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J. J. DUISTERMAAT

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ON THE SIMILARITY BETWEEN THE IWASAWA PROJECTION AND THE DIAGONAL PART

by

J.J. Duistermaat

1. Statement of the result.

Let G be a real connected semisimple Lie group with finite center and G = KAN its Iwasawa decomposition. Via the adjoint representation, and with respect to a suitable basis in $\mathfrak{g}$, K, resp. A, resp. N are the set of matrices in G which are orthogonal, resp. diagonal with positive entries, resp. upper triangular.

The Iwasawa projection H from G onto the Lie algebra $\mathfrak{a}$ of A is defined by

(1.1) $\quad x \in K.\exp H(x).N, \quad x \in G.$

Obviously H factorizes through the projection from G onto the (non-compact Riemannian) symmetric space K\G. If $\mathfrak{s}$ (called $\mathfrak{p}$ by everybody else) denotes the orthogonal complement of $\mathfrak{k}$ in $\mathfrak{g}$ with respect to the killing form, then the Cartan decomposition G = K.exp $\mathfrak{s}$ yields that

(1.2) $\quad \mathfrak{s} \overset{\exp}{\rightarrow} G \rightarrow K\backslash G$

is a diffeomorphism from $\mathfrak{s}$ onto K\G. So the Iwasawa projection can be studied by looking at the mapping

(1.3) $\quad \gamma = H \circ \exp : \mathfrak{s} \rightarrow \mathfrak{a}.$

On the other hand we have the orthogonal projection

(1.4) $\pi : \mathfrak{s} \to \mathfrak{a}$

with respect to the Killing form. In the above matrix terminology, $\mathfrak{s}$ is the space of symmetric matrices in $\mathfrak{g}$ and π is the operation of taking the diagonal part of the symmetric matrix. So this projection has a very simple minded interpretation, whereas the Iwasawa projection is a rather more mysterious object.

Theorem 1.1. There is a real analytic map $\Psi : \mathfrak{s} \to K$ such that

i) $\Phi_X : k \to k.\Psi(\mathrm{Ad}\ k^{-1}(X))$ is a diffeomorphism from K onto K, for each $X \in \mathfrak{s}$.

ii) $\gamma(\mathrm{Ad}\ \Psi(X)^{-1}(X)) = \pi(X)$ for all $X \in \mathfrak{s}$.

That is, we can turn the Iwasawa projection into the orthogonal projection by an action of Ad K, the element of K depending analytically on $X \in \mathfrak{s}$.

It also follows from the theorem that the images of an Ad K-orbit in $\mathfrak{s}$ under γ and π are the same. This was obtained before by Kostant [4] who showed separately that both images are equal to the convex hull of the intersection of the Ad K-orbit in $\mathfrak{s}$ with $\mathfrak{a}$. Since this intersection is equal to a Weyl group orbit in $\mathfrak{a}$, which is finite, this image is a convex polytope. Very remarkable because an Ad K-orbit is such a roundish object!

Later Heckman [3] reduced the convexity theorem for the Iwasawa projection to the convexity theorem for the diagonal part, for which the proof is much simpler, using a homotopy argument. This homotopy argument actually is one of the elements in the proof of Theorem 1.1.

For me the major motivation for wanting the theorem was the study in [2], together with Kolk and Varadarajan, of the asymptotic behaviour of integrals of the form

(1.5) $$I_a(X,\xi) = \int_K e^{i\langle\gamma(\mathrm{Ad}\ k^{-1}(X)),\xi\rangle}.a(X,k)dk$$

as $\|\xi\| \to \infty$, $\xi \in \mathfrak{a}^*$. The matrix coefficients of the principal series representations of G are given by such integrals, the simplest case being the elementary spherical functions where

(1.6) $$a(X,k) = e^{-\langle\gamma(\mathrm{Ad}\ k^{-1}(X)),\rho\rangle}.$$

The idea in [2] was to consider (1.5) as an oscillatory integral, for which the asymptotics is concentrated at the stationary points of the "phase function".

