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MICROLOCAL ANALYSIS OF DIFFRACTION BY A CORNER

BY MOTOO UCHIDA

Introduction

The purpose of the present paper is to study analytic singularities of solutions to mixed type boundary value problems in the exterior domain of a corner. In particular, we prove that the cone of diffracted singularities is produced by an incident ray which hits the corner. This phenomenon has been observed without proof by J.-B. Keller in his geometrical theory of diffraction [11], where he conjectured that his diffracted ray method does yield the leading terms in the asymptotic expansions of solutions of diffraction problems. In this paper, we shall give a proof to his observation in the analytic category.

Let M be a real analytic manifold. Let Ω_1, Ω_2 be open subsets of M given by $\varphi_1>0$, $\varphi_2>0$ respectively for real-valued C^ω-functions φ_1, φ_2 with $d\varphi_1 \wedge d\varphi_2 \neq 0$. Let

$$\Omega=\Omega_1 \cup \Omega_2.$$

To every hyperfunction u defined on Ω is associated the closed conic subset $SS_\Omega(u)$ of $T^*_M X$, with X being a complex neighborhood of M; this set is called the boundary analytic wavefront set of u (which was introduced by P. Schapira [19], [20], [21]). The singular spectrum (or the analytic wavefront set) $SS(u)$ of u over Ω is a closed conic subset of $\Omega \times_M T^*_M X$ and the equality $SS_\Omega(u) \cap \pi^{-1}(\Omega) = SS(u)$ holds, where $\pi : T^*_M X \to M$. [*Cf.* 2.4 for the definition of the set $SS_\Omega(u)$.]

The main purpose of this paper is to prove the following "propagation" and "condensation" results of analytic singularities at a corner. Let $\Omega=\Omega_1 \cup \Omega_2$ be as above; set $K = M \setminus \Omega$, $N_0 = \{ x \in M \mid \varphi_1(x) = \varphi_2(x) = 0 \}$. Let $P = P(x, D)$ be a differential operator with analytic coefficients on M with principal symbol $f = f(z, \zeta)$. Let $p \in T^*_M X$ with $\pi(p) \in N_0$. Assume

(0.1) each boundary $\{\varphi_1=0\}$, $\{\varphi_2=0\}$ of Ω_1, Ω_2 is noncharacteristic for P;

(0.2) $$\operatorname{Im} f \mid T^*_M X \equiv 0;$$

(0.3) $df \wedge \omega(p) \neq 0$, with ω being the fundamental 1-form on $T^* X$.

Let $b^{\pm}(q)$ denote the positive (resp. negative) bicharacteristic curve of P on T_M^*X issued from $q \in T_M^*X$ with $f(q)=0$. Assume in addition

$$(0.4) \qquad b^{\pm}(p) \text{ is transversal to K and } \pi(b^{\pm}(p)) \subset \Omega$$

[*cf.* (6.5) for the precise meaning]. Let $Y_0 = \{ z \in X \mid \varphi_1(z) = \varphi_2(z) = 0 \}$, $\rho_0 : Y_0 \times_X T^*X \to T^*Y_0$ the natural projection. Define a real curve passing through p:

$$C_p = \{ q \in N_0 \times_M T_M^*X \mid f(q) = 0,\ \rho_0(q) = \rho_0(p) \}.$$

Then we have:

THEOREM 0.1. — *Let u be a hyperfunction solution on Ω to the differential equation $Pu=0$. If $b^-(p) \subset SS(u)$ and if $b^-(q) \cap SS(u) = \varnothing$ for every $q \in C_p \setminus \{p\}$ close to p, then $p \in SS_\Omega(u)$ and $b^+(p) \subset SS(u)$.*

THEOREM 0.2. — *Let u be as in Theorem 0.1. If $p \in SS_\Omega(u) \setminus \overline{SS(u) \cap b^-(p)}$, then there is a neighborhood $C_p(\varepsilon)$ of p in C_p such that $C_p(\varepsilon) \subset SS_\Omega(u)$ and $b^+(q) \subset SS(u)$ for all $q \in C_p(\varepsilon)$.*

Theorem 0.1 asserts that an isolated singularity of u propagates beyond the corner along the bicharacteristic curve. On the other hand, Theorem 0.2 proves the appearance of diffracted rays at the corner. These theorems are proved in Section 6 (6.2 and 6.3).

In Section 7, we shall apply Theorems 0.1 and 0.2 to the Dirichlet problem in the region Ω for a second order differential operator P of real principal type. Preparing a lemma of the reflection of singularities at the corner (*cf.* 7.2.2), we prove (*cf.* 7.3.2 for the precise statement):

THEOREM 0.3. — *Let u be a solution to the mixed type Dirichlet boundary value problem for* P *in* Ω. *If u has a single incoming singularity "in general position" at the corner, then u has the outgoing singularities forming the cone of diffracted rays.*

The argument used there clarifies the microlocal geometrical aspect of diffraction problems, and this gives a proof to Keller's geometrical theory [11] of diffraction by a corner from the standpoint of microlocal analysis. *Cf.* also the work of Cheeger and Taylor [1], Rouleux [14], Varrenne [27] for diffraction of a simple progressing wave on a Riemannian manifold by a conical singularity (*cf.* 7.3.4).

Our method is based on the theory of $\mathscr{C}_{A|X}$ of P. Schapira ([19], [20], [21]), which has been propounded as a framework for microlocal study of boundary value problems in a general domain (possibly with non smooth boundary). Summarizing some generalities of sheaves in Section 1, we make a short review on this general theory in Section 2. *Cf.* also, e.g., the work of Kataoka [7], [8], [10], Schapira [17] to [20], Sjöstrand [23] for microlocal analysis in the analytic category of boundary value problems in domains with smooth boundary. Section 3 is a supplement to the general theory of $\mathscr{C}_{A|X}$.

In Section 4, we prove the injectivity of a certain homomorphism induced from inclusion of open subsets; this will be used in the proofs of Theorems 0.1 and 0.2. This section actually constitutes the central part of the proof of diffraction.

Section 5 is a preparatory section to the succeding sections. In Section 6, we prove Theorems 0.1 and 0.2. In Section 7, we state Theorem 0.3 in precise form and prove it by applying Theorems 0.1 and 0.2.

The main results of this paper have been announced in [26].

The author would like to thank P. Schapira and K. Kataoka. Not only is this work based on the theory of $\mathscr{C}_{A|X}$ of P. Schapira for the framework, but also the proof of Theorem 3.1 has been completed on his suggestion. The author also benefitted much from discussions with K. Kataoka at a preliminary stage of this work. The author must thank A. Kaneko for pointing out Keller's paper. Finally, the author would like to express his sincere gratitude to H. Komatsu for his constant encouragement.

1. Generalities of microlocal study of sheaves

Let X be a C^∞-manifold, $\pi : T^* X \to X$ its cotangent bundle. D(X) denotes the derived category of the category of complexes of sheaves of $\mathbb{C}$-vector spaces on X, and $D^b(X)$ denotes the full subcategory consisting of complexes with bounded cohomologies. In this section we recall some basic notions of microlocal study of sheaves; refer to [6] for the details.

1.1. For two subsets S_1, S_2 of X, the normal cone $C_x(S_1; S_2)$ of S_1 along S_2 at $x \in X$ is the subset of $T_x X$ defined by

$$C_x(S_1; S_2) = \{ v \in T_x X \mid \text{there are sequences } \{x_n\} \subset S_1,\ \{y_n\} \subset S_2, \\ \text{and } \{a_n\} \subset \mathbb{R}_+ \text{ such that } x_n \to x,\ y_n \to x,\ a_n(x_n - y_n) \to v \}.$$

For a subset S of X and $x \in X$, we set

$$N_x(S) = T_x X \setminus C_x(X \setminus S; S),$$
$$N_x^*(S) = \{ \theta \in T_x^* X \mid \langle \theta, v \rangle \geqq 0 \text{ for all } v \in N_x(S) \}.$$

We denote by $N^*(S)$ the union of $N_x^*(S)\,(x \in X)$, which is a closed convex conic subset of $T^* X$ and called the conormal cone to S in X.

1.2. The constant sheaf on X is denoted by $\mathbb{C}_X$. For a locally closed subset A of X, we set $\mathbb{C}_A = i_! i^{-1} \mathbb{C}_X$ with $i : A \hookrightarrow X$.

