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# Cherednik algebra for the normalizer

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**Abstract.** Ginzburg, Guay, Opdam and Rouquier established an equivalence of categories between a quotient category of the category $\mathcal{O}$ for the rational Cherednik algebra and the category of finite dimension modules of the Hecke algebra of a complex reflection group $W$. We announce a generalization of this result to the extension of the Hecke algebra associated to the normalizer of a reflection subgroup.

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## Version française abrégée

Dans [14] est définie une extension de l'algèbre de Hecke d'un groupe fini de reflexions complexes $W$. On la note $H(W,W_0)$, elle est réalisée à partir du normalisateur d'un sous groupe de réflexions $W_0$ de $W$. Le but de cet article est d'établir un résultat équivalent à celui de [9, théorème 5.12] pour la catégorie des $H(W_0,W)$-modules de dimension fini. Pour cela nous allons définir différents objets : l'algèbre de Cherednik associée à $H(W,W_0)$, éléments d'Euler, Opérateurs de Dunkl–Opdam, catégorie $\mathcal{O}$. Puis nous construisons explicitement le « foncteur $KZ$ » qui va réaliser l'équivalence de catégorie. Nous donnons ensuite une construction similaire permettant d'omettre le groupe de réflexions $W$.

## 1. Introduction

Let $V$ be a $\mathbb{C}$-vector space of finite dimension $n$. Let $W < GL(V)$ be a finite complex reflection group. Let $W_0 < W$ be a reflection subgroup of $W$. According to [3], we can associate to $W$ a braid group $B(W)$ and a Hecke algebra $H(W)$. In [14] is introduced an extension of $H(W)$ as an algebra associated to the normalizer $N_W(W_0)$, called its Hecke algebra and denoted $H(W,W_0)$, for more results see [10, 11].

We refer to [3] for the general definitions used below. We have a surjection $\pi : B(W) \to W$ sending a braided reflection of hyperplane $H$ to the distinguished reflection $s_H$ of hyperplane $H$. We denote $\mathscr{A}$ the hyperplane arrangement of $W$. Let $\widehat{B_0} := \pi^{-1}(N_W(W_0))$ and $J$ be the two-sided ideal of $\mathbb{C}\widehat{B}_0$ generated by $\langle \sigma_H^{m_H} = 1,\ H \in \mathscr{A} \setminus \mathscr{A}_0$ and $\sigma_H^{m_H} = \sum_{k=0}^{m_H-1} a_{H,k}\sigma_H^k,\ H \in \mathscr{A}_0\rangle$ where $m_H$ is the order of the pointwise stabilizer of $H$ in $W$, denoted $W_H$ and the scalars $(a_{H,k})_{k\in\{0,\dots m_H-1\}}$ are complex numbers invariant under the action of $N_W(W_0)$, $\forall\ w \in N_W(W_0)$, $a_{w(H),k} = a_{H,k}$ for all $k \in \{1,\dots,m_H-1\}$. As in [14] we define the Hecke algebra of the normalizer as the quotient algebra of $\mathbb{C}\widehat{B_0}$ by the ideal $J$.

There is a second equivalent definition. Let $K := \mathrm{Ker}(\pi_1(X/W) \to \pi_1(X_0/W_0))$ and $\widetilde{B_0} := \frac{\widehat{B_0}}{K}$ where $X := V \setminus \bigcup_{H\in\mathscr{A}} H$ and $X_0 = V \setminus \bigcup_{H\in\mathscr{A}_0} H$. Then

$$H(W,W_0) \simeq \frac{\mathbb{C}\widetilde{B_0}}{\langle \sigma_H^{m_H} = \sum_{k=0}^{m_H-1} a_{H,k}\sigma_H^k, \quad H \in \mathscr{A}_0\rangle}$$

We introduce Cherednik algebras in this new context, and we prove

**Theorem 1.** *There exists an equivalence of categories between the quotient category $\mathscr{O}/\mathscr{O}_{tor}$ and the category of $H(W,W_0)$-modules of finite dimension, where $\mathscr{O}$ is a highest weight category associated to the Cherednik algebra of the pair $(W_0,W)$.*

