ANNALES DE L'INSTITUT FOURIER

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Annales de l'institut Fourier, tome 45, nº 2 (1995), p. 565-575

<http://www.numdam.org/item?id=AIF_1995__45_2_565_0>

Ann. Inst. Fourier, Grenoble
45, 2 (1995), 565–575

THE ASYMPTOTICS OF SPHERICAL FUNCTIONS AND THE CENTRAL LIMIT THEOREM ON SYMMETRIC CONES

by Genkai ZHANG

0. Introduction.

In multivariate statistical analysis the spaces of positive definite matrices (real or complex) are of considerable interests. One studies the spherical polynomials, central limit theorem on them. See [M]. A natural generalization of those spaces is the cone of positive elements in a formally real Jordan algebra. This class of cones includes cones of positive definite real, complex and quaternion matrices and the forward light cone. In this paper we will prove a kind of central limit theorem on any such symmetric cones.

In his paper [T2] Terras obtained a central limit theorem for rotation-invariant random variables on the space of $n \times n$ real positive definite (symmetric) matrices in the case $n = 3$. This has been recently generalized to any n by Richards [R]. In this paper we will generalize this central limit theorem to the symmetric cone of positive elements in a Jordan algebra.

The main tools used in [R] are the Binet-Cauchy theorem and certain coefficients in some recurrence formulas of zonal (spherical) polynomials in [Ku]. Recently we have found some recurrence formulas on any simple and formally real Jordan algebra [Z]. To prove our central limit theorem we will

Key words: Spherical functions – Central limit theorem – Jordan algebra – Symmetric spaces – Heat equation – Spherical polynomial.
Math. classification: 43A65 – 62E15.

use those recurrence formulas and a recent result of Faraut and Koranyi [FK] on expansion of reproducing kernels.

Acknowledgment. — The author would like to thank Jonathan Arazy for comments on the paper and Bent Ørsted for constant encouragement. He also thanks Donald. St. P. Richards for drawing his attention to the work of P. Graczyk ([G1] and [G2]). He acknowledges the financial support from Odense University, Denmark, where this work was done.

1. Preliminaries.

In this section we recall some facts about the symmetric cone in a Jordan algebra. Most results in this section can be found, e.g. in [UU] and [KS] (on formally real Jordan algebras) and [T1] (in the special case of real symmetric matrices).

Let X be a simple and formally real Jordan algebra [BK] with product $x \circ y$ and unit element e. Basic examples are the space $X = \mathcal{H}_r(\mathbb{K})$ of all self-adjoint $r \times r$-matrices over $\mathbb{K} = \mathbb{R}, \mathbb{C}$ or $\mathbb{H}$ (quaternions) with the anti-commutator product $x \circ y = (xy + yx)/2$, and the "spin factor" $X = \mathbb{R}^{1+n}$ of all vectors $x = (x_0, x_1, \dots, x_n) = (x_o, x')$, with product $x \circ y = (x_0 y_0 + x'y', x_0 y' + y_0 x')$. The open set

$$\mathcal{P} = \{x^2, x \in X \text{ invertible}\}$$

is a convex cone. For the matrix algebras, $\mathcal{P}$ is the cone of positive definite matrices, whereas for the spin factor $\mathcal{P}$ is the forward light cone. The linear automorphism group

$$GL(\mathcal{P}) = \{g \in GL(X) : g(\mathcal{P}) = \mathcal{P}\}$$

is transitive on $\mathcal{P}$. Let G be the identity component of $GL(\mathcal{P})$. G has Lie algebra

$$\mathfrak{g} = \{M \in \mathfrak{gl}(X) : \exp(tM) \in GL(\mathcal{P}), \text{ for all } t \in \mathbb{R}\}$$

with the commutator bracket $[M, N] = MN - NM$. The Lie algebra $\mathfrak{g}$ has a Cartan decomposition $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, where $\mathfrak{k}$ is the Lie algebra of the "orthogonal" group

$$K = O(\mathcal{P}) = \{g \in G : ge = e\}$$

and $\mathfrak{p} = \{M_x : x \in X\}$ consists of all multiplication operators

$$M_x y = x \circ y$$

on X. Moreover $\mathcal{P} = G/K$ is now a Riemannian symmetric space.