1.3. For an object F of $D^b(X)$, SS(F) denotes the microsupport of F due to Kashiwara-Schapira [6], which is by definition, roughly speaking, the set of codirections of X in which codirections F does not propagate. The microsupport SS(F) is a closed conic involutive subset of $T^* X$.

Let K be a closed subset of X. K is said to be C^α-convex ($1 \leqq \alpha \leqq \infty$) at x if K is convex for a choice of local C^α-coordinates of X in a neighborhood of x.

PROPOSITION 1.1. — *Let* K *be a* C^α-*convex closed subset of* X; *let* $T^*_K X = SS(\mathbb{C}_K)$. *Let* $(x; \xi) \in T^* X$. *Then* $(x; \xi) \in T^*_K X$ *if and only if there is a* C^α-*function* g *on* X *with* $g(x) = 0$, $dg(x) = (x; \xi)$ *and* $K \subset \{g \geqq 0\}$ *in a neighborhood of* x. *In particular, each fibre of* $T^*_K X$ *is convex.*

Considering the above proposition, $T^*_K X$ is called the generalized conormal set of K.

PROPOSITION 1.2. — *Let* U *be an open subset of* X; *let* $T^*_U X = SS(\mathbb{C}_U)$. *Then we have*

$$T^*_U X \subset \bar{U} \times_X N^*(U)^a,$$

where $(\bullet)^a$ *denotes the antipodal map on* $T^* X$. *If* $X \setminus U$ *is* C^1-*convex and* $U = \operatorname{Int} \bar{U}$, *equality holds.*

Let M be a closed submanifold of X, $\varphi : M \to X$ the embedding. Let ρ and ϖ be the natural maps associated with φ:

$$T^* M \xleftarrow{\ \rho\ } M \times_X T^* X \xrightarrow{\ \varpi\ } T^* X.$$

PROPOSITION 1.3. — *Let* G *be an object of* $D^b(M)$. *Then*

$$SS(\varphi_* G) = \varpi\rho^{-1}(SS(G)).$$

In particular, $SS(\varphi_* G)$ *is* $T^*_M X$-*invariant.*

COROLLARY 1.4. — *Let* Ω *be an open subset of* M; *let* $T^*_\Omega X = SS(\mathbb{C}_\Omega)$. *Then*

$$T^*_\Omega X \subset \varpi\rho^{-1}(\bar{\Omega} \times_M N^*(\Omega)^a).$$

1.4. Let M be a closed submanifold of X. Sato's microlocalization along M is denoted by $\mu_M : D^b(X) \to D^b(T^*_M X)$. Refer to Sato-Kawai-Kashiwara [15], chap. 1, and Kashiwara-Schapira [6], chap. 2, for its construction and fundamental properties.

There is also the bifunctor (*cf.* [6])

$$\mu\text{hom}: \quad D^b(X)^\circ \times D^b(X) \to D^b(T^* X),$$

which possesses the following properties:

$$\mu\text{hom}(\mathbb{C}_M, F) \cong \mu_M(F), \tag{1.1}$$

$$\mathbf{R}\pi_* \mu\text{hom}(G, F) \cong \mathbf{R}\mathscr{H}om(G, F), \tag{1.2}$$

$$\operatorname{supp}(\mu\text{hom}(G, F)) \subset SS(G) \cap SS(F). \tag{1.3}$$

In particular,

$$\text{supp}(\mu\text{hom}(\mathbb{C}_\Omega, F)) \subset \text{SS}(F) \cap T^*_\Omega X \tag{1.4}$$

for $\Omega \subset M \subset X$ of Corollary 1.4.

For $p \in T^* X$, we denote by $D^b(X; p)$ the localization of $D^b(X)$ by the null system $\{G \in \text{Ob}(D^b(X)) \mid p \notin \text{SS}(G)\}$. It follows from (1.3) that if G_1 and G_2 are isomorphic in $D^b(X; p)$, then $\mu\text{hom}(G_1, F)$ and $\mu\text{hom}(G_2, F)$ are isomorphic in a neighborhood of p.

2. Review on the theory of $C_{A|X}$

Let M be a real analytic manifold of dimension n, X a complex neighborhood of M, $\pi : T^* X \to X$ the cotangent bundle of X. We use the following notations for sheaves:

$\mathcal{O}_X$: the sheaf of holomorphic functions on X;

$\mathcal{A}_M$: the sheaf of real analytic functions on $M (= \mathcal{O}_X|_M)$;

$\mathcal{B}_M$: the sheaf of Sato's hyperfunctions on M;

$\mathcal{C}_M$: the sheaf of Sato's microfunctions on $T^*_M X$;

$\mathcal{D}_X$: the sheaf of rings of differential operators of finite order on X;

$\mathcal{E}_X$: the sheaf of rings of microdifferential operators of finite order on $T^* X$.

Refer to Sato-Kawai-Kashiwara [15] for the definitions and fundamental properties of $\mathcal{B}_M$, $\mathcal{C}_M$, $\mathcal{D}_X$, $\mathcal{E}_X$. *Cf.* also [4], [22]. We denote by $D^b(\pi^{-1}\mathcal{D}_X)$ the derived category of the category of complexes of $\pi^{-1}\mathcal{D}_X$-modules with bounded cohomologies.

Let A be a locally closed subset of M. Following Schapira ([19], [20]), we define an object $\mathcal{C}_{A|X}$ of $D^b(\pi^{-1}\mathcal{D}_X)$ by

$$\mathcal{C}_{A|X} = \mu\text{hom}(\mathbb{C}_A, \mathcal{O}_X) \otimes \text{or}_{M|X}[n], \tag{2.1}$$

with $\text{or}_{M|X}$ being the relative orientation sheaf of M in X. In particular, for $A = M$, $\mathcal{C}_{M|X}$ is nothing but the sheaf $\mathcal{C}_M$ of Sato's microfunctions.

In [19] and [20], by using the functorial definition (2.1), a general framework is set up for microlocal study of boundary value problems. In this section we quickly recall the notations and results in the theory of $\mathcal{C}_{A|X}$. *Cf.* [19], [20], [21] for the details of this section; *cf.* [25] for Sections 2.8 and 2.9.

2.0. Let A be a locally closed subset of M. Since the flabby dimension of $\mathcal{O}_X$ is $\leqq n$, $H^i(\mathcal{C}_{A|X}) = 0$ for $i > 0$.

2.1. Let K be a closed C^ω-convex (*i.e.* convex for a choice of local coordinates) subset of a real analytic manifold M.

Proposition 2.1. — *The complex of sheaves $\mathcal{C}_{K|X}$ is concentrated in degree* 0:

$$H^i(\mathcal{C}_{K|X}) = 0 \qquad (i \neq 0);$$

therefore $\mathscr{C}_{K|X}$ *in an* $\mathscr{E}_X$*-module, and the natural* $\mathscr{E}_X$*-homomorphism* $\mathscr{C}_{K|X}|_{T^*_M X} \to \mathscr{C}_M$ *is injective.*

Remark. — The $\mathscr{E}_X$-module $\mathscr{C}_{K|X}$ was first introduced in a different way by Kataoka ([7], [8], [9]) in the case where K is a closed half-space of M with C^ω-boundary. The functorial definition (2.1) is due to Schapira ([19], [20]).

Let $\Omega = M \setminus K$. Then there exists a distinguished triangle

$$\mathscr{C}_{K|X} \to \mathscr{C}_M \to \mathscr{C}_{\Omega|X} \xrightarrow{+1}. \tag{2.2}$$

Hence $\mathscr{C}_{\Omega|X}$ is quasi-isomorphic to a complex of $\mathscr{E}_X$-modules, and we have:

COROLLARY 2.2. — $H^0(\mathscr{C}_{\Omega|X})$ *is supported in* $T^*_M X$, *and* $H^i(\mathscr{C}_{\Omega|X})|_{T^*_M X} = 0$ *for* $i \neq 0$.

2.2. Let Ω, Ω' be two open subsets of M with $\Omega \supset \Omega'$. There is a canonical morphism ("microlocal restriction")

$$\mathscr{C}_{\Omega|X} \to \mathscr{C}_{\Omega'|X}. \tag{2.3}$$

2.3. Let Ω be an open subset of M. Assume that $N^*_x(\Omega) \neq T^*_x M$ for all $x \in \bar{\Omega}$. This implies that, for a choice of local coordinates, there is an open convex cone γ such that $\Omega + \gamma \subset \Omega$ locally. We say that Ω has the cone property if Ω satisfies this condition.

