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Corrigendum to "Complete Solutions of a Family of Cubic Thue Equations" [Journal de Théorie des Nombres de Bordeaux 18 (2006), 285-298]

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Résumé. Dans notre article original [4], le lemme 2.2 n'était pas correctement démontré. L'erreur a depuis été réparée par Lee–Louboutin [3]. Cette erreur n'affecte cependant pas le résultat final de l'article.

Abstract. In our original paper [4], Lemma 2.2 was not properly proved. This was done by Lee–Louboutin [3]. The gap doesn't affect the final result of the paper.

In [4], using Baker's method we studied the family of Thue equations

$$\begin{aligned}\Phi_n(x,y) &= x^3 + (n^8+2n^6-3n^5+3n^4-4n^3+5n^2-3n+3)x^2y \\ &\qquad - (n^3-2)n^2xy^2 - y^3 \\ &= \pm 1,\end{aligned} \tag{1}$$

for $n \geq 0$. To do so, we considered the number field $\mathbb{K}_n$ related with $\phi_n(x)$ defined by

$$\begin{aligned}\phi_n(x) &= x^3 + (n^8+2n^6-3n^5+3n^4-4n^3+5n^2-3n+3)x^2 \\ &\qquad - (n^3-2)n^2x - 1.\end{aligned} \tag{2}$$

See also [2], pages 100-103. One can see that ϕ_n has three real roots $\theta^{(1)}$, $\theta^{(2)}$, $\theta^{(3)}$. For any solution (x,y) of (1), we have

$$\Phi_n(x,y) = \prod_{j=1}^{3}\left(x-\theta^{(j)}y\right) = N_{\mathbb{Q}(\theta^{(1)})/\mathbb{Q}}\left(x-\theta^{(j)}y\right) = \pm 1. \tag{3}$$

This means theat $x-\theta^{(j)}y$ is a unit in the order $\mathcal{O} := \mathbb{Z}[\theta^{(1)}, \theta^{(2)}]$. We proved the following result (see Lemma 2.2 of [4]).

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Lemma 2.2. *Let us consider* $\mathcal{O} := \mathbb{Z}[\theta^{(1)}, \theta^{(2)}]$ *and* $< -1, \theta^{(1)}, \theta^{(2)} >$ *a subgroup of the unit group. We have*

$$I := [\mathcal{O}^\times :< -1, \theta^{(1)}, \theta^{(2)} >] < 3, \tag{4}$$

for $n \geq 29$*.*

There was a gap in the proof of Lemma 2.2 due to the misinterpretation of Cusick's result, see Proposition 2.1 of [1]. Lee and Louboutin filled the gap and confirmed Lemma 2.2, for any integer n by proving Lemma 6.4, page 293 of [3]. As Lemma 2.2 is correct, this doesn't affect the main result obtained in our paper.

Remark. The example given on page 288 of [3] has no link with polynomial (2).

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References

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