Let $e_1, \ldots, e_r$ be a maximal orthogonal system of idempotents in X. Then we have

$$e = e_1 + \ldots + e_r.$$

Let

$$X = \sum_{1 \le i \le j \le r} X_{ij} \tag{1.1}$$

be the Peirce decomposition induced by $\{e_1, \ldots, e_r\}$. Here

$$X_{i,j} = X_{ji} = \left\{ x \in X : e_k \circ x = \frac{\delta_{jk} + \delta_{ik}}{2} x \right\}$$

are the joint eigenspaces of the multiplication operators $\{M_{e_j}\}_{j=1}^r$ on X. The integer r is independent of the choice of the maximal orthogonal system of idempotents and is called the rank of X.

Let

$$\mathfrak{a} = \mathbb{R}M_{e_1} + \ldots + \mathbb{R}M_{e_r}$$

be the subspace of $\mathfrak{p}$ generated by $M_{e_1}, \ldots, M_{e_r}$. Then $\mathfrak{a}$ is maximal abelian in $\mathfrak{p}$. So r is the rank of the Riemannian symmetric space $\mathcal{P}$. We have thus a root space decomposition of $(\mathfrak{g}, \mathfrak{a})$ [H1]. We let $\{\gamma_j\}_{j=1}^r$ be the dual basis in $\mathfrak{a}^*$ of $\{M_{e_j}\}_{j=1}^r$, i.e.

$$\gamma_j(M_{e_k}) = \delta_{jk}.$$

Then the corresponding root system consists of $\dfrac{\gamma_j - \gamma_k}{2}$, $j \neq k$ with common multiplicity a. The Weyl group is the symmetric group S_r. For example in the case of matrix algebra $\mathcal{H}_r(\mathbb{K})$ the rank is r and the multiplicity is $a = 1, 2$ or 4 according to $\mathbb{K} = \mathbb{R}, \mathbb{C}$, or $\mathbb{H}$ respectively.

We fix an ordering of $\mathfrak{a}^*$ by letting

$$\gamma_1 < \gamma_2 < \cdots < \gamma_r.$$

Therefore, the positive roots are $\dfrac{\gamma_j - \gamma_k}{2}$, $j > k$. Let $\underline{\rho} = \sum_{j=1}^r \rho_j \gamma_j$ be, as usual, the half sum of positive roots

$$\underline{\rho} = \frac{a}{2} \sum_{j \ge k} \frac{\gamma_j - \gamma_k}{2} = \frac{a}{2} \sum_{j=1}^r \left(j - \frac{r+1}{2} \right) \gamma_j.$$

We will now describe the spherical functions and the heat equation on the symmetric space $\mathcal{P}$. We need the notion of Jordan algebra determinants.

Let $P(x) := 2(M_x)^2 - M_{x^2}$. The determinant of it is up to a constant a unique (integer) power of an irreducible polynomial $\Delta(x)$ of degree r on X, normalized so that $\Delta(e) = 1$. $\Delta(x)$ is called the determinant of the Jordan algebra X. It can also be defined as the unique irreducible polynomial given by the "Cramer's rule":

$$x^{-1} = \frac{\text{cofactor of } x}{\Delta(x)}, \ (x \in X \text{ invertible})$$

normalized so that $\Delta(e) = 1$.

Let X_{ij} be given in the Peirce decomposition (1.1) we put

$$X_j = \sum_{k,\ell \le j} X_{k\ell}$$

and let P_j be the orthogonal projection of X on X_j; we define

$$\Delta_j(x) = \Delta_{X_j}(P_j x)$$

where Δ_{X_j} is the corresponding determinant in X_j.