(1.7) $$F_{X,\xi} : k \to \langle\gamma(\mathrm{Ad}\ k^{-1}(X)),\xi\rangle$$

On the similarity between the Iwasawa projection and the diagonal part

on K. We then observed that $F_{X,\xi}$ had exactly the same critical points and critical values as its "infinitesimal counterpart"

$$(1.8)\qquad f_{X,\xi} = \lim_{t\to 0} \frac{1}{t} F_{tX,\xi} : k \to \langle \pi(\mathrm{Ad}\, k^{-1}(X)), \xi\rangle.$$

These critical points in turn had such a special, rigid structure that the asymptotics of (1.5) could be obtained by a repeated application of the method of stationary phase.

It had already been observed in [2] that the equality of critical points and critical values of $F_{X,\xi}$ and $f_{X,\xi}$ leads to the existence of a diffeomorphism $\Phi_{X,\xi} : K \to K$ such that $F_{X,\xi} \circ \Phi_{X,\xi} = f_{X,\xi}$.

However, the diffeomorphism is not unique and at that time I could not find $\Phi_{X,\xi}$ depending smoothly on X and ξ. Already continuous dependence on ξ would imply, replacing ξ by $t\xi$, dividing by t, and letting $t \to 0$, that $F_{X,\xi} \circ \Phi_{X,0} = f_{X,\xi}$. That is, one could find a diffeomorphism Φ_X not depending on ξ. Then, using the substitution of variables

$$(1.9)\qquad k = \Phi_X(l), \quad l \in K,$$

the integral (1.5) can be rewritten as $(X \in \mathfrak{s},\ \xi \in \mathfrak{a}^*)$

$$(1.10)\qquad I_a(X,\xi) = \int_K e^{i\langle \pi(\mathrm{Ad}\, k^{-1}(X)),\xi\rangle} a(X,\Phi_X(k)) . \left|\det \frac{\partial \Phi_X}{\partial k}(k)\right| dk.$$

In this way the study of the asymptotic behaviour would be reduced to doing stationary phase with the simpler $f_{X,\xi}$ as the phase function, rather than $F_{X,\xi}$. (Such asymptotics has been done before by Clerc and Barlet [1].)

It is one of the applications of Theorem 1.1, that the integral representation (1.10) actually holds with a $\Phi_X(k)$ which depends analytically on X and k simultaneously. For instance, for the elementary spherical functions this leads to an integral formula of the form

$$(1.11)\qquad \phi_\xi(\exp X) = \int_K e^{i\langle \pi(\mathrm{Ad}\, k^{-1}(X)),\xi\rangle} b(\mathrm{Ad}\, k^{-1}(X))dk,$$

for some analytic function $b : \mathfrak{s} \to \mathbb{R}$. As an application of the analyticity of b, one can note that replacing ξ, resp. X by $i\xi$, resp. iX, one obtains the elementary spherical functions for the *compact* symmetric space which is dual to K\G. (In this case ξ has to be taken in a weight lattice.) So also for these functions an integral formula like (1.10) holds, at least for small $\|X\|$. I owe this observation to Richard van den Dries (T.H. Delft), who is using this integral formula in his characterization of invariant pseudo-differential operators on compact symmetric spaces in terms of their eigenvalues.

2. SL(2,R).

For $G = SL(2,\mathbb{R})$, $\dim K = \dim \mathfrak{a}$ (= 1), so the substitution of variables is unique up to a flip. In order to determine it explicitly, write the elements of K as

$$(2.1)\qquad k = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}, \theta \in \mathbb{R}/2\pi\mathbb{Z},$$

and the elements of $\mathfrak{a}$ as

$$(2.2)\qquad X = \begin{pmatrix} t & 0\\ 0 & -t\end{pmatrix}, \quad t \in \mathbb{R}.$$

Then $Y = \mathrm{Ad}\, k^{-1}(X) = k^{-1}Xk$ is the general element of $\mathfrak{s}$, and $\Phi_X(k)$ is the element of K with the coordinate μ given implicitly by

$$(2.3)\qquad e^{2t}\cos^2\mu + e^{-2t}\sin^2\mu = e^{2t\cos 2\theta}.$$

From this one can determine $\Psi(Y) = k^{-1}.\Phi_X(k)$. It is not entirely trivial to verify that this defines a real analytic mapping $\Psi : \mathfrak{s} \to \mathfrak{a}$!