## 2. Construction of the $KZ_0$ -functor

### 2.1. *The Cherednik algebra of the pair* $(W_0,W)$

We denote $A(W_0,W)$ this algebra, and we define it as an algebra admitting a triangular decomposition in the sense of [13]. As a vector space, $A(W_0,W)$ is $\mathbb{C}[V]\otimes\mathbb{C}N_W(W_0)\otimes\mathbb{C}[V^*]$ and we add the following relations on the generators of $\mathbb{C}[V]$, $\mathbb{C}[V^*]$ and $\mathbb{C}N_W(W_0)$,

$$[x',x] = 0 \text{ for all } (x,x') \in V^*\times V^*$$
$$[y,y'] = 0 \text{ for all } (y,y') \in V\times V$$
$$[y,x] = tx(y) + \sum_{H\in\mathscr{A}_0} \frac{\alpha_H(y)x(v_H)}{\alpha_H(v_H)} \sum_{j=0}^{m_H-1} m_H(k_{H,j+1}-k_{H,j})\epsilon_{H,j}$$

where $\epsilon_{H,j} = \frac{1}{m_H}\sum_{w\in W_H\setminus\{\mathrm{id}\}} \det(w)^j w$ is a primitive orthogonal idempotent of $\mathbb{C}W_H$, $\alpha_H \in V^*$ such that $\mathrm{Ker}(\alpha_H) = H$. The vector $v_H \in V$ is such that $\mathbb{C}.v_H$ is a $W_H$-stable complement of $H$. The set $(k_{H,j})_{j\in\{0,\ldots,m_H-1\}}$ is a set of complex number such that $k_{w(H),j} = k_{H,j}$ and $t \in \mathbb{C}$. In order to define a $KZ$ functor, we need to assume $t \neq 0$. Therefore, up to renormalization we can assume $t = 1$ which we do from now on.

As noticed by the referee, this algebra is a special case of a symplectic reflection algebra as in [6], for $N_W(W_0)$ acting on $V\oplus V^*$ in natural way.

### 2.2. *Dunkl–Opdam operators*

We denote by $\mathscr{D}(X)$ the algebra of differential operators over $X$. In [11] is introduced a differential 1-form, $N_W(W_0)$-equivariant and integrable,

$$\omega_0 = \sum_{H\in\mathscr{A}_0} a_H \frac{\mathrm{d}\alpha_H}{\alpha_H} \in \Omega^1(X)\otimes\mathbb{C}W_0$$

where $a_H = \sum_{j=0}^{m_H-1} m_H k_{H,j}\epsilon_{H,j}$. We build a connection on a trivial vector bundle over $X$, by $\nabla := d+\omega_0$. This connection is flat and $N_W(W_0)$-equivariant. The covariant derivative of this connection in the direction of $y \in V$ is a differential operator called Dunkl–Opdam operator, noted $T_y$.

**Proposition 2.** *For all $y \in V$, $T_y := \partial_y + \sum_{H\in\mathscr{A}_0} \frac{\alpha_H(y)}{\alpha_H} a_H \in \mathscr{D}(X)\rtimes N_W(W_0)$. This family of differential operators satisfies two properties $\forall\,(y,y') \in V\times V$,*

$$[T_y, T_{y'}] = 0$$

*and $\forall\, y \in V$, $\forall\, w \in N_W(W_0)$, $w.T_y.w^{-1} = T_{w(y)}$.*

We introduce the algebra $A(W,W_0)_{reg} = \mathbb{C}[X]\otimes_{\mathbb{C}[V]} A(W,W_0)$. We can define a faithful representation of $A(W_0,W)$.

**Theorem 3 (Dunkl embedding).**

(1)
$$\begin{array}{rrcl} \Phi: & A(W_0,W) & \longrightarrow & \mathscr{D}(X)\rtimes N_W(W_0) \\ & x\in V^* & \longmapsto & x \\ & w\in N_W(W_0) & \longmapsto & w \\ & y\in V & \longmapsto & T_y \end{array}$$

*is an injective morphism of algebras.*

(2) *By localization, the morphism $\Phi$ becomes an isomorphism of algebra. We note $\Phi_{reg}$ the isomorphism between $A(W_0,W)_{reg}$ and $\mathscr{D}(X)\rtimes N_W(W_0)$.*

## 2.3. *The category $\mathscr{O}$ -*

Let $\mathrm{eu}_0=\sum_{y\in\mathscr{B}} y^*.y-\sum_{H\in\mathscr{A}_0} a_H$, where $\mathscr{B}$ is a basis of $V$. This operator is called the Euler element. It induces an inner graduation on $A(W_0,W)$, $A(W_0,W)^i:=\{a\in A(W_0,W)\mid[\mathrm{eu}_0,a]=ia\}$ for all $i\in\mathbb{Z}$, because $[\mathrm{eu}_0,x]=x$, $[\mathrm{eu}_0,y]=-y$, $[\mathrm{eu}_0,w]=0$.