For any $\underline{\mathbf{t}} = (t_1, \dots, t_r) \in \mathbb{C}^r$ we define

$$\Delta_{\underline{s}}(x) = \prod_{j=1}^{r} \Delta_j^{s_j}(x). \tag{1.2}$$

Let $\boldsymbol{\lambda} = \sum_{j=1}^{r} \lambda_j \gamma_j \in \mathfrak{a}_{\mathbb{C}}^*$, the complexification of $\mathfrak{a}^*$, we let $\underline{s} = (s_1, s_2, \dots, s_r)$ be defined by

$$s_j = \lambda_j + \rho_j - \lambda_{j+1} - \rho_{j+1}, \tag{1.3}$$

where we put $s_r = \lambda_r + \rho_r$. The spherical function $h_{\boldsymbol{\lambda}}$ is then

$$h_{\boldsymbol{\lambda}}(x) = \int_K \Delta_{\underline{s}}(kx)dk = \int_K \prod_{j=1}^{r} \Delta_j^{s_j}(kx)dk.$$

Then $h_{\boldsymbol{\lambda}}(x)$ are K-invariant eigenfunctions of all the G-invariant differential operators on $\mathcal{P}$. The Helgason-Fourier transform of a K-invariant function f on $\mathcal{P}$ is defined by

$$\widehat{f}(\boldsymbol{\lambda}) = \int_{\mathcal{P}} f(x) h_{\boldsymbol{\lambda}}(x) d^*x,$$

where d^* is the G-invariant measure on $\mathcal{P}$.

For later use we also introduce the spherical polynomials, which is the analytic continuation of $h_{\boldsymbol{\lambda}}$ (in $\boldsymbol{\lambda}$). Let $\underline{\mathbf{m}} = (m_1, \dots, m_r)$ be a r-tuple of integers with $m_1 \ge m_2 \ge \dots \ge m_r \ge 0$. The functions

$$\phi_{\underline{\mathbf{m}}}(x) = \int_K \prod_{j=1}^{r} \Delta_j^{m_j - m_{j+1}}(kx)dk,$$

where $m_{r+1} = 0$, are the spherical polynomials on $\mathcal{P}$. In particular if we put

$$\underline{\mathbf{1^j}} = \sum_{k=1}^{j} \gamma_k,$$

the functions

$$\phi_{\underline{\mathbf{1^j}}}(x) = \int_K \Delta_{\underline{\mathbf{1^j}}}(kx)dk = \int_K \Delta_j(kx)dk$$

are called the fundamental spherical polynomials on $\mathcal{P}$.

Finally we recall the heat equation on $\mathcal{P}$. Let L be the Laplace-Beltrami operator on $\mathcal{P}$. We normalize the K-invariant norm on $\mathfrak{p}$ so that norm M_{e_1} has norm 1. Then $M_{e_1}, \ldots, M_{e_r}$ are orthonormal basis of $\mathfrak{a}$. This also gives a normalization of the Riemannian metric on the cone $\mathcal{P} = G/K$.

Then $\phi_{\boldsymbol{\lambda}}$ are eigenfunctions of L:

$$L\phi_{\boldsymbol{\lambda}} = \Big(\sum_{j=1}^{r} \lambda_j^2 - \frac{a(r^3 - r)}{48}\Big)\phi_{\boldsymbol{\lambda}}.$$

The heat equation is defined

$$\begin{cases} \partial_t u(x,t) = Lu(x,t),\ (x,t) \in \mathcal{P} \times \mathbb{R}^+ \\ u(x,0) = f(x). \end{cases}$$

2. The main theorem and the proof.

Before stating our main theorem we introduce some notation. Let $H = \sum_{j=1}^{r} h_j M_{e_j} \in \mathfrak{a}$. We let $J(\exp H)$ be the function defined on A by

$$J(\exp H) = \frac{1}{2^{\binom{n}{2}}} \prod_{i \geq j} (e^{h_i} - e^{h_i})^a \prod_{i=1}^{r} e^{-a\frac{(n-1)}{2} h_k}.$$

Our main theorem is the following central limit theorem.

