



## Winter Braids Lecture Notes

### **Participants & Programme**

Vol. 4 (2017), p. i-vi.

[<http://wbln.cedram.org/item?id=WBLN\\_2017\\_\\_4\\_\\_r1\\_0>](http://wbln.cedram.org/item?id=WBLN_2017__4__r1_0)

**cedram**

*Texte mis en ligne dans le cadre du  
Centre de diffusion des revues académiques de mathématiques  
<http://www.cedram.org/>*

## Participants & Programme

This volume of *Winter Braids Lecture Notes* contains the lecture notes for the four mini-courses given at Winter Braids VII, which took place in Caen from February 27th to March 2nd, 2017. This seventh edition of the school was held in honour of Patrick Dehornoy, for his 65th birthday.

### Participants

Audoux	Benjamin	(Université Aix-Marseille)
Bahayou	Amine	(Kasdi Merbah)
Banfield	Ian	(Boston College)
Baranowski	Adam	(Warsaw University)
Bellingeri	Paolo	(Université de Caen Normandie)
Belletti	Giulio	(Scuola Normale Superiore di Pisa)
Ben Aribi	Fathi	(Université de Genève)
Bénard	Léo	(Université Paris 7)
Benheddi	Mounir	(Université de Genève)
Blasco	Rubén	(Universidad de Zaragoza)
Bode	Benjamin	(University of Bristol)
Callegaro	Filippo	(Università di Pisa)
Casejuane	Adrien	(Université Grenoble Alpes)
Chavli	Eirini	(Université d'Amiens)
Cohen-Solal	Quentin	(Université de Caen)
Cubides Kovacsics	Pablo	(Université de Caen)
Cumplido Cabello	Mar'a	(Universidad de Sevilla)
Damiani	Celeste	(Osaka City University)
Dehornoy	Patrick	(Université de Caen)
Esterle	Alexandre	(Université de Picardie)
Fenn	Roger	(University of Sussex)
Ffitch	Edward	(University of Warwick)
Florens	Vincent	(Université de Pau)
Földvári	Viktória	(Eötvös Loránd University)
Fromentin	Jean	(Université du Littoral Côte d'Opale)
Gaudreau	Robin	(McMaster University)
Gervais	Sylvain	(Université de Nantes)
Gobet	Thomas	(Université de Lorraine)
Godelle	Eddy	(Université de Caen)

## Participants & Programme

González-Meneses	Juan	(Universidad de Sevilla)
Gonzalez	Pagotto	(Université Grenoble Alpes)
Guaschi	John	(Université de Caen)
Gügümcü	Neslihan	(National Technical University of Athens)
Kitano	Teruaki	(Soka University)
Krammer	Daan	(University of Warwick)
Lebed	Victoria	(Trinity College Dublin)
Levitt	Gilbert	(Université de Caen)
Leturcq	David	(Université Grenoble Alpes)
Limouzineau	Maÿlis	(Université Paris 6)
Lopez	Daniel	(Université Paris 6)
Martel	Jules	(Université de Toulouse)
Marin	Ivan	(Université de Amiens)
Martinez Metzmeier	Cesar	(Université de Caen)
McLeay	Alan	(University of Glasgow)
McCammond	Jon	(UC Santa Barbara)
Meilhan	Jean-Baptiste	(Université Grenoble Alpes)
Meumertzheim	Johanna	(University of Regensburg)
Mignard	Michaël	(Université de Bourgogne)
Mihajlovic	Stefan	(Central European University)
Misev	Filip	(Université Aix Marseille)
Mulazzani	Michele	(Università di Bologna)
Naisse	Grégoire	(Université Catholique de Louvain)
Nasybullov	Timur	(University of Bologna)
Neaime	Georges	(Université d'Amiens)
Queffelec	Hoel	(CNRS / Université de Montpellier)
Paris	Luis	(Université de Bourgogne)
Picantin	Matthieu	(Université Paris 6)
Poineau	Jerome	(Université de Caen)
Porti	Joan	(Universitat Autònoma de Barcelona)
Ricard	Riba	(Universitat Autònoma de Barcelona)
Robert	Louis-Hadrien	(Universität Hamburg)
Rolfesen	Dale	(University of British Columbia)
Satgé	Philippe	(Université de Caen)
Serrano	Juan	(University of Zaragoza)
Shimizu	Tatsuro	(Kyoto University)
Silvero	Marithania	(Universidad de Sevilla)
Spano	Gilberto	(Université de Grenoble Alpes)
Stathis	Antoniou	(National Technical University of Athens)
Szymik	Markus	(NUST Trondheim)
Taipe	Frank	(Université de Caen)
Tesson	Emilie	(Université de Caen)
Thiel	Anne-Laure	(Universität Stuttgart)
Thumm	Alexander	(Universität Stuttgart)
Toffoli	Enrico	(Universität Regensburg)
Vainerman	Leonid	(Université de Caen)
Vershinin	Vladimir	(Université de Montpellier)
Vera	Anderson	(Université de Strasbourg)
Vértesi	Vera	(CNRS / Université de Strasbourg)
Wagner	Emmanuel	(Université de Bourgogne)