Let N be a closed submanifold of M of codimension d. If $N \subset \bar{\Omega}$, then we have the boundary value morphism due to Schapira:

$$\mathrm{bv}: \quad \mathscr{C}_{\Omega|X} \to \mathscr{C}_{N|X} \otimes \mathrm{or}_{N|M}[d], \tag{2.4}$$

where $\mathrm{or}_{N|M}$ denotes the relative orientation sheaf of N in M.

Remark. — There is a topological boundary value morphism $\mathbb{C}_N \otimes \omega_{N|M} \to \mathbb{C}_\Omega$, where $\omega_{N|M} = \mathrm{or}_{N|M}[-d]$. Schapira [19] constructed morphism (2.4) by applying the functor $\mu\mathrm{hom}(\bullet, \mathscr{O}_X)$ to the topological boundary value morphism.

2.4. Let Ω be an open subset of M. We have the spectral map

$$\alpha: \quad \pi^{-1}\Gamma_\Omega(\mathscr{B}_M) \to H^0(\mathscr{C}_{\Omega|X}).$$

Let $u \in \Gamma(\Omega, \mathscr{B}_M)$. We set $SS_\Omega(u) = \mathrm{supp}(\alpha(u))$; this set is a closed conic subset of $T^*_M X$ and called the boundary analytic wavefront set of u.

2.5. Let $\mathscr{M}$ be a coherent $\mathscr{D}_X$-module (*i.e.* a system of differential equations); $\mathrm{Char}(\mathscr{M})$ denotes the characteristic variety of $\mathscr{M}$. Let Ω be an open subset of M and let $T^*_\Omega X$ denote the microsupport of $\mathbb{C}_\Omega$. We have the spectral map

$$\alpha: \quad \pi^{-1}\Gamma_\Omega \mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{B}_M) \to H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega|X}).$$

Let $u \in \Gamma(\Omega, \mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{B}_M))$. We set $SS^{\mathscr{M}}_\Omega(u) = \mathrm{supp}(\alpha(u))$, which is a closed conic subset of $T^*_\Omega X \cap \mathrm{Char}(\mathscr{M})$.

Remark. — Let $\mathcal{M}=\mathcal{D}_X/\mathcal{D}_X P$. If $M\setminus\Omega$ is C^ω-convex in M, we have the equality $SS_\Omega^{\mathcal{M}}(u)\cap T_M^* X=SS_\Omega(u)$ for any hyperfunction solution u to P on Ω. This follows from Corollary 2.2.

2.6. Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module. Let Ω, Ω' be open subsets of M with $\Omega\supset\Omega'$. Then we have the map [*cf.* (2.3)].

$$\beta:\quad H^0\,\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M},\mathcal{C}_{\Omega|X})\to H^0\,\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M},\mathcal{C}_{\Omega'|X}). \tag{2.5}$$

Let u be a section of $H^0\,\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M},\mathcal{C}_{\Omega|X})$. We set $SS_{\Omega'}^{\mathcal{M}}(u)=\operatorname{supp}(\beta(u))$.

2.7. Let N be a closed C^ω-submanifold of M, Y the complexification of N in X. Let ρ and ϖ be the natural maps associated with $Y\hookrightarrow X$:

$$T^*Y\xleftarrow{\ \rho\ }Y\times_X T^*X\xrightarrow{\ \varpi\ }T^*X.$$

Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module and suppose that Y is non characteristic for $\mathcal{M}$. The tangential system of $\mathcal{M}$ on Y is denoted by $\mathcal{M}_Y$.

PROPOSITION 2.3. — *For a locally closed subset* A *of* N, *there is a natural isomorphism*

$$\rho_*\varpi^{-1}\,\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M},\mathcal{C}_{A|X})\otimes or_{N|M}[d]\cong\mathbf{R}\mathcal{H}om_{\mathcal{D}_Y}(\mathcal{M}_Y,\mathcal{C}_{A|Y}), \tag{2.6}$$

with $d=\operatorname{codim} N$, *where* $\mathcal{C}_{A|Y}$ *denotes the complex of* $\mathcal{D}_Y$*-modules defined in the same way as is* $\mathcal{C}_{A|X}$ *in* (2.1).

Let Ω be an open subset of M having the cone property. If $N\subset\bar{\Omega}$, then, using (2.4) and (2.6) with $A=N$, we obtain the boundary value morphim

$$bv:\quad \rho_*\varpi^{-1}\,\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M},\mathcal{C}_{\Omega|X})\to\mathbf{R}\mathcal{H}om_{\mathcal{D}_Y}(\mathcal{M}_Y,\mathcal{C}_N). \tag{2.7}$$

PROPOSITION 2.4 (Schapira [19]). — *Assume that the boundary of* Ω *is a* C^ω*-hypersurface and* Ω *lies on one side of it. Let* $N=\partial\Omega$. *If* N *is non characteristic for* $\mathcal{M}$, *then the homomorphism* [cf. (2.4)]

$$bv:\quad H^0\,\mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M},\mathcal{C}_{\Omega|X})\to H^1\,\mathbf{R}\Gamma_{T_\Omega^*X\cap T_N^*X}\,\mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M},\mathcal{C}_{N|X})\otimes or_{N|M}$$

is injective on $T_\Omega^*X\cap T_N^*X$; *therefore* (2.7) *is also injective in the* 0*-th cohomology.*

2.8. We also need the following lemma (*cf.* [25], Lemma 4.1). Let N, N_0 be closed C^ω-submanifolds of M with $N\supset N_0$. Let $d=\operatorname{codim} N$, $d_0=\operatorname{codim} N_0$. Let Y denote the complexification of N in X.

LEMMA 2.5. — *If* Y *is non characteristic for* $\mathcal{M}$, *then*

$$H^i\,\mathbf{R}\Gamma_{T_N^*X\cap T_{N_0}^*X}\,\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M},\mathcal{C}_{N_0|X})=0\qquad(i<d_0),$$

and the natural homomorphism

(2.8) $\quad \mathrm{H}^{d}\mathbf{R}\mathscr{H}om_{\mathscr{E}_{\mathrm{X}}}(\mathscr{M}, \mathscr{C}_{\mathrm{N}|\mathrm{X}})\otimes \mathrm{or}_{\mathrm{N}|\mathrm{M}} \to \mathrm{H}^{d_0}\mathbf{R}\Gamma_{\mathrm{T}^*_{\mathrm{N}}\mathrm{X}\cap \mathrm{T}^*_{\mathrm{N}_0}\mathrm{X}}\mathbf{R}\mathscr{H}om_{\mathscr{E}_{\mathrm{X}}}(\mathscr{M}, \mathscr{C}_{\mathrm{N}_0|\mathrm{X}})\otimes \mathrm{or}_{\mathrm{N}_0|\mathrm{M}}$

is injective on $\mathrm{T}^*_{\mathrm{N}}\mathrm{X}\cap \mathrm{T}^*_{\mathrm{N}_0}\mathrm{X}$.

2.9. Let Ω be an open subset of M with C^{ω}-boundary N and lying on one side of N. Let $\mathscr{M}$ be a coherent $\mathscr{D}_{\mathrm{X}}$-module and assume that N is non characteristic for $\mathscr{M}$. Then we have an injectivity theorem of microlocal restriction homomorphism:

PROPOSITION 2.6 [25]. — *Let* Ω' *be an open subset of* M *with* C^1*-boundary contained in* Ω *and tangent to* Ω *at* $x\in\partial\Omega$. *Then the homomorphism*

$$\beta:\quad \mathrm{H}^0\mathbf{R}\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{C}_{\Omega|\mathrm{X}}) \to \mathrm{H}^0\mathbf{R}\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{C}_{\Omega'|\mathrm{X}})$$

is injective on $\pi^{-1}(x)$.

This implies that the analytic wavefront set of the boundary value of a hyperfunction solution to $\mathscr{M}$ does not change on the fibre of x when one deforms the boundary (*cf.* also Kataoka [10], Lebeau [13]).

3. The theory of $\mathrm{C}_{\mathrm{A}\,|\,\mathrm{X}}$ (2)

This section is a supplement to te preceding section; we prove a basic property of the sheaf $\mathscr{C}_{\mathrm{K}\,|\,\mathrm{X}}$ for a C^{ω}-convex closed subset K of M.