THEOREM 1. — *Let Y_m, $m \geq 1$ be a sequence of independent, identical distributed, K-invariant random variables in $\mathcal{P}$, with common density function $f(Y)$. Assume that $f(Y)$ satisfies*

$$\int_{\mathfrak{a}} h_i f(\exp H) J(\exp H) dh_1 \cdots dh_r = 0$$

$$\int_{\mathfrak{a}} h_i h_j f(\exp H) J(\exp H) dh_1 \cdots dh_r = 0, \quad i \neq j$$

and the convolution $S_m = Y_1 \circ Y_2 \circ \cdots \circ Y_m$ *is normalized as in (3.39) of [T1] and that* S_m *has density function* $f_m^\sharp$. *Then for any measurable subset* $S \subset \mathcal{P}$

$$\int_S f_m^\sharp(Y) d^*(Y) \approx \exp \frac{a^3(r^3 - r)}{96(2 + ar)} \times \int_S G_{\frac{(2+a)}{2(2+ar)}} * F_{\frac{a}{2+ar}}(Y) d^*(Y)$$

as $m \to \infty$.

In the above, G_t *is the fundamental solution of the heat equation on* $\mathcal{P}$ *with Helgason-Fourier transform*

$$\widehat{G_t}(\boldsymbol{\lambda}) = \exp\left(t\Big(\sum_{i=1}^r \lambda_i^2 - \frac{a(r^3 - r)}{48} \Big) \right).$$

Further F_t *has Helgason-Fourier transform*

$$\widehat{F_t}(\boldsymbol{\lambda}) = \exp\left(t\Big(\sum_{1 \le k < j \le r} \lambda_i \lambda_j \Big) \right).$$

Following [T1] and [R] we need to find the expansion of the spherical functions $h_{\boldsymbol{\lambda}}(\exp H)$ on $\mathfrak{a}$ around $H = 0$. Recall that spherical functions $h_{\boldsymbol{\lambda}}(\exp H)$ are Weyl group invariant in $\boldsymbol{\lambda}$ and $H \in \mathfrak{a}$. The expansion is (recalling that $H = \sum_{j=1}^r h_j M_{e_j}$)

$$\begin{aligned} h_{\boldsymbol{\lambda}}(\exp H) =& 1 + \Big(\alpha_{11} \sum_{i=1}^r \lambda_i + \alpha_{12}\Big)\Big(\sum_{i=1}^r h_i\Big) \\ &+ \Big(\alpha_{21} \sum_{i=1}^r \lambda_i^2 + \alpha_{22} \sum_{1 \le i < j \le r} \lambda_i \lambda_j + \alpha_{23} \sum_{i=1}^r \lambda_i + \alpha_{24}\Big)\Big(\sum_{i=1}^r h_i^2\Big) \\ &+ P(\lambda_1, \dots, \lambda_r)\Big(\sum_{1 \le i < j \le r} h_i h_j \Big) + \text{ higher terms;} \end{aligned}$$

namely we need to find the six coefficients α_{11}, α_{12}, α_{21}, α_{22}, α_{23}, α_{24}.

We fix $\boldsymbol{\lambda} \in \mathfrak{a}_{\mathbb{C}}^*$ and $\underline{s}$ as in (1.3). Consider the formula (1.2) in the polar coordinates

$$\Delta_{\underline{s}}(ka \cdot e) = \prod_{j=1}^r \Delta_j^{s_j}(ka \cdot e) \tag{2.1}$$

and put $a = \exp H$. Following [R] we take logarithm differentiation to obtain

$$\frac{1}{\Delta_{\underline{s}}(ka \cdot e)} \frac{\partial}{\partial h_1} \Delta_{\underline{s}}(ka \cdot e) = \sum_{j=1}^r s_j \frac{1}{\Delta_j(ka \cdot e)} \frac{\partial}{\partial h_1} \Delta_j(ka \cdot e). \tag{2.2}$$

Integrating over K and putting $H=0$ we get

$$\frac{\partial}{\partial h_1} h_{\boldsymbol{\lambda}}(a\cdot e)\bigg|_{H=0} = \sum_{j=1}^{r} s_j \frac{\partial}{\partial h_1}\phi_{\underline{\mathbf{1}_\mathbf{j}}}(a\cdot e)\bigg|_{H=0}. \tag{2.3}$$

We adopt the notation [R] to denote

$$f_j'(k) = \frac{\partial}{\partial h_1}\Delta_j(ka\cdot e)\bigg|_{H=0}$$

$$f_j''(k) = \frac{\partial^2}{\partial h_1^2}\Delta_j(ka\cdot e)\bigg|_{H=0}$$

and

$$\mathcal{E}(f) = \int_K f(k)dk$$

for a function f on K.

