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Foreword

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Foreword

The reader of this volume holds in his hands the proceedings of the workshop on function fields, zeta functions and Drinfeld modular forms held from 22th to 26th June, 2015, at Imperial College London, honoring the 61st birthday of David Goss. David, who was also a close personal friend of the organisers, passed away tragically on the 4th of April, 2017, so we would like to dedicate this volume to his memory, and write a few words about the significance of his work in function field arithmetic.

David received his PhD in 1977 from Harvard under the supervision of Barry Mazur. He held positions at Princeton, Berkeley, and Brandeis before joining Ohio State University in 1982, where he retired in 2013 and became Professor Emeritus. David was generous with his time and knowledge and was full of enthusiasm and support for his colleagues, especially junior researchers in number theory. He was considered by many as a mentor and a friend, and his broad and deep knowledge as well as his kindness and warmth will be missed by many.

David's research was in algebraic number theory and algebraic geometry, where he made extensive contributions to the arithmetic theory of function fields, publishing some 40 academic papers. He was also the author of "Basic Structures of Function Field Arithmetic", which is one of the most popular and accessible references in this topic. Starting with his Harvard thesis, David introduced a theory of modular forms for (k, ∞), with k a global field of positive characteristic and ∞ a fixed place of k. In 1979, while at Princeton, he accomplished another important advance, by introducing zeta and L functions for (k, ∞) (now called Goss' zeta and L-functions). In this seminal work, he also introduced the famous character group S_∞ and the zeta-phenomenology of analytic continuation, the negative values, the trivial zeroes, and other highly inspirational structures. He later continued developing his theory of zeta and L-functions in the path of Euler and Riemann, and pioneered this vast area, such as the analogue of the Riemann hypothesis, the functional equation, the Gamma factors.

The main stay of the conference was one of the primary interests of David, namely the study of special values of L-functions of characteristic p

values. The broader subject of special values of L-functions has significant connections to other important areas of mathematics such as the study of Galois representations, algebraic cycles, motives, K-theory, transcendence theory, modular forms and Iwasawa theory. In the last few years, the study of special values of L-functions of objects defined over positive characteristic fields, such as algebraic varieties and t-motives, experienced particularly rapid growth. Some of these advances used powerful cohomological trace formulas available in this setting; however other methods are more exotic, and hence mysterious, including deformations of Drinfeld modular forms. These in turn introduced a host of new problems about the latter objects and spurred a great deal of activity. As mentioned above, characteristic p-valued L-functions attached to Drinfeld modules and t-motives, as well as Drinfeld modular forms, were first introduced by Goss. These objects have familiar flavour if compared to their counterparts over number fields, that is, they exhibit similar analytic behaviour and arithmetic behaviour of their special values; at least at first sight. Many of the basic properties of these L-functions are now well understood. In particular Taguchi and Wan proved, using analytic techniques introduced by Dwork, that these L-functions have entire continuation to S_∞. In another major development Böckle and Pink found a cohomological proof of this analytic behaviour, via a trace formula. Of course we would like to have an interpretation of the special values of these L-functions in terms of global arithmetic objects, a main theme in number theory, which is perhaps the most fascinating, and enigmatic, and important progress has been made in the last couple of years, notably including variants of the class number formula. And so much remains to be done!

In the present volume, the reader will find many faces of David's arithmetic dream. The theory of modular forms with values in complete fields of positive characteristic, including v-adic interpolation, higher rank modular forms, is considered in the contributions of Papanikolas and Zeng, Breuer and Basson, Gekeler, Perkins. Aspects of zeta and L-functions, as well as their values, but also more classical questions of algebraic number theory or transcendence, are investigated in the works of Rosen, Anglès, Ngo Dac and Tavares Ribeiro, as well as Boeckle and Thakur, and Pellarin. Studies with more geometric flavour, related to characteristic p Galois representations, crystals, rigid cohomology, are tackled by Lazda, Heuer and Pál(1). An extension of zeta values (introduced by Thakur) is given by multiple zeta values which are considered, in the volume, through analogues in positive characteristic which appear as a not fully understood *alter ego* of the real ones, by Chen, Thakur, Pellarin, and Chang and Mishiba. The volume begins with a paper of David, containing some of his views on

(1) Pál's article is currently under review and is not included in this issue.

measures, distributions, digit calculus, Riemann's hypothesis, and his "crazy group $S_{(q)}$," as he liked to call it. Some of the structures he introduced, like this one, remain mysterious. May this volume be an inspiration, especially to younger researchers, to pursue the challenges that he put forward.

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Note of the editor As for all proceedings published by JTNB, the articles in this issue followed the usual editorial process and are referred as normal submissions.