The Jacobian of Φ_X is equal to

$$(2.4)\qquad \frac{2|t\sin 2\theta|.e^{t\cos 2\theta}}{\sqrt{2}\sqrt{\cosh(2t) - \cosh(2t\cos 2\theta)}},$$

leading to the following formula for the elementary spherical function:

$$(2.5)\qquad \phi_\xi(\exp X) = \frac{1}{2\pi}\int_{\mathbb{R}/2\pi\mathbb{Z}} e^{it\tau\cos 2\theta}\,\frac{|t\sin 2\theta|}{\sqrt{\frac{\cosh(2t) - \cosh(2t\cos 2\theta)}{2}}}\,d\theta.$$

Here we have written $\langle X,\xi\rangle = t\tau$. This can also be written as

$$(2.6)\qquad \phi_\xi(\exp X) = \frac{4}{\pi}\int_0^t \cos\tau s.\frac{ds}{\sqrt{\frac{1}{2}(\cosh(2t) - \cosh(2s))}}.$$

A similar formula for all rank one symmetric spaces can be found in Koornwinder [8], formula (2.16) and (2.18).

I prefer (2.5) over (2.6), because there are no boundary points nor singularities for the integrand as in (2.6). To see the analyticity of the integrand in (2.5) we write

On the similarity between the Iwasawa projection and the diagonal part

$$(2.7) \qquad \cosh(2t) - \cos(2t \cos 2\theta) = (2t \sin 2\theta)^2 \sum_{n=1}^{\infty} \frac{(2t)^{2n-2}}{(2n!)} \sum_{k=0}^{n-1} (\cos 2\theta)^{2k},$$

from which

$$(2.8) \qquad \phi_\xi(\exp X) = \frac{1}{2\pi} \int_{\mathbb{R}/2\pi\mathbb{Z}} e^{it\tau\cos 2\theta} \Big[\sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{2.(2t)^{2(n-k)}}{(2n+2)!} (t \cos 2\theta)^{2k}\Big]^{-\frac{1}{2}} d\theta.$$

In turn this allows us to write

$$(2.9) \qquad \phi_\xi(\exp X) = \sum_{k=0}^{\infty} c_k(t^2).(\frac{1}{i}\frac{\partial}{\partial\tau})^{2k} \frac{1}{2\pi} \int_{\mathbb{R}/2\pi\mathbb{Z}} e^{it\tau \cos 2\theta} d\theta,$$

where the c_k are suitable power series in t^2 with some positive radius of convergence. So the elementary spherical function, which is a hypergeometric function, can be obtained from the Bessel function

$$(2.10) \qquad \psi_\xi(\exp X) = \frac{1}{2\pi} \int_{\mathbb{R}/2\pi\mathbb{Z}} e^{it\tau\cos 2\theta} d\theta,$$

which is the elementary spherical function for the Cartan motion group, by applying an infinite order differential operator with respect to the eigenvalue (= character) parameter τ, with coefficients which are Ad K-invariant functions on $\mathfrak{s}$. This is the strategy in Stanton and Tomas [7]. That such a description is possible for all real rank one spaces can be derived from the previously mentioned explicit formulae of Koornwinder [8], but can also be read of from (1.11).

This description would generalize to arbitrary symmetric spaces if the amplitude $b(\mathrm{Ad}\, k^{-1}(X))$ in (1.11) could be written as

$$(2.11) \qquad b(\mathrm{Ad}\, k^{-1}(X)) = \sum_m c_m(X).\pi(\mathrm{Ad}\, k^{-1}(X))^m$$

($m = (m_1,\dots,m_{\dim \mathfrak{a}})$ a multi-index), where the c_m are Ad k-invariant functions on $\mathfrak{s}$. This however is one of the open questions which I have on this subject.

3. Proof of the theorem.

We begin by recalling some facts about the functions $F_{X,\xi}$, $f_{X,\xi}$ from [2].

Lemma 3.1. ([2], Lemma 5.9). For $x \in G$, write

$$(3.1) \qquad x \in \kappa(x).AN, \ \kappa(x) \in K.$$

Then, for every $X \in \mathfrak{s}$, $\xi \in \mathfrak{a}^*$:

$$(3.2) \qquad dF_{X,\xi}(1) = df_{\tilde{X},\xi}(1) \circ L_X, \text{ where}$$

(3.3) $\tilde{X} = \mathrm{Ad}\, \kappa(\exp X)^{-1}(X)$

and L_X is the linear isomorphism: $\mathfrak{k} \to \mathfrak{k}$ given by

(3.4) $L_X = \frac{\sinh \mathrm{ad}\, \tilde{X}}{\mathrm{ad}\, \tilde{X}} \circ \mathrm{Ad}\, \kappa(\exp X)^{-1}$.