We need a result of Faraut and Koranyi [FK].

Lemma 1. — *For $a=\exp H$ we have*

$$\prod_{j=1}^{r}(1-te^{h_j}) = \sum_{j=0}^{r}(-1)^j\binom{r}{j}\phi_{\underline{\mathbf{1}_\mathbf{j}}}(a\cdot e)t^j. \tag{2.4}$$

Proof. — This is a special case of Theorem 3.8 in [FK]. See also Remark 2 after the theorem. We give here a simple independent proof. We know that the polynomial $\Delta(e-z)$ is a polynomial of degree r on X and is K invariant. Thus we have an expansion

$$\Delta(e-tz) = \sum_{j=0}^{r} c_j\phi_{\underline{\mathbf{1}_\mathbf{j}}}(z)t^j,$$

for some constants c_j. Now take $z=a\cdot e$ the above equation reads

$$\prod_{j=1}^{r}(1-te^{h_j}) = \sum_{j=0}^{r} c_j\phi_{\underline{\mathbf{1}_\mathbf{j}}}(z)t^j.$$

Evaluating the equation at $H=0$ and noting that $\phi_{\underline{\mathbf{1}_\mathbf{j}}}(e)=1$ we find the coefficients c_j. □

Lemma 1 has also been proved previously by J. Arazy and Z. Yan [A].

Now we differentiate the expansion in Lemma 1 and set $H=0$,

$$-t(1-t)^{r-1} = \sum_{j=0}^{r}(-1)^j\binom{r-1}{j-1}\frac{r}{j}\frac{\partial}{\partial h_1}\phi_{\underline{\mathbf{1}_\mathbf{j}}}(a\cdot e)\bigg|_{H=0} t^j;$$

however the LHS of the above is

$$-t(1-t)^{r-1} = \sum_{j=0}^{r}(-1)^j \binom{r-1}{j-1} t^j,$$

thus we find

$$\mathcal{E}(f_1') = \frac{\partial}{\partial h_1}\phi_{\underline{1^j}}(a\cdot e)\Big|_{H=0} = j/r.$$

Using this we see

$$\alpha_{11} = \frac{1}{r}, \quad \alpha_{12} = 0.$$

Note that if we differentiate (2.4) twice we get

$$\mathcal{E}(f_1'') = \frac{\partial^2}{\partial h_1^2}\phi_{\underline{1^j}}(a\cdot e)\Big|_{H=0} = j/r.$$

Now the coefficients of $2\sum_{i=1}^{r} h_i^2$ is

$$Q(s_1,\dots,s_r) = \frac{\partial^2}{\partial h_1^2} h_{\boldsymbol{\lambda}}(a\cdot e)\Big|_{H=0}. \tag{2.5}$$

We have

$$Q(s_1,\dots,s_r) = \mathcal{E}\Big(\Big(\sum_{j=1}^{r} s_j f_j'\Big)^2\Big) + \sum_{j=1}^{r}(s_j\mathcal{E}(f_j'') - s_j^2\mathcal{E}((f_j')^2)).$$

We need to find

$$\mathcal{E}(f_1'f_2'),\ \mathcal{E}((f_1')^2).$$

We start to calculate $\mathcal{E}(f_1'f_j')$. It follows from §1 that

$$\Delta_{\underline{1^1}+\underline{1^j}} = \Delta_1\Delta_j.$$

Differentiating at $H=0$ and integrating we find

$$\frac{\partial^2}{\partial h_1^2}\phi_{\underline{1^1}+\underline{1^j}}(ka\cdot e)\Big|_{H=0} = \mathcal{E}(f_1'') + \mathcal{E}(f_j'') + 2\mathcal{E}(f_1'')\mathcal{E}(f_j''). \tag{2.6}$$

To find out the LHS we need the a result proved in [Z] (see also [St]).