3.1. Let M, X be as in Section 2. Let K be a closed C^{ω}-convex subset of M and let $\mathrm{T}^*_{\mathrm{K}}\mathrm{X}$ denote the microsupport of $\mathbb{C}_{\mathrm{K}}$. Let $\mathscr{C}_{\mathrm{K}\,|\,\mathrm{X}}$ be the sheaf of microfunctions along $\mathrm{T}^*_{\mathrm{K}}\mathrm{X}$ due to Schapira ([19], [20]): $\mathscr{C}_{\mathrm{K}\,|\,\mathrm{X}}=\mathscr{H}^n(\mu\mathrm{hom}(\mathbb{C}_{\mathrm{K}}, \mathscr{O}_{\mathrm{X}}))\otimes\mathrm{or}_{\mathrm{M}}$ (*cf.* also Kataoka [7], [8], [9]; *cf.* Sect. 2.1).

Let $x\in\mathrm{K}$. Let $(\mathrm{T}^*_{\mathrm{K}}\mathrm{M})_x=(\mathrm{T}^*_{\mathrm{K}}\mathrm{X})_x/(\mathrm{T}^*_{\mathrm{M}}\mathrm{X})_x$; then $(\mathrm{T}^*_{\mathrm{K}}\mathrm{M})_x\subset\mathrm{T}^*_x\mathrm{M}$. Let N be a C^{ω}-submanifold of M with $x\in\mathrm{N}$ such that

$$\mathrm{T}_x\mathrm{N}=\{v\in\mathrm{T}_x\mathrm{M}\,|\,\langle\theta, v\rangle=0\ \text{ for all}\ \theta\in(\mathrm{T}^*_{\mathrm{K}}\mathrm{M})_x\},$$

Y the complexification of N in X. For $p\in\mathrm{T}^*_{\mathrm{K}}\mathrm{X}\cap\pi^{-1}(x)$, denote by $\mathscr{L}_p$ the fibre passing through p of the composition

$$(\mathrm{T}^*_{\mathrm{K}}\mathrm{X})_x \hookrightarrow (\mathrm{T}^*_{\mathrm{N}}\mathrm{X})_x \xrightarrow{\mathrm{pr}} (\mathrm{T}^*_{\mathrm{N}}\mathrm{Y})_x.$$

The fibre $\mathscr{L}_p$ is defined independently of a choice of N and called be ρ-leaf of $\mathrm{T}^*_{\mathrm{K}}\mathrm{X}$ passing through p (*cf.* [26], Sect. 2.1). Note that $\mathscr{L}_p\subset\pi^{-1}(x)$.

THEOREM 3.1. — *Let* K *be a closed* C^{ω}*-convex subset of* M, $x\in\mathrm{K}$, *and let* u *be a section of* $\mathscr{C}_{\mathrm{K}\,|\,\mathrm{X}}|_{\pi^{-1}(x)}$ *in a neighborhood of* $p\in\mathrm{T}^*_{\mathrm{K}}\mathrm{X}\cap\pi^{-1}(x)$. *Let* $\mathscr{L}$ *be the* ρ*-leaf of* $\mathrm{T}^*_{\mathrm{K}}\mathrm{X}$ *passing through* p. *Then* $\mathrm{supp}(u)\cap\mathscr{L}=\mathscr{L}$ *in a neighborhood of* p *if* $u\not\equiv 0$; i.e., *u has the unique continuation property along* $\mathscr{L}$.

To prove the theorem, we may assume that K is a closed convex subset of $M=\mathbb{R}^n$. Let $x \in K$. Let $\{\xi_1, \ldots, \xi_d\}$ be a set of linearly independent vectors of $(T^*_K M)_x$, and set $L=\{x' \in \mathbb{R}^n \mid \langle x'-x, \xi_j\rangle \geqq 0 \ (\forall j \in \{1, \ldots, d\})\}$. Then we easily see that $K \subset L$, and we have the following two lemmas:

LEMMA 3.1.1. — *The natural $\mathscr{E}_X$-homomorphism $\mathscr{C}_{K\mid X}|_{T^*_L X} \to \mathscr{C}_{L\mid X}$ is injective.*

Sketch of Proof (cf. [19]). — By the trick of a dummy variable due to Kashiwara and for a choice of affine coordinate system $(x_1, \ldots, x_n)$ of $\mathbb{R}^n$, we may assume that $M=\mathbb{R}^n$, $L=\{x_1 \geqq 0, \ldots, x_d \geqq 0\}$ $(d<n)$, $K=K' \times \mathbb{R}_{x_n}$ for a closed convex subset K' of $\mathbb{R}^{n-1}$ and we have only to prove the injectivity at $p=(0; \zeta) \in T^*_L X$ with $\operatorname{Im}\zeta_n \neq 0$. Then, by using a quantized complex contact transformation (*cf.* [6], chap. 11), the problem is reduced to the following:

Let $z=x+iy\,(=(z_1, \ldots, z_n))$ be the affines coordinates of $\mathbb{C}^n$. Set

$$D_1=\left\{z \in \mathbb{C}^n \mid y_n > \sum_{j=1}^{n-1} y_j^2 - d(y', K')^2\right\}, \qquad D_2=\left\{z \in \mathbb{C}^n \mid y_n > \sum_{j=1}^{d} (y_j)_-^2 + \sum_{k=d+1}^{n-1} y_k^2\right\},$$

where $d(y', K')$ denotes the distance between $y'=(y_1, \ldots, y_{n-1})$ and K' in $\mathbb{R}^{n-1}$. Define the sheaves $\mathscr{C}_{\partial D_k}$ $(k=1, 2)$ on ∂D_k by $\mathscr{C}_{\partial D_k}=\Gamma_{D_k}(\mathscr{O}_{\mathbb{C}^n})/\mathscr{O}_{\mathbb{C}^n})|_{\partial D_k}$. Note that $D_1 \subset D_2$. Then $\partial D_1 \cap \partial D_2$ corresponds to $T^*_K X \cap T^*_L X$, and the injectivity of $\mathscr{C}_{K\mid X}|_{T^*_L X} \to \mathscr{C}_{L\mid X}$ is equivalent to that of the natural restriction map $\mathscr{C}_{\partial D_1}|_{\partial D_1 \cap \partial D_2} \to \mathscr{C}_{\partial D_2}|_{\partial D_1 \cap \partial D_2}$.

Let $z=(z_1, \ldots, z_n)$ be the affine coordinate system of $\mathbb{C}^n$ which is real on $\mathbb{R}^n$, $(z; \zeta)$ the associated coordinates of $T^*\mathbb{C}^n$.

LEMMA 3.1.2. — *Let* $L=\{x \in \mathbb{R}^n \mid x_1 \geqq 0, \ldots, x_d \geqq 0\}$, *and let u be a section of* $\mathscr{C}_{L\mid X}|_{\pi^{-1}(0)}$. *Then u has the unique continuation property in the variables* $(\zeta_1, \ldots, \zeta_d)$ *on* $\{\zeta=(\zeta_1, \ldots, \zeta_d, \zeta'') \in \pi^{-1}(0) \mid \operatorname{Re}\zeta_i \geqq 0 \ (i=1, \ldots, d), \operatorname{Re}\zeta''=0\}$.

Sketch of Proof. — We may assume that $d<n$, and we shall work on $\operatorname{Im}\zeta_n \neq 0$. Set

$$D=\left\{z \in \mathbb{C}^n \mid y_n > \sum_{j=1}^{d} (y_j)_-^2 + \sum_{d<k<n} y_k^2\right\} \qquad \text{and} \qquad \mathscr{C}_{\partial D}=\Gamma_D(\mathscr{O}_{\mathbb{C}^n})/\mathscr{O}_{\mathbb{C}^n\,\partial D}.$$

Then, by using a quantized complex contact transformation, it is sufficient to prove that $\mathscr{C}_{\partial D}$ has the unique continuation property in the variables $(z_1, \ldots, z_d)$ on $\{z \in \partial D \mid y_1 \geqq 0, \ldots, y_d \geqq 0\}$. We shall see this property in the variable z_1 on $\{z \in \partial D \mid y_1 \geqq 0, y_j=y_j^0 \ (j=2, \ldots, d)\}$ for fixed $y_j^0 \geqq 0\,(j=2, \ldots, d)$. Let us set

$$D'=\left\{z \in \mathbb{C}^n \mid y_n > (y_1)_-^2 + \sum_{j=2}^{d} (y_j - y_j^0)^2 + \sum_{d<k<n} y_k^2\right\}$$

and $\mathscr{C}_{\partial D'}=\Gamma_{D'}(\mathscr{O}_{\mathbb{C}^n})/\mathscr{O}_{\mathbb{C}^n}|_{\partial D'}$. Since $D' \subset D$, $\mathscr{C}_{\partial D}|_{\partial D \cap \partial D'} \to \mathscr{C}_{\partial D'}|_{\partial D \cap \partial D'}$ is injective; therefore it is reduced to proving that $\mathscr{C}_{\partial D'}$ has the unique continuation property in z_1 on $\{z \in \partial D' \mid y_1 \geqq 0\}$, which is easy to see by using a local version of Bochner's tube theorem (*cf.* the proof of [18], Theorem 2.1).