## Abstracts of Courses

**Jon McCammond** (UC Santa Barbara)

*The mysterious geometry of Artin groups*

Artin groups are easy to define but are, in most cases, notoriously hard to understand. In particular, for most Artin groups we do not even know how to solve the word problem. And what makes the situation a bit mysterious is that it can be a little difficult to pinpoint the exact reason why they are currently hard to understand. The primary goal of this short course is to highlight exactly where the problems begin.

In the first talk I review the close connection between Artin groups and Coxeter groups and the associated topological spaces used to investigate them. In the second talk I summarize exactly which Artin groups we understand (meaning we have an explicit solution to the word problem) and which ones we don't. And in the third talk I turn my attention to those Artin groups (and their relatives) that are not currently understood but which we are likely to understand sometime soon.

**Joan Porti**(UA Barcelona)

*Character varieties and knot symmetries*

The set of homomorphisms of the fundamental group of a three manifold in  $SL(2, \mathbb{C})$  is called the variety of representations. The variety of characters encodes the conjugacy classes of the representations. Both varieties are key tools to study the geometry and topology of three manifolds.

I will start introducing the basic definitions and properties. I plan to put emphasis on explicit computations and examples. I will also survey some of its applications. Then I will focus in a joint work with Luisa Paoluzzi, on the different behavior of the variety of characters of knots with symmetries according to the kind of symmetry, whether it is free or has fixed points (also called period).

**Dale Rolfsen** (University of British Columbia)

*Ordering braids, knot groups and beyond*

This will be a series of three talks on application of ordered groups to topology and vice-versa. My interest in the subject began when I learned about Patrick Dehornoy's beautiful proof that the braid groups are left-orderable. This has the nice consequence (for some calculations I was doing at the time) that the group ring  $\mathbb{Z}[B_n]$  has no zero divisors. Because of this proof I became hooked on orderable groups for many years since then!

**Vera Vertesi** (Université de Strasbourg)

*Braids in contact structures*

This will be a series of three lectures on the properties of braids respecting contact structures.

A contact structure on a 3-manifold is a plane field that is not tangent to any open surface. Contact structures can be traced back to the works of Sophus Lie in 1872. The standard contact structure in  $\mathbb{R}^3$  is the plane field given as the kernel of the one form  $dz - r^2 dv$ . In the beginning of the lecture series I will explain the famous theorem of Bennequin stating that  $\mathbb{R}^3$  has nonstandard contact structures. The proof uses braids that are transverse to the contact structure, such knots are called transverse knots. Braid theory for transverse knots is interesting in its own right; I will describe a transverse version of Markov and Alexander theorem. If time permits I will talk about our recent work with J. Etnyre concerning braid representations of knots that are tangent to the contact structure, called Legendrian knots.

## Abstracts of Short Talks

**Léo Bénard** (Univ. Paris 7)

*Reidemeister torsion form on the character variety*

We define the Reidemeister torsion as a rational differential form on the character variety of a 3-manifold  $M$  with toric boundary. Then we study the singularities of this differential form, there are two cases : the singularities at finite points and at ideal points. The latter are related, by the Culler-Shalen theory, to incompressible surfaces in the variety  $M$ . The main result provides a relation between the vanishing order of the torsion at an ideal point and the Euler characteristic of the related surface.

**Mounir Benheddi** (Univ. Genève)

*On Khovanov homology of infinite torus links*

Khovanov homology is a homology theory that associates to any link a bi-graded vector space and whose Euler characteristic is the Jones polynomial. There exists an explicit formula for the Jones polynomial of any torus link  $T_{p,q}$ , and it is thus natural to try and compute their respective Khovanov homology. However such precise computations quickly become unmanageable due to the size of the chain complex, which grows exponentially on the number of crossings. By fixing  $p$  and letting  $q$  grow to infinity, one obtains an 'infinite  $p$ -torus link' and, as shown by Stosic, a corresponding well-defined 'limit' vector space. More often than not, these limit homologies tend to have more structure than their finite counterparts. In this talk, we will endow the Khovanov homology of the infinite  $p$ -torus link with a structure of commutative algebra and describe it explicitly for  $p = 2, 3, 4$ .

**Rubén Blasco** (Univ. Zaragoza)

*Some properties on even Artin groups*

Artin groups are an interesting family of groups from both an algebraic and a topological point of view. In my talk I will focus on a special subfamily: even Artin groups, and I will present some interesting results about their algebraic and topological properties.