For every simple $\mathbb{C}N_W(W_0)$ module $E$, $\sum_{H\in\mathscr{A}_0} a_H\in Z(\mathbb{C}N_W(W_0))$ acts on $E$ by multiplication by a scalar $c_E$. We define a partial ordering on $\mathrm{Irr}(N_W(W_0))$: $E<E'$ if $c_E-c_{E'}\in\mathbb{Z}_{>0}$.

For each $E\in\mathrm{Irr}(N_W(W_0))$ we define a $A(W_0,W)$ module called standard object or Verma module,

$$\Delta(E)=\mathrm{Ind}_{\mathbb{C}[V^*]\otimes\mathbb{C}N_W(W_0)}^{A(W_0,W)} E$$

The category $\mathscr{O}$ is a full sub category of the category of $A(W_0,W)$ modules, where the modules are finitely generated, locally nilpotent for the action of $\mathbb{C}[V^*]$ and isomorphic to the direct sum of the generalized $\mathrm{eu}_0$-eigenspaces. According to [1,2,9], the category $\mathscr{O}$ is Abelian, Artinian. The object $\Delta(E)$ is indecomposable. The category $\mathscr{O}$ is highest weight with $\{\Delta(E)\}_{E\in\mathrm{Irr}(N_W(W_0))}$ as the set of standard object. Every standard object $\Delta(E)$ admits a simple head $L(E)$. Every simple object in $\mathscr{O}$ is isomorphic to some $L(E)$ and $L(E)$ admits a projective cover. Every object $M$ of $\mathscr{O}$ admits a finite composition series. The B.G.G reciprocity law is satisfied inside $\mathscr{O}$.

## 2.4. *Functor $KZ_0$*

Let $\delta:=\prod_{H\in\mathscr{A}}\alpha_H\in\mathbb{C}[V]$. Let $(A(W_0,W)\text{-mod})_{tor}$ be the subcategory of $A(W_0,W)$-mod with $\delta$-torsion, i.e. $M\in A(W_0,W)$-mod, $M_{tor}:=\{m\in M\mid \exists\, n\geq 0\ \delta^n.m=0\}$, then $M\in(A(W_0,W)\text{-mod})_{tor}$ if $M_{tor}=M$. Let $\mathscr{O}_{tor}:=\mathscr{O}\cap(A(W_0,W)\text{-mod})_{tor}$

We have a localization functor,

$$\begin{array}{rrcl} \mathrm{Loc}: & A(W_0,W)\text{-mod} & \longrightarrow & A(W_0,W)_{reg}\text{-mod} \\ & M & \longmapsto & A(W_0,W)_{reg}\otimes_{A(W_0,W)} M \end{array}$$

This functor induces a fully faithful functor $\frac{\mathscr{O}}{\mathscr{O}_{tor}}\to A(W_0,W)_{reg}$-mod.

The Dunkl embedding gives an equivalence of categories between $A(W_0,W)_{reg}$-modules and $\mathscr{D}(X)\rtimes N_W(W_0)$-modules. We also have the following equivalence of categories between $\mathscr{D}(X)\rtimes N_W(W_0)$-modules and $e.(\mathscr{D}(X)\rtimes N_W(W_0)).e$-modules and with $\mathscr{D}(X)$-modules where $e=\frac{1}{|N_W(W_0)|}\sum_{g\in N_W(W_0)} g$ it is an idempotent of $\mathbb{C}N_W(W_0)$. From the results of [4] we get an isomorphism of algebras $\mathscr{D}(X)^{N_W(W_0)}\simeq\mathscr{D}(X/N_W(W_0))$, thanks to the fact that $N_W(W_0)$ acts without fixed points on $X$.

Let us examine the structure of $\mathscr{D}(X)\rtimes N_W(W_0)$-modules for the case of a localized standard object. The localized Verma module $\Delta(E)_{reg}$ is a free $\mathbb{C}[X]$-module of dimension $\dim(E)$, so it corresponds to an algebraic vector bundle over $X$. We endow this vector bundle with a connection by considering the action of $T_y$ on an element of $\Delta(E)_{reg}$. This leads to the formula

$$\nabla_y(P\otimes v):=\partial_y(P\otimes v)=\partial_y(P)\otimes v+\sum_{H\in\mathscr{A}_0}\frac{\alpha_H(y)}{\alpha_H}.(P\otimes a_H v)$$