Proof of Theorem 3.1. — Let $\zeta=(\zeta', \zeta'')(=\xi+i\eta)$ denote the fibre coordinates of $T_x^* X$ with $\zeta' \in \mathbb{C}^d$, $\zeta'' \in \mathbb{C}^{n-d}$ such that $\{\xi=(\xi', \xi'') \in T_x^* M \mid \xi''=0\}$ is the subspace of $T_x^* M$ generated by $(T_K^* M)_x$. Then $\mathscr{L}=\{\zeta=(\xi'+i\eta', i\mathring{\eta}'') \mid (\xi', 0) \in (T_K^* M)_x, \eta' \in \mathbb{R}^d\}$ for some fixed $\mathring{\eta}'' \in \mathbb{R}^{n-d}$. For a set $\{\xi_1, \ldots, \xi_d\}$ of d linearly independent vectors of $(T_K^* M)_x$, we set $P(\xi_1, \ldots, \xi_d)=\left\{\xi=\sum_{j=1}^{d} a_j \xi_j \mid a_j \geqq 0\ (\forall j)\right\}$, which is a polyhedral cone contained in $(T_K^* M)_x$. Let u be a section of $\mathscr{C}_{K \mid X}|_{\pi^{-1}(x)}$ on an open subset $U \subset \pi^{-1}(x) \cap T_K^* X$. Then it follows from Lemma 3.1.1 and 3.1.2 that the support of u is open (and closed) in $U \cap \{\zeta=(\xi'+i\eta', i\mathring{\eta}'') \mid (\xi', 0) \in P(\xi_1, \ldots, \xi_d), \eta' \in \mathbb{R}^d\}$ for any fixed $\mathring{\eta}'' \in \mathbb{R}^{n-d}$ and for any linearly independent subset $\{\xi_1, \ldots, \xi_d\}$ of $(T_K^* M)_x$. Since $(T_K^* M)_x$ is a d-dimensional convex cone, this completes the proof.

Remark. — In the case where K has C^ω-boundary, Theorem 3.1 was first proved by Kataoka ([7], [9]) (*cf.* [7], Prop. 1.8).

3.2. We give a few corollaries of Theorem 3.1. We shall refer to these results in Section 5.1 only in a very special case.

Let K be a closed C^ω-convex subset of M, $\Omega=M \setminus K$.

PROPOSITION 3.2. — *Let $x \in K$ and let $\mathscr{M}$ be a coherent $\mathscr{D}_X$-module. Assume that there is a real-valued C^ω-function g on M with $g(x)=0$ satisfying:*

(a-1) $K \subset \{g \leqq 0\}$ *in a neighborhood of x;*

(a-2) *considering g a holomorphic function on* X, $(x; dg(x)) \notin \operatorname{Char}(\mathscr{M})$.

Then we have: (i) $\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{K \mid X})|_{\pi^{-1}(x)}=0$, (ii) $H^i \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega \mid X})|_{\pi^{-1}(x)}=0$ $(i<0)$.

Proof. — Let $p \in T_K^* X \cap \operatorname{Char}(\mathscr{M}) \cap \pi^{-1}(x)$. Let $\mathscr{L}_p$ be the ρ-leaf of $T_K^* X$ passing through p. Let $Z=\{z \in X \mid g(z)=0\}$, $\rho: T^* X \times_X Z \to T^* Z$ the natural projection; then $\mathscr{L}_p \supset \rho^{-1} \rho(p) \cap T_K^* X$ by the very definition of $\mathscr{L}_p$, and $\rho^{-1} \rho(p) \cap T_K^* X \neq \{p\}$ in every neighborhood of p. On the other hand, since Z is noncharacteristic for $\mathscr{M}$, $\rho^{-1} \rho(p) \cap \operatorname{Char}(\mathscr{M})=\{p\}$ in a neighborhood of p. Let $u \in \mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{K \mid X})_p$; then $\operatorname{supp}(u) \subset \{p\}$. Hence it follows from Theorem 3.1 that $u=0$. This completes the proof of (i). The second assertion follows immediately from the first one, by using the distinguished triangle (2.2).

PROPOSITION 3.3. — *Let x, K, $\mathscr{M}$ be as in Proposition* 3.2. *Let u be a hyperfunction solution to $\mathscr{M}$ defined on* $M \setminus K$. *If $SS_\Omega^{\mathscr{M}}(u) \cap \pi^{-1}(x) \subset \{0\}$, then there exist a neighborhood* U *of x in* M *and analytic solutions $\tilde{u}_\alpha$ to $\mathscr{M}$ defined on* U *such that $\tilde{u}_\alpha|_\Omega=u$ on the component U_α of $U \setminus K$, where α indexes the set $\{U_\alpha\}$ of the components of $U \setminus K$.*

Proof. — Note that, since $SS(u|_{\Omega \cap U}) \subset T_M^* M$ for a neighborhood U of x, u is analytic on $\Omega \cap U$. By the assumption of existence of a C^ω-function g satisfying (a-1) and (a-2), the problem is reduced to the case $K=\{g \leqq 0\}$.

Let $K=\{g \leqq 0\}$ with $dg \neq 0$. Let $F \in \operatorname{Ob}(D^b(X))$. Then we have (*cf.* [6] and [19])

$$\mathbf{R}\pi_! \mu\mathrm{hom}(\mathbb{C}_\Omega, F) \cong \mathbf{R}\mathscr{H}om(\mathbb{C}_\Omega, \mathbb{C}_X) \otimes F \cong F \otimes \mathbb{C}_{\bar{\Omega}} \otimes \mathrm{or}_{M \mid X}[-n],$$

$$\mathbf{R}\pi_* \mu\mathrm{hom}(\mathbb{C}_\Omega, F) \cong \mathbf{R}\mathscr{H}om(\mathbb{C}_\Omega, F) \cong \mathbf{R}\Gamma_\Omega(F).$$

Hence we have the distinguished triangle

$$\mathrm{F}|_{\mathrm{M}} \otimes \mathbb{C}_{\bar{\Omega}} \to \mathbf{R}\Gamma_{\Omega}(\mathrm{F}) \otimes or_{\mathrm{M} \mid \mathrm{X}}[n] \to \mathbf{R}\,\mathring{\pi}_{*}\,\mu hom(\mathbb{C}_{\Omega}, \mathrm{F}) \otimes or_{\mathrm{M} \mid \mathrm{X}}[n] \overset{+1}{\to},$$

where $\mathring{\pi}: \mathrm{T}^*\mathrm{X} \setminus \mathrm{X} \to \mathrm{X}$. Putting $\mathrm{F} = \mathbf{R}\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{O}_{\mathrm{X}})$ and taking the induced long exact sequence, by Proposition 3.2 (ii), we get an exact sequence

$$0 \to \mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{A}_{\mathrm{M}}) \otimes \mathbb{C}_{\bar{\Omega}} \to \Gamma_{\Omega}\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{B}_{\mathrm{M}}) \to \mathring{\pi}_{*}\mathrm{H}^0\,\mathbf{R}\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{C}_{\Omega \mid \mathrm{X}}).$$

This completes the proof.

4. A preliminary theorem

4.1. Notations. — Let M, X be as in Section 2. Let $\Omega = \{\varphi > 0\}$ be an open subset of M with analytic boundary $\mathrm{N} = \{\varphi = 0\}$ $(d\varphi \neq 0)$. Let $x_0 \in \mathrm{N}$.

Let (φ_1, φ_2) be a pair of real-valued C^{ω}-functions on M with $\varphi_1(x_0) = \varphi_2(x) = 0$, $d\varphi_1 \wedge d\varphi_2 \neq 0$. Let $\Omega' = \{\varphi_1 > 0, \varphi_2 > 0\}$ and assume $\Omega' \subset \Omega$.

