Winter Braids Lecture Notes

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Introduction

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Introduction

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Winter Braids is an international school on low dimensional topology, with a particular emphasis on the multiple aspects of braids: algebraic, geometrical, topological or algorithmic. More generally, it is devoted to the various thematics connected with these objects, such as mapping class groups, singularity theory and plane algebraic curves, quantum topology and TQFTs, symplectic topology, categorification or theoretical physics. The school is held each winter since 2010, in a different french university.

The *Winter Braids Lecture Notes* is a free open access electronic and peer-reviewed journal, which contains the lecture notes of each school, since the fourth edition. The main aim of this journal is to provide a body of original introductory texts for young researchers on classical and emerging topics in low dimensional topology and its ramifications.

This volume of *Winter Braids Lecture Notes* contains lecture notes for the four mini-courses given at Winter Braids VI, which took place in Lille from February 22th to 25th, 2016. Below is a short description for each of these courses.

- The course of Maciej Borodzik (Warsaw) dealt with Heegaard Floer homology, which were defined around 2000 by Ozsváth and Szabó. Since then, the subject was successfully applied to many aspects of knot theory and the topology of 3-manifolds. In particular, it is known that there is an isomorphism between Heegaard Floer homology groups and the corresponding Seiberg-Witten Floer homology groups of a 3-manifold. The main advantage is that Heegaard Floer problems can often be reduced to combinatorics of Heegaard diagrams, which make this theory more accessible to an inexperienced reader.
  The aim of these notes is to introduce this theory, with a specific focus on applications to algebraic geometry. Indeed, some of the main recent developments of the theory use tools such as L-space knots and d-invariants to the study of cuspidal singularities. Through this focus, these lecture notes represent a valuable complement to the recent excellent general surveys on the theory.

- After reviewing the combinatorics of triangulations, Rinat Kashaev (Geneva) introduced a new type of Topological Quantum Field Theory (TQFT), the so called Teichmüller TQFT. Recall that a classical three dimensional TQFT, like the Turaev-Viro or Reshetikhin-Turaev TQFT, provides invariants for closed three manifolds and finite dimensional representations of the mapping class group of closed surfaces out of combinatorial descriptions of the manifolds and an algebraic input, like 6j symbols and representations of quantum groups.
  The Teichmüller TQFT also builds on combinatorial descriptions of manifolds, but the algebraic datas are replaced by integrals and distributions. One special feature of this new TQFT is that it provides infinite dimensional representations of the mapping class group. They are directly linked to hyperbolic geometry and are related to Chern-Simons theory with gauge group $SL(2,\mathbb{C})$.

- The course of Patrick Popescu-Pampu (Lille) covered two distinct domains: the world of surface singularities and the world of contact geometry, with the aim of presenting results at the intersection of these two areas, in particular the connections between Stein fillings and Milnor fibers.
  The chosen approach was to introduce these two domains and to make them accessible to non-specialists using examples, basic facts and intuitions in both fields; in the text this aspect was greatly expanded and enriched with illuminating drawings and an extended bibliography.

- Takuya Sakasai (Tokyo) presented an overview of the so–called Johnson-Morita theory, which is an approach to the study of the mapping class group via a nested sequence of subgroups refining the Torelli group, and a family of homomorphisms defined on these successive subgroups. After reviewing some basics of the mapping class group and the Torelli group, the lectures gave a detailed exposition of these Johnson homomorphisms, as well as their extensions to the realm of 3-manifolds and applications to knot theory.

The organisers also selected 10 short talks, providing a wide spectrum of topics in topology and interactions. Some of these short talks presented recent advances in the understanding of the various invariants currently at play in low-dimensional topology – from classical invariants (such as the signature or Alexander polynomial) to quantum invariants and link homologies, or L^2-invariants. This gave a great overview of some of the deep questions concerning the individual and global understanding of these numerous invariants. Other short talks addressed topics such as mapping class group representations derived from TQFTs, representations of the braid group, triangulations of hyperbolic knot complements, knotted surfaces in 4-space or character varieties. The abstracts of these short talks can be found at the end of this introduction.