**Proposition 4.** *$\nabla_y$ is a flat, $N_W(W_0)$-equivariant connection with regular singularities over $V$.*

Since this property is true for every standard object, it is also true for every object in $\mathscr{O}$. Applying the Riemann–Hilbert–Deligne correspondance, we get a horizontal sections functor $\frac{\mathscr{O}}{\mathscr{O}_{tor}} \to \mathbb{C}\pi_1(X/N_W(W_0))$-mod, $M \to ((M_{reg}^{N_W(W_0)})^{an})^{\nabla}$. According to [11, Proposition 2.6], this action by monodromy factorizes through $H(W,W_0)$. So we get a functor $KZ_0 : \frac{\mathscr{O}}{\mathscr{O}_{tor}} \to H(W,W_0)$-mod which is exact and fully-faithful. From classical results (see [15]), we get that $KZ_0$ is representable by a projective object noted $P_{KZ_0}$. We prove the following

**Theorem 5.** *$KZ_0$ is fully faithful and essentially surjective from the category $\frac{\mathscr{O}}{\mathscr{O}_{tor}}$ to the category of $H(W,W_0)$-modules of finite dimension.*

## 3. Forgetting $W$

In this section we provide a related result involving only $W_0$, and not the ambient group $W$. The general setting is as follows. Let $G$ be a finite subgroup of $GL(V)$. Let $G_0$ be a normal subgroup of $G$ generated by reflexions. Let $\mathscr{R}_0$ be the set of reflexions of $G_0$ and $\mathscr{A}_0$ the arrangement of reflecting hyperplanes of $G_0$. The first goal is to build up a Hecke algebra for $G$ from the Hecke algebra of $G_0$ generalizing $H(W_0,W)$ for $G = N_W(W_0)$.

Let $X^+$ be the subspace of $V$ on which $G$ acts freely and let $X_0$ be the subspace of $V$ on which $G_0$ acts freely. The manifold $X_0 \setminus X^+$ is of codimension $> 2$ then $\pi_1(X^+) \simeq \pi_1(X_0)$ [12, Theorem 2.3]. Since $G_0$ acts freely on $X_0$, it also acts freely on $X^+$ therefore the projection maps $X_0 \to X_0/G_0$ and $X^+ \to X^+/G_0$ are covering maps and we get two short exact sequences.

$$\begin{array}{ccccccccc} 1 & \longrightarrow & \pi_1(X^+) & \longrightarrow & \pi_1(X^+/G_0) & \longrightarrow & G_0 & \longrightarrow & 1 \\ \uparrow{\scriptstyle =} & & \uparrow{\scriptstyle \simeq} & & \uparrow & & \uparrow{\scriptstyle =} & & \uparrow{\scriptstyle =} \\ 1 & \longrightarrow & \pi_1(X_0) & \longrightarrow & \pi_1(X_0/G_0) & \longrightarrow & G_0 & \longrightarrow & 1 \end{array}$$

The exactness and the commutativity of the diagram together imply $\pi_1(X^+/G_0) \simeq \pi_1(X_0/G_0)$.

The braid group $B_0$ of $G_0$ is a normal subgroup of $B := \pi_1(X^+/G)$, we get a short exact sequence

$$1 \longrightarrow B_0 := \pi_1(X_0/G_0) \longrightarrow \pi_1(X^+/G) \longrightarrow G/G_0 \longrightarrow 1$$

Let $I$ be the ideal of $\mathbb{C}B_0$ generated by the relations $\sigma_H^{m_H} = \sum_{k=0}^{m_H-1} a_{H,k}\sigma_H^k$ for $\sigma_H$ a braided reflection associated to $H \in \mathscr{A}_0$. Then the Hecke algebra of $G_0$ is the quotient $H_0 := \frac{\mathbb{C}B_0}{I}$. According to the now proven BMR freeness conjecture (see the references of [11] or its weaker version in Characteristic 0 [5]) it is an algebra finitely generated of dimension $|G_0|$. Let $I^+ = \mathbb{C}B \otimes_{\mathbb{C}B_0} I$ be the ideal which define the Hecke algebra of $G$, $H(G) := \frac{\mathbb{C}B}{I^+} \simeq \mathbb{C}B \otimes_{\mathbb{C}B_0} H_0$ is of dimension $|G|$.