Set $\mathrm{N}_0 = \{x \in \mathrm{M} \mid \varphi_1(x) = \varphi_2(x) = 0\}$; then $x_0 \in \mathrm{N}_0 \subset \partial\Omega'$. Let Y_0 (resp. Y) denote the complexification of N_0 (resp. of N) in X,

$$\rho_0 : \mathrm{T}^*\mathrm{X} \times_{\mathrm{X}} \mathrm{Y}_0 \to \mathrm{T}^*\mathrm{Y}_0, \qquad \rho : \mathrm{T}^*\mathrm{X} \times_{\mathrm{X}} \mathrm{Y} \to \mathrm{T}^*\mathrm{Y}$$

the natural projections.

4.2. Statement of theorem. — Let Ω, Ω' be as in 4.1.

Let $p \in \mathrm{T}^*_{\mathrm{N}}\mathrm{X} \cap \pi^{-1}(x_0)$. Let f be a homogeneous holomorphic function on $\mathrm{T}^*\mathrm{X}$ defined in a neighborhood of p with $f(p) = 0$. Let $\mathscr{M}$ be a coherent $\mathscr{D}_{\mathrm{X}}$-module. Assume the following:

(4.1) $\quad$ Y is non characteristic for $\mathscr{M}$;

(4.2) $\quad$ $\mathrm{Char}(\mathscr{M}) \subset \{f = 0\}$ in a neighborhood of p;

(4.3) $\quad$ $\{f, \varphi\}(p) \neq 0$,

with $\{*, *\}$ denoting the canonical Poisson bracket on $\mathrm{T}^*\mathrm{X}$.

Let $\mathrm{C}_p = \{q \in \rho_0^{-1}\rho_0(p) \cap \mathrm{T}^*_{\mathrm{N}}\mathrm{X} \mid f(q) = 0\}$; then C_p is a real nonsingular curve passing through p and contained in $\rho_0^{-1}\rho_0(p)$.

Theorem 4.1. — *Assume* (4.1), (4.2) *and* (4.3). *Let* U *be an open connected interval of* C_p *on which* $\{f, \varphi\} \neq 0$. *Then the homomorphism* [*cf.* (2.3)]

$$\mathrm{H}^0\,\mathbf{R}\Gamma_{\mathrm{Z}}(\mathrm{U}, \mathbf{R}\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{C}_{\Omega \mid \mathrm{X}}) \mid \mathrm{C}_p) \to \mathrm{H}^0\,\mathbf{R}\Gamma_{\mathrm{Z}}(\mathrm{U}, \mathbf{R}\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{C}_{\Omega' \mid \mathrm{X}}) \mid \mathrm{C}_p)$$

is injective for any closed proper subset Z *of* U.

4.3. PROOF. — Since there is an open subset $\Omega_1 \subset \Omega$ with analytic boundary N_1 with $N_0 \subset N_1$, on account of Proposition 2.6, we may replace Ω with Ω_1 and assume from the beginning that $N_0 \subset N$. Let us then choose a system of local coordinates $x=(x_1, \ldots, x_n) \in \mathbb{R}^n$ of M so that $\Omega=\{x_1>0\}$, $N_0=\{x_1=x_2=0\}$.

Let $z=x+iy$ be the complexification of x, $(z;\zeta)=(z_1, z_2, z''; \zeta_1, \zeta_2, \zeta'')$ $(\zeta=\xi+i\eta)$ be the associated coordinates of T^*X. We may assume that $N_x^*(\Omega') \subset \bar{\Omega}' \times (-G)$ for a closed convex proper cone G of $\mathbb{R}^2$ with $\{\xi_1 \leqq 0, \xi_2=0\} \subset G$; therefore, by Corollary 1.4, we have

$$T_{\Omega}^* X \subset \{\xi_1 \leqq 0, \xi_2=0, \xi''=0\}, \qquad T_{\Omega'}^* X \subset \{(\xi_1, \xi_2) \in G, \xi''=0\}.$$

We then have the following commutative diagram [*cf.* (2.3) and (2.4)]:

$$\begin{array}{ccc} \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega|X}) & \to & \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega'|X}) \\ \downarrow{\scriptstyle bv} & & \downarrow{\scriptstyle bv} \\ \mathbf{R}\Gamma_{\{\xi_1 \leqq 0, \xi_2=0\}} F[2] & \xrightarrow{\delta} & \mathbf{R}\Gamma_{\{(\xi_1,\xi_2)\in G\}} F[2], \end{array}$$

where $F=\mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{N_0|X}) \otimes or_{N_0|M}$. The left vertical arrow bv is factored as follows (*cf.* Sect. 2.3):

$$\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega|X}) \xrightarrow{b_1} \mathbf{R}\Gamma_{T_N^*X \cap T_\Omega^*X} \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{N|X}) \otimes or_{N|M}[1]$$

$$\xrightarrow{b_2} \mathbf{R}\Gamma_{T_{N_0}^*X \cap T_\Omega^*X} \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{N_0|X}) \otimes or_{N_0|M}[2].$$

Here $H^0(b_1)$ is injective by Proposition 2.4, and so is $H^0(b_2)$ by Lemma 2.5; therefore, since $H^i\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega|X})=0$ $(i<0)$, the map $H^0\mathbf{R}\Gamma_Z(U, b_2 \circ b_1|_{C_p})$ is also injective. Hence it is sufficient to prove the injectivity of the map

$$H^0\mathbf{R}\Gamma_Z(U, \delta|_{C_p}) : \mathbb{H}_Z^2(U, \mathbf{R}\Gamma_{\{\xi_1 \leqq 0, \xi_2=0\}} F) \to \mathbb{H}_Z^2(U, \mathbf{R}\Gamma_G F).$$

Let us take the long exact sequence induced from δ:

$$\ldots \to \mathbb{H}_Z^1(U, \mathbf{R}\Gamma_{G\setminus\{\xi_2=0\}} F) \to \mathbb{H}_Z^2(U, \mathbf{R}\Gamma_{\{\xi_1 \leqq 0, \xi_2=0\}} F) \xrightarrow{\delta} \mathbb{H}_Z^2(U, \mathbf{R}\Gamma_G F) \to \ldots$$

We shall prove that the first term is trivial [*i.e.*, $\mathbb{H}_Z^1(U, (\mathbf{R}\Gamma_{G\setminus\{\xi_2=0\}} F)|_{C_p})=0$]. Since $H^i F=0$ $(i<1)$ by Proposition 2.3, we have

$$\begin{aligned} \mathbb{H}_Z^1(U, (\mathbf{R}\Gamma_{G\setminus\{\xi_2=0\}} F)|_{C_p}) &= \Gamma_Z(U; (\Gamma_{G\setminus\{\xi_2=0\}} H^1 F)|_{C_p}) \\ &= \varinjlim_{O \supset U} \{u \in \Gamma(O \cap \{\xi_2 \neq 0\}; H^1 F) \mid \operatorname{supp}(u) \subset G, \overline{\operatorname{supp}(u)} \cap U \subset Z\}, \end{aligned}$$

where O runs through the family of open neighborhoods of U in $T^*X \cap \{z_1=0\}$ and $\overline{\operatorname{supp}(u)}$ denotes the closure of $\operatorname{supp}(u)$ in O. Take a section u. Suppose that $\overline{\operatorname{supp}(u)} \cap U \neq \varnothing$, and we shall see the contradiction. Since $Z \neq U$ and U is connected, there is a boundary point $q=(\mathring{\zeta}_1; q')$ $(q'=\rho(q)\in T^*_N Y)$ of $\mathrm{supp}(u)\cap U$ in U. Condition (4.3) implying $\partial f/\partial\zeta_1(q)\neq 0$, there exist $\varepsilon>0$, an open neighborhood O′ of q' in T* Y and a homogeneous holomorphic function g on O′ such that

$$O_\varepsilon\cap\{f=0\}=\{\zeta_1=g(z',\zeta'),\ (z',\zeta')\in O'\}$$

with $O_\varepsilon=\{z_1=0,\ |\zeta_1-\mathring{\zeta}_1|<\varepsilon\}\times O'$. Set

$$O_\varepsilon^{(\pm)}=O_\varepsilon\cap\{\pm\xi_2>0\},\qquad O'^{(\pm)}=O'\cap\{\pm\xi_2>0\}.$$

Then $\rho|_{O_\varepsilon^{(\pm)}\cap\{f=0\}}$ gives a holomorphic isomorphism from $O_\varepsilon^{(\pm)}\cap\{f=0\}$ onto $O'^{(\pm)}$. Thus it follows the division theorem of microdifferential operators for microfunctions with holomorphic parameters (*cf*. [3], [5], [15]) that there is an isomorphism

$$(\rho|_{O_\varepsilon^{(\pm)}})_*(H^1 F|_{O_\varepsilon^{(\pm)}})\cong\mathscr{H}om_{\mathscr{E}_Y}(\mathscr{M}_Y^{O_\varepsilon^{(\pm)}},\mathscr{C}_{N_0|Y}),\tag{4.4}$$

where $\mathscr{M}_Y^{O_\varepsilon^{(\pm)}}=(\rho|O_\varepsilon^{(\pm)})_*(\mathscr{E}_{Y\to X}\otimes_{\mathscr{E}_X}\mathscr{M}|O_\varepsilon^{(\pm)})$, which is a coherent $\mathscr{E}_Y$-module on $O'^{(\pm)}$. Since every section of $\mathscr{C}_{N_0|Y}$ has the variable ζ_2 as a holomorphic parameter (*cf*. [3], [5], [15]), (4.4) implies that every section of $H^1 F|_{O_\varepsilon^{(\pm)}}$ has the variable ζ_2 as a holomorphic parameter on the set $O_\varepsilon^{(\pm)}\cap T^*_{N_0}X\cap\{f=0\}$.