We are very grateful to the four authors of these lecture notes, for their remarkable work and for their contribution to the Winter Braids Lecture Notes.

This edition in Lille was made possible thanks to Arnaud Bodin, whose hard work as a local organisor is deeply and sincerely acknowledged. Thanks are also due for his help in the preparation of this volume of WBLN.

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Abstracts of Courses

Maciej Borodzik (Warsaw University)
Heegaard Floer homology with a view towards geometry of complex plane curves

The lecture will be an introduction to Heegaard–Floer homology focused on applications in algebraic geometry. I will give definitions of the Heegaard-Floer homology, but the exposition will be mostly based on examples, rather than on giving rigorous definitions. The tentative plan is as follows.

1. Introduction. A few exemplary complexes are presented and their properties are given. A formal definition of knot Floer homology and Heegaard Floer homology will is given.

2. I will define d-invariants in Heegaard Floer homology and show their basic applications. I will show how to calculate d-invariants of large surgeries on some families of knots.

3. Two applications in algebraic geometry will be given. First one will be the proof of semicontinuity of semigroups and the second one will be the proof of the conjecture of Fernandez de Bobadilla, Luengo, Melle-Hernandez and Nemethi. Some further generalizations will be shown.

Rinat Kashaev (Université de Genève)
Triangulations and quantum knot invariants

The main goal of this course is to describe an approach to construction of quantum knot invariants based on the combinatorial framework of triangulations. This approach is motivated by the the fact that comparing to knot diagrams triangulations present the following advantages:
- they offer the possibility of studying knots in arbitrary three-manifolds
- they display intrinsic three-dimensionality
- they make direct connection to the topology and geometry of knot complements.
Additionally, some 4–dimensional aspects of the considered constructions will also be discussed.

Plan:
1. Triangulations and Pachner moves.
2. Shaped triangulations.
3. H-triangulations.
4. Definition of the partition function of a shaped triangulation within Teichmüller TQFT.
5. Examples of calculation.

Patrick Popescu Pampu (Université Lille I)
Contact topology and singularities

This course will be an introduction to the study of interactions between singularity theory of complex analytic varieties and contact topology. We will concentrate on the relation between the smoothings of singularities and the Stein fillings of their contact boundaries.

Takuya Sakasai (University of Tokyo)
Johnson-Morita theory

The role of the mapping class group and its Torelli subgroup in low dimensional topology is widely accepted to be important and structures of these mysterious groups have been studied for a long time. In 1980's, Dennis Johnson introduced a homomorphism, now called the Johnson homomorphism,from the Torelli subgroup to a symplectic module and used it to prove several fundamental (but deep) facts on the Torelli subgroup. After that, Shigeyuki Morita clarified and extended this theory. In a series of his works, he revealed close relationships to invariants of 3-dimensional manifolds and cohomology of the mapping class group. Since then, this theory has been further extended by many people in various directions. In this series of talks, starting from the review of works of Johnson and Morita, we discuss recent developments of the theory of Johnson homomorphisms and its applications:

1. Mapping class group, Torelli subgroup and the first Johnson homomorphism
2. Higher Johnson homomorphisms
3. Extensions and Applications

Abstracts of Short Talks

Fathi Ben Aribi (Univ. de Genève)
L^2-Alexander invariants in knot theory

The L^2-Alexander invariant of a knot is a continuous real function defined by Li and Zhang in 2006 as an infinitedimensional version of the Alexander polynomial. It contains classical invariants of geometry and topology like the simplicial volume and the genus. Using 3–dimensional topology, we will explain how this invariant detects several knots among the set of all knots.

Celoria Daniele (Univ. di Firenze)
Grid homology in lens spaces

After a short introduction on grid homology for knots in lens spaces, we will give some application to concordances and rational genus.