LEMMA 2. — *We have the recurrence formula of the product $\phi_{\underline{1^1}}\phi_{\underline{1^j}}$ of the spherical polynomials*

$$\phi_{\underline{1^1}}\phi_{\underline{1^j}} = \frac{j(1+\frac{a}{2}r)}{r(1+\frac{a}{2}j)}\phi_{\underline{1^1}+\underline{1^j}} + \frac{r-j}{r(1+\frac{a}{2}j)}\phi_{\underline{1^{j+1}}}$$

for the product $\phi_{\underline{1^1}}\phi_{\underline{1^j}}$.

The lemma can also be written as

$$\frac{j(1+\frac{a}{2}r)}{r(1+\frac{a}{2}j)}\phi_{\underline{1}^1+\underline{1}^j} = \phi_{\underline{1}^1}\phi_{\underline{1}^j} - \frac{r-j}{r(1+\frac{a}{2}j)}\phi_{\underline{1}^{j+1}}.$$

Thus

$$\frac{\partial^2}{\partial^2 h_1}\phi_{\underline{1}^1+\underline{1}^j}(ka\cdot e)\Big|_{H=0} = \frac{(1+\frac{a}{2}j)(2j+r(j+1))-(r-j)(j+1)}{(rj(1+\frac{a}{2}r)}.$$

Now we substitute this to (2.6) to get

$$\mathcal{E}(f_1'f_j') = \frac{2+aj}{2r+ar^2}.$$

In particular

$$\mathcal{E}\left((f_1')^2\right) = \frac{2+a}{2r+ar^2}.$$

Thus the coefficient of $\lambda_1\lambda_2$ in (2.5) is

$$\begin{aligned} 2\alpha_{22} &= 2(\mathcal{E}((f_1'f_2') - \mathcal{E}\left((f_1')^2\right)) \\ &= \frac{2a}{2r+ar^2} \end{aligned}$$

and the coefficient of λ_1^2 is

$$\begin{aligned} 2\alpha_{21} &= \mathcal{E}\left((f_1')^2\right) \\ &= \frac{2+a}{2r+ar^2}. \end{aligned}$$

The coefficient of λ_1 in (2.5) is

$$\begin{aligned} 2\alpha_{23} &= \mathcal{E}(f_1') - \mathcal{E}\left((f_1')^2\right) - a\sum_{j=2}^{r-1}\mathcal{E}(f_1'f_j') + \frac{a}{2}(r-1)\mathcal{E}(f_1') \\ &= 0. \end{aligned}$$

On the other hand, we have $Q(s_1,\ldots,s_r) = 0$ for $\underline{s} = 0$ or for $\lambda_j = -\frac{a}{2}(j - \frac{r+1}{2})$, that is

$$0 = \alpha_{21}\sum_{j=1}^{r}\lambda_j^2 + \alpha_{22}\sum_{1\le i<j\le r}\lambda_i\lambda_j + \alpha_{24}$$

which gives us

$$\alpha_{24} = -\frac{a(r^2-1)}{48(2+ar)}.$$

Finally we get the required expansion

$$
\begin{aligned}
h_{\boldsymbol{\lambda}}(\exp H) =&1 + \Big(\frac{1}{r}\sum_{i=1}^{r}\lambda_i\Big)\Big(\sum_{i=1}^{r}h_i\Big) \\
&+ \Big(\frac{2+a}{2(2r+ar^2)}\sum_{i=1}^{r}\lambda_i^2 + \frac{a}{2r+ar^2}\sum_{1\leq i<j\leq r}\lambda_i\lambda_j \\
&\qquad\qquad - \frac{a(r^2-1)}{48(2+ar)}\Big)\Big(\sum_{i=1}^{r}h_i^2\Big) \\
&+ P(\lambda_1,\dots,\lambda_r)\Big(\sum_{\leq i<j\leq r}h_ih_j\Big) + \text{ higher terms.}
\end{aligned}
$$

The rest of the proof is similar to that in [T1].

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Manuscrit reçu le 18 octobre 1994.

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