**Thomas Gobet** (Univ. Lorraine)

*On simple dual braids and Mikado braids of type  $D_n$*

We explain how to relate simple dual braids and Mikado braids of type  $D_n$  to those of type  $B_n$  by using a suitable embedding of Artin-Tits groups of type  $D_n$  inside a quotient of an Artin-Tits group of type  $B_n$ . This allows to show that any simple dual braid of type  $D_n$  is a Mikado braid. Joint with B. Baumeister.

**Maÿlis Limouzineau** (Univ. Paris 7 )

*Cobordism and concordance of Legendrian knots contour of generating functions*

I will describe some explicit constructions related with cobordisms of Legendrian knots (in the sense of V.I. Arnold). I will focus on those knots which possess a generating function. After recalling the basic definitions and the main motivations, I will show how to define a concordance group of Legendrian knots equipped with a generating function.

## Participants & Programme

**Alan McLeay** (Univ. Glasgow)

*Ivanov's Metaconjecture: Automorphism Groups of Sufficiently Rich Complexes of Regions for Surfaces with Punctures*

It is a well-known and fundamental result of Ivanov that the curve complex of an orientable surface with punctures has automorphism group isomorphic to the extended mapping class group of the surface. It was subsequently shown that the equivalent statement is true for a number of other complexes, among them the pants complex (Margalit) and the separating curve complex (Brendle-Margalit, Kida). Such results led Ivanov to make a meta-conjecture: all sufficiently rich complexes related to the surface will have automorphism group isomorphic to the extended mapping class group. A result by Brendle-Margalit shows this to be true for a broad class of complexes for closed surfaces. In this talk I will give the more general result for complexes relating to surfaces with punctures.

**Louis-Hadrien Robert** (Univ. Hamburg)

*Categorification of MOY calculus*

MOY calculus has been introduced in the 90s to compute combinatorially the quantum link invariant associated with the Hopf algebra  $U_q(\mathfrak{sl}_N)$ . It associates to any decorated graph a Laurent polynomial in  $q$ . I will describe a TQFT-like functor which categorifies the MOY calculus and provides a new description of the  $\mathfrak{sl}_N$ -homology. Finally, I will explain how this sheds a new light on the structures of cohomology rings of partial flag varieties. Joint with E. Wagner

**Juan Serrano** (Univ. Zaragoza)

*A Functorial Extension of the Magnus Representation to the Category of three-dimensional Cobordisms*

Let  $R$  be an integral domain and  $G$  be a subgroup of its group of units. We consider the category  $\text{Cob}_G$  of 3-dimensional cobordisms between oriented surfaces with connected boundary, equipped with a representation of their fundamental group in  $G$ . Under some mild conditions on  $R$ , we construct a monoidal functor from  $\text{Cob}_G$  to the category  $\text{pLagr}_R$  consisting of "pointed Lagrangian relations" between skew-Hermitian  $R$ -modules. We call it the "Magnus functor" since it contains the Magnus representation of mapping class groups as a special case. Our construction is inspired from the work of Cimasoni and Turaev on the extension of the Burau representation of braid groups to the category of tangles. It can also be regarded as a  $G$ -equivariant version of a TQFT-like functor that has been described by Donaldson. The study and computation of the Magnus functor is carried out using classical techniques of low-dimensional topology. Joint with V. Florens and G. Massuyeau.

**Marithania Silvero Casanova** (Univ. Sevilla)

*A new realization of extreme Khovanov homology*

Khovanov homology is a link invariant introduced in 2000 by Mikhail Khovanov. This bigraded homology categorifies Jones polynomial and it has been proved to detect the unknot. In this talk we present a new approach to extreme Khovanov homology in terms of a specific graph constructed from the link diagram. With this point of view, we pose a conjecture related to the existence of torsion in extreme Khovanov homology and show some examples where the conjecture holds.

**Gilberto Spano** (Univ. Grenoble Alpes)

*Twisted Gromov and Lefschetz invariants associated with bundles*

Given a 3-manifold  $Y$ , we define a twisted version of the Reidemeister torsion of  $Y$  associated with a choice of a surface bundle over  $Y$ . We show how this bundle-twisted Reidemeister torsion is related to some standard twisted Reidemeister torsions of  $Y$  associated with a choice of a representation of its fundamental group. We define then a twisted version of the Gromov invariants for symplectic 4-manifolds

## Participants & Programme

and we give examples that relate these new invariants to some twisted Reidemeister torsions in dimension 3.

**Enrico Toffoli** (Univ. Regensburg)

*Multivariable signatures and link concordance*

In 2005, Cimasoni and Florens introduced a multivariable generalization of the Levine-Tristram signature of a link. This invariant can be defined using generalized Seifert surfaces, but a more abstract approach is often useful. In this talk, we will present a way to define the multivariable signature of a given link as the twisted signature of a suitable class of 4-manifolds. We will then discuss applications to link concordance.