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**Introduction**


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# INTRODUCTION

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This topical volume of Confluentes Mathematici grew out of the lectures of the Journées de Physique Mathématique 2014 on "BPS States, Hitchin Systems and Quivers". The Journées de Physique Mathématique is an annual advanced school organised jointly by the École Normale Supérieure Lyon and the Université Claude Bernard Lyon 1, which focuses each year on a different subject in mathematical physics. The 2014 edition took place at the Institut Camille Jordan of the UCBL from September 3–5 and consisted of three lecture courses of four lectures each, around the subjects mentioned in the title.

The mathematical subjects of Hitchin systems and quiver representations may sound rather disconnected at first. However, in recent advances on supersymmetric quantum field theories both subjects appear prominently in the study of the quantum states with lowest energy, the so-called Bogomol'nyi-Prasad-Sommerfield (BPS) states. This class of states has been the key to numerous advances in field theory and gravity and may be useful to classify supersymmetric theories in the future. Hitchin systems occur in particular in the string theoretic description of such quantum field theories in terms of an auxiliary Riemann surface, while moduli spaces of quiver representations have been indispensable for our understanding of the spectrum of BPS bound states in these theories. Reversely, the interactions with physics have led to many new insights for the mathematics of both Hitchin systems and quivers.

The aim of the meeting was to bring together these different research directions so as to give an overview of the state of the art and to encourage further developments among these rich subjects. The following lecture courses were presented during the workshop:

- Philip Boalch, "Wild Character Varieties and Wild Mapping Class Groups,"
- Gregory W. Moore, "Algebraic Structures in $1+1$ Dimensional Massive $(2,2)$ Theories,"
- Markus Reineke, "Quiver Moduli and BPS State Counts."

Three seminars complemented the program:

- Iosif Bena, "Multi-center solutions, quivers, and their implication for black hole physics and the information paradox,"
- Frédéric Chapoton, "On cluster varieties associated with tree-shaped quivers,"
- Michele Del Zotto, "Progress about 4D $\mathcal{N}=2$ Quivers."

The organizing committee consisted of Johannes Kellendonk (UCBL), Jan Manschot, (UCBL and Trinity College Dublin), Henning Samtleben (ENS Lyon) and Thomas Strobl (UCBL).

We have collected four articles related to the topic of the school in this volume of Confluentes Mathematici, which represent the interdisciplinary nature of this

research field. We have ordered the four articles by the dimension of the underlying physical theory (namely two, four, six, and once more six dimensions):

- Davide Gaiotto, Gregory Moore and Edward Witten (p. 5–48) discuss boundary conditions in a class of supersymmetric two-dimensional field theories. They provide an introduction to the web-based formalism developed by the authors, which describes the category associated to half-supersymmetric boundary conditions. The web-based category is equivalent to a version of the Fukaya-Seidel $A_\infty$ category and has applications for knot homologies, categorified wall-crossing and surface defects.
- Jan Manschot, Boris Pioline and Ashoke Sen (p. 49–69) review their work on the spectrum of BPS bound states with an emphasis on a formula, which conjecturally determines the Hodge polynomial of moduli spaces of quiver representations in terms of a set of so-called 'single-center' invariants. This formula is known as the Coulomb Branch Formula, and is inspired by the physics of bound states of supersymmetric black holes in four dimensions.
- Michele Cirafici (p. 71–99) provides a thorough introduction to recent developments for BPS solutions of six-dimensional supersymmetric field theories considered on a (possibly non-compact) Calabi-Yau three-fold, which includes works by the author. The article explains in detail how moduli spaces of BPS solutions including defects can be analyzed using algebraic-geometric techniques such as moduli spaces of representations of framed quivers and sheaves.
- Sven Meinhardt (p. 101–158) gives a comprehensive introduction to recent advances in the subject of Donaldson-Thomas theory of quivers with potential. This theory provides a formal framework for invariants of moduli spaces in algebraic geometry, to which the author has made various contributions. Sheaves supported on a Calabi-Yau three-fold can in many cases be related to a quiver with potential and provide in this way a formal mathematical approach to BPS invariants discussed in other articles in this volume.

Let us conclude by thanking the UCBL, the ENS Lyon and the Labex MILYON (ANR-10-LABX-0070 and ANR-11-IDEX-0007) for their generous support to realize the meeting. We would also like to thank Frank Wagner for his assistance with preparing this special volume, and the anonymous referees for their time and energy to review the articles.

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