Lemma 3.2. ([2], Lemma 1.1). Let $X \in \mathfrak{s}$ and let $\xi \in \mathfrak{a}^*$ correspond to $H = H_\xi \in \mathfrak{a}$ via the Killing form. Then

(3.5) $df_{X,\xi}(1) = 0 \Leftrightarrow [X,H] = 0$.

If $[X,H] = 0$ then $\exp X \in G_H^0$, a connected reductive subgroup with an Iwasawa decomposition, the components of which are contained in K, resp. A, resp. N. So $\kappa(\exp X) \in G_H^0$ and $[\tilde{X},H] = 0$ if $\tilde{X}$ is as in (3.3). Using Lemma 3.1 we conclude that $dF_{X,\xi}(1) = 0 \Leftrightarrow df_{X,\xi}(1) = 0$. Using that

(3.6) $\frac{d}{dt} F_{X,\xi}(k.\exp tY)_{t=0} = dF_{\mathrm{Ad}\, k^{-1}X,\xi}(1)(Y)$, $k \in K$, $Y \in \mathfrak{k}$,

and the same formula with F replaced by f, it follows that $F_{X,\xi}$ and $f_{X,\xi}$ have the same set of critical points.

Lemma 3.3. ([2], Cor. 5.2). If $X \in \mathfrak{s}$, $\xi \in \mathfrak{a}^*$, then

(3.7) $\frac{d}{dt} F_{tX,\xi}(1) = f_{X,\xi}(\kappa(\exp tX))$.

Now we look at the 1-parameter family of functions

(3.8) $F_{X,\xi}^{(t)} = \frac{1}{t} F_{tX,\xi}$, $F_{X,\xi}^{(0)} = f_{X,\xi}$, $F_{X,\xi}^{(1)} = F_{X,\xi}$.

We see that the set of critical points of $F_{X,\xi}^{(t)}$ is equal to the set of critical points of $\frac{1}{t} f_{tX,\xi} = f_{X,\xi}$ so to the set of critical points of $f_{X,\xi}$, for all $t \in \mathbb{R}$. Moreover

(3.9) $F_{X,\xi}^{(t)}(1) = \frac{1}{t} \int_0^t f_{X,\xi}(\kappa(\exp sX))ds$,

If $dF_{X,\xi}^{(t)}(1) = 0$ then $\kappa(\exp sX)$ is a critical point for $f_{X,\xi}$ for all $s \in [0,t]$, so $f_{X,\xi}(\kappa(\exp sX)) = f_{X,\xi}(1)$, that is

(3.10) $F_{X,\xi}^{(t)}(1) = f_{X,\xi}(1)$ if $dF_{X,\xi}^{(t)}(1) = 0$.

Using that $F_{X,\xi}^{(t)}(k) = F_{\mathrm{Ad}\, k^{-1}X,\xi}^{(t)}(1)$, we get that $F_{X,\xi}^{(t)}$ and $f_{X,\xi}$ have the same values at the critical points. Now we try to find a diffeomorphism $\Phi_{X,\xi}^{(t)} : K \to K$ depending smoothly on t, such that $\Phi_{X,\xi}^{(0)}$ = identity and

On the similarity between the Iwasawa projection and the diagonal part

$$F^{(t)}_{X,\xi}(\Phi^{(t)}_{X,\xi}(k)) = f_{X,\xi}(k) \text{ for all } t \in [0,1]. \tag{3.11}$$