Since q is a boundary point of $\overline{\mathrm{supp}(u)\cap U}$, there is a sequence $\{q_\nu;\ \nu\in\mathbb{N}\}$ $(q_\nu=(\zeta_1^{(\nu)},\ i\eta_2^{(\nu)},\ i\eta''))$ of C_p converging to q with $q_\nu\notin\mathrm{supp}(u)$. Then, perturbing $i\eta_2^{(\nu)}$ of q_ν, we get a sequence $\{q_\nu^{(\pm)}\}$ $(q_\nu^{(\pm)}=(\zeta_1^{(\pm,\nu)},\ \zeta_2^{(\pm,\nu)},\ i\eta''))$ of $\rho_0^{-1}\rho_0(p)\cap\{\pm\xi_2>0\}$ (for each $\pm$) so that $q_\nu^{(\pm)}\to q$, $f(q_\nu^{(\pm)})=0$, $q_\nu^{(\pm)}\notin\mathrm{supp}(u)$. Hence, by the unique continuation property in the variable ζ_2 of the sheaf $H^1 F|O_\varepsilon^{(\pm)}\cap\{f=0\}$, we have $u=0$ on $O_\varepsilon^{(\pm)}\cap T^*_{N_0}X$ (by shrinking O_ε if necessary), which is a contradiction. This completes the proof.

5. The complex $C_{\Omega|X}$ for the exterior domain Ω of a corner

Henceforth we shall work on a fixed real analytic manifold M of dimension n and use the notations prepared in Section 2.

5.1. NOTATIONS. — Let Ω_1, Ω_2 be open subsets of M with C^ω-boundary N_1, N_2 respectively. Suppose that N_1 and N_2 intersect transversally. We set

$$\begin{cases}\Omega=\Omega_1\cup\Omega_2,\\ K=M\setminus\Omega,\\ N_0=N_1\cap N_2\end{cases}\tag{5.1}$$

(cf. *Fig.* 5.1); K is a C^ω-convex closed subset of M with piecewise smooth boundary having a corner along N_0. Let Y_1, Y_2 be the complexification in X of N_1, N_2 respectively; set $Y_0=Y_1\cap Y_2$.

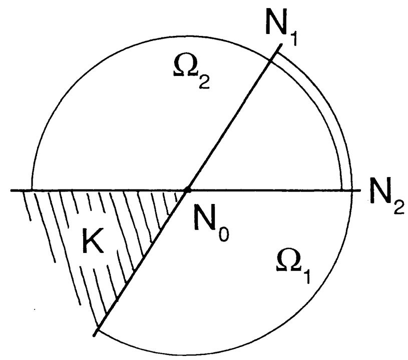

Fig. 5.1.

5.2. Let $\Omega=\Omega_1\cup\Omega_2$ be as in 5.1. Let $T^*_\Omega X$ denote the microsupport of $\mathbb{C}_\Omega$, and let $\mathscr{C}_{\Omega\mid X}$ be the complex of microlocalization of $\mathscr{O}_X$ along $T^*_\Omega X$ (*cf*. Sect. 2.1):

$$\mathscr{C}_{\Omega\mid X}=\mu\mathrm{hom}(\mathbb{C}_\Omega,\ \mathscr{O}_X)\otimes\mathrm{or}_M[n].$$

PROPOSITION 5.1 (Schapira [21]). — *The sheaf* $H^0(\mathscr{C}_{\Omega\mid X})$ *is supported in* $T^*_M X$, *and the homomorphism* [*cf*. (2.3)]

$$H^0(\mathscr{C}_{\Omega\mid X})\to H^0(\mathscr{C}_{\Omega_1\mid X})\oplus H^0(\mathscr{C}_{\Omega_2\mid X}) \tag{5.2}$$

is injective.

Sketch of Proof. — The first part is already proved in Corollary 2.2. Let $K_i=M\setminus\Omega_i$ $(i=1,\ 2)$; then it follows from Proposition 2.1 that $\mathscr{C}_{K\mid X}|_{T^*_M X}$ and $\mathscr{C}_{K_i\mid X}|_{T^*_M X}$ $(i=1,\ 2)$ are subsheaves of $\mathscr{C}_M$. It is sufficient for the second part to prove that, at $p\in T^*_M X$,

$$\mathscr{C}_{K\mid X,\,p}=\mathscr{C}_{K_1\mid X,\,p}\cap\mathscr{C}_{K_2\mid X,\,p}\quad\text{in }\mathscr{C}_{M\,p}. \tag{5.3}$$

By using a quantized complex contact transformation (*cf*. [6], chap. 11), the problem is reduced to the following:

Let $z=x+iy\,(=(z_1,\ \ldots,\ z_n))$ be the affine coordinates of $\mathbb{C}^n$. Set

$$D_0=\left\{z\in\mathbb{C}^n\,|\,y_n>\sum_{j=1}^{n-1}y_j^2\right\},$$

$$D_1=\left\{z\in\mathbb{C}^n\,|\,y_n>(y_1)_-^2+\sum_{j=3}^{n-1}y_j^2\right\},$$

$$D_2=\left\{z\in\mathbb{C}^n\,|\,y_n>(y_2)_-^2+\sum_{j=3}^{n-1}y_j^2\right\},$$

$$D_3=\left\{z\in\mathbb{C}^n\,|\,y_n>(y_1)_-^2+(y_2)_-^2+\sum_{j=3}^{n-1}y_j^2\right\},$$

and define the sheaves $\mathscr{C}_{\partial D_i}$ on ∂D_i $(0 \leqq i \leqq 3)$ by $\mathscr{C}_{\partial D_i} = \Gamma_{D_i} = \Gamma_{D_i}(\mathscr{O}_{\mathbb{C}^n})/\mathscr{O}_{\mathbb{C}^n}|_{\partial D_i}$. Then (5.3) is equivalent to the equality

$$(5.4) \qquad \mathscr{C}_{\partial D_3, 0} = \mathscr{C}_{\partial D_1, 0} \cap \mathscr{C}_{\partial D_2, 0} \quad \text{in}\, \mathscr{C}_{\partial D_0, 0}.$$

Since D_3 is the convex hull of $D_1 \cup D_2$, this follows from a local version of Bochner's tube theorem (*cf.* e. g. [4], Prop. 3.8.6). ∎

Let $\mathscr{M}$ be a coherent $\mathscr{D}_X$-module defined on an open subset of M. The following proposition is a special case of Proposition 3.2.

PROPOSITION 5.2. — *Assume that* Y_1 *and* Y_2 *are non characteristic for* $\mathscr{M}$. *Then we have* $H^i \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega \mid X}) = 0\ (i<0)$.

COROLLARY 5.3. — *Assume that* Y_1 *and* Y_2 *are non characteristic for* $\mathscr{M}$. *Let u be a hyperfunction solution to* $\mathscr{M}$ *on* Ω. *If* $SS_\Omega^{\mathscr{M}}(u) \subset T_X^* X$, *then* $u \in \mathscr{A}_M(\bar{\Omega})$.

Let Ω_3 be another open subset with C^ω-boundary N_3 such that $\Omega_3 \subset \Omega$ and $N_0 \subset N_3$. Note that $T_{\Omega_3}^* X \cap \pi^{-1}(N_0) \subset T_\Omega^* X$. Let Y_3 be the complexification of N_3 in X. Then we have:

PROPOSITION 5.4. — *If* Y_3 *is non characteristic for* $\mathscr{M}$, *then the homomorphism*

$$H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega \mid X})_p \to H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega_3 \mid X})_p$$

is injective at $p \in T_{\Omega_3}^* X \setminus T_M^* X$.