Let us make a link between this new algebra and the algebra $H(W_0,W)$. We defined $H(W_0,W)$ as a quotient of the algebra $\mathbb{C}\widetilde{B}_0$. We defined $\widetilde{B}_0$ as the quotient of $\pi_1(X/N_W(W_0))$ by $K := \mathrm{Ker}(\pi_1(X) \to \pi_1(X_0))$. Since $X_0 \setminus X^+$ has codimension $> 2$

$$K = \mathrm{Ker}(\pi_1(X) \longrightarrow \pi_1(X_0)) \simeq \mathrm{Ker}(\pi_1(X/N_W(W_0)) \longrightarrow \pi_1(X^+/N_W(W_0)))$$

And $\widetilde{B}_0 \simeq \pi_1(X^+/N_W(W_0))$ is our group $\pi_1(X^+/G) =: B$. As a result, the algebra $H(W_0,W)$ is the same as $H(G)$.

Let us consider the category $\mathscr{O}_{tor}^0$ the full subcategory of $\mathscr{O}$ of module annihilated by a power of $\delta_0 := \prod_{H\in\mathscr{A}_0} \alpha_H$. We have

**Theorem 6.** *$KZ_0$ is fully faithful and essentially surjective from the category $\frac{\mathscr{O}}{\mathscr{O}_{tor}^0}$ to the category of finite dimension $H(G)$-modules.*

A priori $\mathcal{O}_{tor}$ and $\mathcal{O}^0_{tor}$ are differents. Actually, we can prove that these two categories are the same. Let $M \in \mathcal{O}^0_{tor}$ then $\mathrm{Loc}(M) = \mathbb{C}[X] \otimes_{\mathbb{C}[X_0]} \underbrace{(\mathbb{C}[X_0] \otimes_{\mathbb{C}[V]} M)}_{=0}$, so $M \in \mathcal{O}_{tor}$.

Conversely, let $M$ be a module inside $\mathcal{O}_{tor}$, we would like to prove $M_{reg^0} := \mathbb{C}[X_0] \otimes_{\mathbb{C}[V]} M = 0$. Let $i : X^+ \hookrightarrow X_0$ be a continuous injection of the open set $X^+$ inside $X_0$. We denote by $\mathcal{O}_{X^+}$ the structural sheaf of $X^+$ and $\mathcal{O}_{X_0}$ the structural sheaf of $\mathcal{O}_{X_0}$. We denote by $\mathcal{D}_{X_0}$ the sheaf of algebraic differential operators over $X_0$ and $\mathcal{D}_{X^+}$ the sheaf of algebraic differential operators over $X^+$, [8, definitions 2.1.5 and 2.1.12].

We have a morphism of ringed space $(i, i^\sharp) : (X^+, \mathcal{O}_{X^+}) \to (X_0, \mathcal{O}_{X_0})$ where $i^\sharp : i^{-1}\mathcal{O}_{X_0} \to \mathcal{O}_{X^+}$ is the identity map, then $i^\sharp_x : \mathcal{O}_{X_0,x} \to \mathcal{O}_{X^+,x}$ is the identity too. The pull back functor is

$$\begin{aligned} i^* : \mathcal{D}_{X_0}\text{-mod} &\longrightarrow \mathcal{D}_{X^+}\text{-mod} \\ (M, \nabla_0) &\longmapsto (i^*M, i^*\nabla_0) \end{aligned}$$

We have two functors $A(N_W(W_0))^0_{reg}$-mod $\to \mathcal{D}_{X_0}$-mod, $M \to (M, \nabla_0)$ and $A(N_W(W_0))_{reg}$-mod $\to \mathcal{D}_{X^+}$-mod, $M \to (M, \nabla_0)$.

We need to prove $i^*M_{reg^0} = 0$. We have for all $x \in X \subset X_0$, $M_{reg,x} = 0$ it is due to $M \in \mathcal{O}_{tor}$. Since $M_{reg^0}$ and $M_{reg}$ are locally free $\mathcal{O}_{X_0}$-module, respectively $\mathcal{O}_{X_0}$-module, $(i^*M_{reg^0})_x \simeq M_{reg,x}$. Therefore, $(i^*M_{reg^0})_x \simeq M_{reg^0,x} \simeq \mathcal{O}^n_{X_0,x} \simeq 0$ then $n = 0$.

Since $i^*M_{reg^0}$ is a locally free $\mathcal{O}_X$ module, there exists an open affine covering $(U_i)_{i\in I}$ of $X$ such that $(i^*M_{reg^0})_{|U_i} \simeq (\mathcal{O}_{X_{|U_i}})^n = 0$, thus $i^*M_{reg^0} = 0$ so $M_{reg^0} = 0$ then $M \in \mathcal{O}^0_{tor}$. The categories $\mathcal{O}_{tor}$ and $\mathcal{O}^0_{tor}$ are equals. The proof of the equivalence of categories induced by $KZ_0$ uses the same arguments as for 5.

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