therefore, the propagation of full retarded gravitational radiation is self-consistent in steady state model but not in Einstein-de Sitter model while the situation is reversed with full advanced gravitational radiation.

4. — CONCLUSION

In the steady state theory since matter is being created continuously to maintain constant density inspite of expansion there is enough of matter in a proper volume to absorb retarded radiation. The future absorber of the Einstein-de Sitter model suffers from lack of matter in it. Therefore the steady state model produces only retarded solutions whereas the Einstein-de Sitter model does the exact opposite. Finally, we see that if we start with two arrows of time, one determined by the expansion of the Universe and the other by local observations which demand retarded solutions, the two arrows agree in steady state model but not in Einstein-de Sitter universe.

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