Chavli Eirini (Univ. d'Amiens)
The irreducible representations of B_3 of dimension at most 5

In 1999 I. Tuba and H. Wenzl classified the irreducible representations of the braid group B_3 on 3 strands of dimension at most 5, by giving explicit matrices of a triangular form whose coefficients are completely determined by the eigenvalues and a rth root of their determinant. Moreover, they proved that such irreducible representations exist if and only if the eigenvalues do not annihilate some polynomials in the eigenvalues and r. In this talk we will explain the reason we have matrices in this neat form, as well as, the origin of the above polynomials by reproving this classification as a consequence of the freeness conjecture for the generic Hecke algebra of the finite quotient of B_3.

Conway Anthony (Univ. de Genève)
Colored tangles and signatures

I will discuss signatures of links, relate these invariants to the Burau representation of the braid group and discuss twisted generalisations of these objects.

Damiani Celeste (Univ. de Caen Normandie)
On the Alexander polynomial of a ribbon tangle

Ribbon 2–knotted objects are locally flat 2–dimensional submanifolds of R^4 that bound immersed 3–manifolds with only ribbon singularities. They appear as topological realizations of welded knotted objects, where welded knot theory is a quotient of virtual knot theory. We construct a functorial extension of the Alexander polynomial to ribbon tangles. At a combinatorial level this gives rise to a generalization of the Alexander polynomial of links to welded tangles.

Detcherry Renaud (Michigan State Univ.)
Asymptotics of curve operators in TQFT

Curve operators are endomorphisms associated to curves on a curve by Reshetikhin-Turaev TQFTs. We express the asymptotic of their matrix coefficients in terms of trace functions on the moduli space.

Kohli Ben-Michael (Univ. de Bourgogne)
Some interesting evaluations of the Links-Gould polynomials

The Links-Gould polynomials $LG^{m;n}(t^0;t^1)$ are two variable polynomial link invariants. Each one of them is derived from a highest weight representation of quantum supergroup $U_q(gl(m|n))$. In 2005, De Wit, Ishii and Links proved that the Alexander-Conway polynomial of a link could be recovered as an evaluation of certain of these Links-Gould polynomials. I shall highlight another specialization that allows us to recover powers of the Alexander invariant, expressing representations of the braid groups derived from the R-matrices of $LG^{m;n}$ in terms of Burau representations. These evaluations together with tests on values of LG for small prime knots encourage us to believe that the Links-Gould invariant should have a classical interpretation, in quite the same manner as the Alexander polynomial has.

Moussard Delphine (Univ. de Bourgogne)
Braid group orbits in Aff(C)-character varieties of the punctunred sphere

Joint work with Gaël Cousin. The group $PB_n(S^2)$ of n-strands pure braids of the sphere acts naturally on the representations of the fundamental group of the n-punctured sphere. Gaël Cousin has shown that finite orbits of such actions provide interesting flat connections on vector bundles over projective ruled varieties. Motivated by this result, we consider the representations of the fundamental group of the *n*-punctured sphere in the complex affine group. We will describe the finite orbits of the action of the group $PB_n(S^2)$ on these representations.

Morvan Xavier (Univ. de Genève)
Non-commutative generalisation of Thurston's gluing equations

In his famous Princeton Notes, Thurston introduces the so-called gluing equation, defined from ideal triangulations of knot complements. Those are algebraic equations with complex parameters encoding the hyperbolic structures of the knot complement. In this talk, we will define algebraically a non-commutative generalisation of those equation from H-triangulation of the couple $(S^3;K)$.

Stylianakis Charalampos (Univ. of Glasgow)
The normal closure of a power of a half-twist has infinite index in the mapping class group of the punctured sphere

The Jones representation of the mapping class group of the punctured sphere is constructed by formulating irreducible linear representations of braid groups that factor through Hecke algebras. In this talk we introduce the Jones representation and we show that the normal closure of the mth power of a half-twist has infinite index in the mapping class group of a punctured sphere. As a corollary we show that the normal closure of a power of a Dehn twist has infinite index in the hyperelliptic mapping class group of a closed surface of genus at least two.