Differentiating (3.11) with respect to t gives

$$\frac{\partial}{\partial t} F^{(t)}_{X,}(\Phi^{(t)}_{X,\xi}(k)) + dF^{(t)}_{X,\xi}(\Phi^{(t)}_{X,\xi}(k)) \circ \frac{\partial\Phi^{(t)}_{X,\xi}}{\partial t}(k) = 0, \text{ which} \tag{3.12}$$

in fact is equivalent to (3.11) in view of the initial condition $\Phi^{(0)}_{X,\xi}$ = identity. The idea is now to find a vector field $v^{(t)}_{X,\xi}$ on K depending analytically on t, X, ξ such that

$$\frac{\partial}{\partial t} F^{(t)}_{X,\xi}(k) + dF^{(t)}_{X,\xi}(k) \circ v^{(t)}_{X,\xi}(k) = 0 \tag{3.13}$$

and then obtain $\Phi^{(t)}_{X,\xi}$ by solving the ordinary differential equation

$$\frac{\partial}{\partial t}\Phi^{(t)}_{X,\xi}(k) = v^{(t)}_{X,\xi}(\Phi^{(t)}_{X,\xi}(k)),\ \Phi^{(0)}_{X,\xi}(k) = k. \tag{3.14}$$

I learned this idea from Moser [6] and Mather [5], but it might have a much older history.

In any case, for (3.13) it is a necessary condition that $\frac{\partial}{\partial t} F^{(t)}_{X,\xi}(k) = 0$ if $dF^{(t)}_{X,\xi}(k) = 0$, but this follows from $dF^{(t)}_{X,\xi}(k) = 0 \Leftrightarrow df_{X,\xi}(k) = 0$, in which case $F^{(t)}_{X,\xi}(k) = f_{X,\xi}(k)$, constant in t, as observed above. In Lemma 3.1, 3.2 we have seen that $dF^{(t)}_{X,\xi}(k)$ is proportional to $[\mathrm{Ad}\, k^{-1}(X), H_\xi]$ by a linear isomorphism depending analytically on t, X, H_ξ. In view of these observations the existence of $v^{(t)}_{X,\xi}$ with the desired properties is ensured by the following

<u>Lemma</u> 3.4. Let $\psi : \mathfrak{s}\times\mathfrak{a} \to \mathbb{R}$ be analytic such that $\psi(X,H) = 0$ whenever $[X,H] = 0$. Then there exists an analytic map $\chi : \mathfrak{s}\times\mathfrak{a} \to \mathfrak{k}$ such that

$$\psi(X,H) = \langle [X,H], \chi(X,H)\rangle \text{ for all } X \in \mathfrak{s},\ H \in \mathfrak{a}. \tag{3.15}$$

If ψ is linear in H for each X then χ can be chosen not depending on H and if ψ depends smoothly on additional parameters then ψ can be chosen to do the same.

Actually χ is obtained by an explicit formula from ψ, from which these properties can be read off. The construction is based on the observation that in $\mathfrak{s}\times\mathfrak{a}$ the relation $[X,H] = 0$ has a reasonably simple description. For $X \in \mathfrak{s}$ write

$$X = X_0 + \sum_{\alpha\in\Delta^+} X_\alpha,\ X_0 \in \mathfrak{a},\ X_\alpha \in \mathfrak{s} \cap (\mathfrak{g}_\alpha + \mathfrak{g}_{-\alpha}). \tag{3.16}$$

Then

$$[X,H] = - \sum_{\alpha\in\Delta^+} \alpha(H).JX_\alpha \tag{3.17}$$

where J is the linear isomorphism: $\mathfrak{s} \ominus \mathfrak{a} \to \mathfrak{k} \ominus \mathfrak{m}$ which sends $Y - \theta Y$ to $Y + \theta Y$ (for $Y \in \mathfrak{n}$). It follows that $[X,H] = 0$ if and only if for each $\alpha \in \Delta^+$ either $X_\alpha = 0$ or $\alpha(H) = 0$.

For $I \subset \Delta^+$, write now

$$\Pi_I(X) = X_0 + \sum_{\alpha\in\Delta^+\setminus I} X_\alpha. \tag{3.18}$$

Then, based on Newton's binomial formula, we can write

$$\psi(X,H) = \sum_{I\subset\Delta^+} \sum_{J\subset\Delta^+\setminus I} (-1)^{|J|}\psi(\Pi_{I\cup J}(X),H). \tag{3.19}$$

Observing that $\psi(X_0,H) = 0$ by assumption, we concentrate our attention on the term

$$\psi_I(X,H) = \sum_{J\subset\Delta^+\setminus I} (-1)^{|J|}\psi(\Pi_{I\cup J}(X),H). \tag{3.20}$$

