C I R M

# *Courbure discrète : théorie et applications*

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18-22 novembre 2013

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**Introduction**

Vol. 3, n° 1 (2013), p. 1-2.

<http://acirm.cedram.org/item?id=ACIRM_2013__3_1_1_0>

Centre international de rencontres mathématiques
U.M.S. 822 C.N.R.S./S.M.F.
Luminy (Marseille) France

Actes des rencontres du C.I.R.M.
Vol. **3** n° 1 (2013) 1-2

# Introduction

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The present volume contains the proceedings of the 2013 Meeting on discrete curvature, held at CIRM, Luminy, France. The aim of this meeting was to bring together researchers from various backgrounds, ranging from mathematics to computer science, with a focus on both theory and applications. With 27 invited talks and 8 posters, the conference attracted 70 researchers from all over the world. The challenge of finding a common ground on the topic of discrete curvature was met with success, and these proceedings are a testimony of this work.

Discrete curvature has been a fast-growing topic and common denominator in many fields in the past decade. Indeed, many applications and theoretical constructions are related to or rely on some notion of discrete curvature or one of its avatars, such as the Laplace-Beltrami operator. More interestingly, new concepts and new approaches have emerged, sometimes from seemingly disconnected fields, allowing a better understanding of curvature in the discrete realm, as well as new ways to tackle applications. In parallel, new challenges arise in computer science that require more sophisticated theoretical apparatus, often using the latest theoretical developments.

Discrete curvature may arise on discretized surfaces, raising the problem of convergence to the smooth model. In that case, the curvature—or more appropriately the curvatures—need to be defined, and are the goal as well as an obstacle to convergence (Morvan & Sun, Fu, Tai). Its definition allows countless applications (Bac *et al.*, Olsson & Boykov). Discrete curvature may also appear as geometrically relevant quantity in a discrete space otherwise disconnected from actual smooth geometry, such as a graph (Keller). What we have seen in this meeting are different definitions and concepts of discrete curvature, suited to different problems and settings. However, they all share the common trait of defining a notion according to its geometric consequences and properties.

One example of recent development is the increasing role played by optimal transportation, an old problem with new aspects which applies particularly well to the discrete setup. The Wasserstein metric may be used to define curvature and deduce combinatorial, functional and topological information (Bauer *et al.*, Maas), but also to compare shapes (Alliez *et al.*, Memoli). This comparison principle is found also on polytopes (Baird), and analogous ideas appear in deformation and shape matching using various energies (Cremers *et al.*, Sorkine). And it relates to the metric approach to curvature (Saucan).

The Laplace-Beltrami operator and energy functional are linked to the curvature and play a key role in understanding the geometry and spectrum, and of course in countless applications, such as segmentation, inpainting (Stuehmer), curvature flows (Aflalo *et al.*, Boykov). A more profound approach to this operator can be achieved through exterior differential calculus (Memari, Leok), which seeks to preserve (some of) the structural properties of the continuum. Other functionals may intervene such as the Willmore functional or the Hilbert-Einstein functional (Izmestiev) to characterize the geometry.

A similar albeit different approach to differential calculus comes from integrable theory, developed in particular by the German school. The choice of a particular mesh (circular, quad-based) implies analogous properties to those of special surfaces (e.g. minimal or constant mean curvature surfaces) or special parametrization such as conformal maps. This yields natural and robust definitions for the curvature (Hoffmann) and applies remarkably to architectural constructions (Pottmann); at the same time, it relates to discrete complex analysis (Bobenko & Günther, Skopenkov).

Digital geometry also leads back to curvature, which can be defined on pixels and voxels in a very geometric way (Lachaud), and applies to topological problems (Kenmochi *et al.*) and flows (Imiya). One of the crucial issues there, as in many of the above cases (but not all), is the one of consistency or convergence properties, thus allowing to compute smooth quantities by refinement.

The success of this meeting is an encouragement to go further ahead. In addition to these proceedings, we plan to edit in the near future a survey of the different approaches to discrete curvature, based on what was presented here at CIRM, and reaching beyond. Such a resource will be very valuable to both researchers and students wishing to enter this rich field, and will help to disseminating and expanding the theory.

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