Every term in the right hand side is equal to zero if $\alpha(H) = 0$ for all $\alpha \in \Delta^+\setminus I$. Let $\alpha_1,\dots,\alpha_p \in \Delta^+\setminus I$ be a basis of $\sum_{\alpha\in\Delta^+\setminus I} \mathbb{R}.\alpha$. Write

$$\mathfrak{a}_j = \{H \in \mathfrak{a};\ \alpha_i(H) = 0 \text{ for } i < j\},\quad \mathfrak{a}_0 = \mathfrak{a}, \tag{3.21}$$

and let π_i be a linear projection from $\mathfrak{a}_{j-1}$ to $\mathfrak{a}_j$. Write

$$\pi_j = \tilde{\pi}_j \circ \dots \circ \tilde{\pi}_1 : \mathfrak{a} \to \mathfrak{a}_j, \tag{3.22}$$

$$\psi_I(X,H) = \sum_{j=1}^{p} \psi_I(X,\pi_{j-1}(H)) - \psi_I(X,\pi_j(H)), \tag{3.23}$$

and finally

$$\begin{aligned} &\psi_I(X,\pi_{j-1}(H)) - \psi_I(X,\pi_j(H)) \\ &= \sum_{J\subset\Delta^+\setminus I} (-1)^{|J|}[\psi(\pi_{I\cup J}(X),\pi_{j-1}(H)) - \psi(\pi_{I\cup J}(X),\pi_j(H))] \\ &= \sum_{J\subset\Delta^+\setminus I\cup\{\alpha_j\}} (-1)^{|J|}[\psi(\pi_{I\cup J}(X),\pi_{j-1}(H)) - \psi(\pi_{I\cup J\cup\{\alpha_j\}}(X,\pi_{j-1}(H)) \\ &\quad - \psi(\pi_{I\cup J}(X),\pi_j(H)) + \psi(\pi_{I\cup J\cup\{\alpha_j\}}(X),\pi_j(H))]. \end{aligned} \tag{3.24}$$

The last expression between square brackets is equal to zero if $X_{\alpha_j} = 0$ or $\alpha_j(H) = 0$. So this expression is of the form

On the similarity between the Iwasawa projection and the diagonal part

$$\alpha_j(H).\langle JX_{\alpha_j}, \chi_{I,j}(X,H)\rangle$$

for some analytic mapping $\chi_{I,j} : \mathfrak{s}\times\mathfrak{a} \to \mathfrak{k}_{\alpha_j} = \mathfrak{k} \cap (\mathfrak{g}_{\alpha_j} + \mathfrak{g}_{-\alpha_j})$. Summing all the terms gives the desired mapping χ.

From χ we get an analytic vector field $v^{(t)}_{X,\xi}$ on K, depending analytically on t, X, ξ satisfying (3.13). Now, observing that

$$F^{(t)}_{\mathrm{Ad}\, l^{-1}(X),\xi}(k) = F^{(t)}_{X,\xi}(lk),\ k \in K, \tag{3.25}$$

it follows that $\lambda_l^* v^{(t)}_{\mathrm{Ad}\, l(X),\xi}$ satisfies (3.13) as well, here $\lambda_l : k \to l.k$ denotes left multiplication by l. Because the equation (3.13) is linear in v, also

$$\bar{v}^{(t)}_{X,\xi} = \int_K \lambda_l^* v^{(t)}_{\mathrm{Ad}\, l(X),\xi}\, dl \tag{3.26}$$

will satisfy (3.13). This vectorfield has the additional symmetry

$$\bar{v}^{(t)}_{\mathrm{Ad}\, k^{-1}(X),\xi} = \lambda_k^* \bar{v}^{(t)}_{X,\xi}, \tag{3.27}$$

which for the solution $\Phi^{(t)}_{X,\xi}$ of (3.14), with v replaced by $\bar{v}$, will lead to

$$\Phi_X(lk) = l.\Phi_{\mathrm{Ad}\, l^{-1}(X)}(k), \quad k,l \in K. \tag{3.28}$$

This proves Theorem 1.1, with $\Psi(X) = \Phi_X(1)$, $X \in \mathfrak{s}$.

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J.J. Duistermaat
Rijksuniversiteit Utrecht
Mathematisch Instituut
Budapestlaan 6
Post bus 80.010
3508 TA UTRECHT

(Pays-Bas)