Proof. — Let us choose a system of local coordinates $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ of M such that $\Omega_i = \{x_i > 0\}$ $(i=1, 2)$. Let z be the complexification of x, $(z, \zeta) = (x+iy, \xi + i\eta)$ the associated coordinate system of $T^* X$. We set

$$\Lambda_0 = \{(x+iy, \xi+i\eta) \mid x_1 = x_2 = 0, y = 0, \xi_1 < 0, \xi_2 < 0, \xi_3 = \ldots = \xi_n = 0\},$$
$$\Lambda_1^- = \{(x+iy, \xi+i\eta) \mid x_1 = 0, x_2 \leqq 0, y = 0, \xi_1 < 0, \xi_2 = \xi_3 = \ldots = \xi_n = 0\},$$
$$\Lambda_2^- = \{(x+iy, \xi+i\eta) \mid x_1 \leqq 0, x_2 = 0, y = 0, \xi_2 < 0, \xi_1 = \xi_3 = \ldots = \xi_n = 0\};$$

then $T_\Omega^* X \setminus T_M^* X = \Lambda_0 \cup \Lambda_1^- \cup \Lambda_2^-$.

(*Case* 1) $p \in \Lambda_0$. — Since $SS(\mathbb{C}_{K \setminus N_0}) \cap \Lambda_0 = \varnothing$, $\mathbb{C}_K$ and $\mathbb{C}_{N_0}$ are isomorphic in $D^b(X; p)$; therefore the $\mathscr{E}_X$-homomorphism $\mathscr{C}_{N_0 \mid X} \to \mathscr{C}_{K \mid X}$ is an isomorphism in a neighborhood of p [*cf*. (1.3)]. On the other hand, since $p \notin T_M^* X$, we have an isomorphism $\mathscr{C}_{\Omega \mid X} \xrightarrow{\sim} \mathscr{C}_{K \mid X}[1]$ in a neighborhood of p [*cf*. (2.2)]. Thus, putting $K_3 = M \setminus \Omega_3$, we have a commutative diagram

$$\begin{array}{ccc}
H^0 \mathbf{R}\mathscr{H}om_{\mathscr{E}_X}(\mathscr{M}, \mathscr{C}_{\Omega \mid X})_p & \to & H^0 \mathbf{R}\mathscr{H}om_{\mathscr{E}_X}(\mathscr{M}, \mathscr{C}_{\Omega_3 \mid X})_p \\
\wr\| & & \wr\| \\
H^1 \mathbf{R}\mathscr{H}om_{\mathscr{E}_X}(\mathscr{M}, \mathscr{C}_{K \mid X})_p & \to & H^1 \mathbf{R}\mathscr{H}om_{\mathscr{E}_X}(\mathscr{M}, \mathscr{C}_{K_3 \mid X})_p \\
\wr\| & & \wr\| \\
H^1 \mathbf{R}\mathscr{H}om_{\mathscr{E}_X}(\mathscr{M}, \mathscr{C}_{N_0 \mid X})_p & \to & H^1 \mathbf{R}\mathscr{H}om_{\mathscr{E}_X}(\mathscr{M}, \mathscr{C}_{N_3 \mid X})_p.
\end{array}$$

Let $\rho_3 : T^*X \times_X Y_3 \to T^*Y_3$. Let $\mathcal{M}_{Y_3}^{\{p\}} = \mathcal{E}_{Y_3 \to X, p} \otimes_{\mathcal{E}_X} (\mathcal{E}_X \otimes_{\mathcal{D}_X} \mathcal{M})_p$; then $\mathcal{M}_{Y_3}^{\{p\}}$ is an $\mathcal{E}_{Y_3, \rho_3(p)}$-module of finite type. By the division theorem of microdifferential operators for microfunctions with holomorphic parameters (*cf.* [3], [5], [15]), we have a commutative diagram

$$\begin{array}{ccc} H^1 \mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{N_0|X})_p & \to & H^1 \mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{N_3|X})_p \\ \wr\!\| & & \wr\!\| \\ \mathrm{Hom}_{\mathcal{E}_{Y_3}}(\mathcal{M}_{Y_3}^{\{p\}}, \mathcal{C}_{N_0|Y_3, \rho_3(p)}) & \to & \mathrm{Hom}_{\mathcal{E}_{Y_3}}(\mathcal{M}_{Y_3}^{\{p\}}, \mathcal{C}_{N_3 \rho_3(p)}), \end{array}$$

Hence the injectivity to be proved is equivalent to that of the last horizontal arrow, which is a conclusion of Proposition 2.1 since $\rho_3(p) \in T_{N_3}^* Y_3$.

(*Case* 2) $p \in \Lambda_1^- \cup \Lambda_2^-$. — Let $p \in \Lambda_1^-$. Since N_3 is tangent to N_1, by replacing Ω_3 with a smaller one, we may assume that $\Omega_3 \subset \Omega_1$. Then we have the decomposition of the morphism $\beta = \beta_1 \circ \beta_2$:

$$\begin{array}{ccc} H^0 \mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{\Omega|X}) & \xrightarrow{\beta} & H^0 \mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{\Omega_3|X}) \\ \downarrow \beta_2 & \nearrow \beta_1 & \\ H^0 \mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{\Omega_1|X}). & & \end{array}$$

Set $N_1^- = N_1 \cap K$. Since $SS(\mathbb{C}_{K \setminus N_1^-}) \cap \Lambda_1^- = \varnothing$, $\mathbb{C}_K$ and $\mathbb{C}_{N_1^-}$ are isomorphic in $D^b(X; p)$; therefore $\mathcal{C}_{N_1^-|X} \xrightarrow{\sim} \mathcal{C}_{K|X}$ in a neighborhood of p. Thus, in the same way as in Case 1, the injectivity of β_2 is reduced to that of the homomorphism of sheaves on $T_{N_1}^* Y_1 : \mathcal{C}_{N_1^-|Y_1}|_{T_{N_1}^* Y_1} \to \mathcal{C}_{N_1}$. On the other hand, it follows from Proposition 2.6 that β_1 in injective, which achieves the proof.

5.3. The case where Y_0 is non characteristic.

Proposition 5.5. — *Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module. If Y_0 is non characteristic for $\mathcal{M}$, then we have*

$$H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega|X})|_{\pi^{-1}(N_0) \setminus T_M^* X} = 0. \tag{5.5}$$

Proof. — We choose a local coordinate system as in the proof of Proposition 5.4 and use the same notations. Let $p \in \Lambda_0$. Then we have the isomorphism

$$H^0 \mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{\Omega|X})_p \cong \mathcal{E}xt^1_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{N_0|X})_p. \tag{5.6}$$

Let $\rho_0 : Y_0 \times_X T^*X \to T^*Y_0$. Let $\mathcal{M}_{Y_0}^{\{p\}} = \mathcal{E}_{Y_0 \to X, p} \otimes_{\mathcal{E}_X} (\mathcal{E}_X \otimes_{\mathcal{D}_X} \mathcal{M})_p$. Then, since Y_0 is noncharacteristic for $\mathcal{M}$, $\mathcal{M}_{Y_0}^{\{p\}}$ is an $\mathcal{E}_{Y_0 \rho_0(p)}$-module of finite type and the division theorem of microdifferential operators gives the isomorphism

$$\mathrm{Ext}^j_{\mathcal{E}_{X\,p}}(\mathcal{M}_p, \mathcal{C}_{N_0|X\,p}) \cong \mathrm{Ext}^{j-2}_{\mathcal{E}_{Y_0 \rho_0(p)}}(\mathcal{M}_{Y_0}^{\{p\}}, \mathcal{C}_{N_0 \rho_0(p)}). \tag{5.7}$$

(5.6) and (5.7) prove that $H^0 \mathbf{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}, \mathcal{C}_{\Omega|X})_p = 0$.

Let $p\in\Lambda_1^-\cap\pi^{-1}(\mathrm{N}_0)$. Let $\rho_1:\mathrm{T}^*\mathrm{X}\times_{\mathrm{X}}\mathrm{Y}_1\to\mathrm{T}^*\mathrm{Y}_1$, $q=\rho_1(p)$. Let $\mathscr{M}_{\mathrm{Y}_1}^{\{p\}}$ be the $\mathscr{E}_{\mathrm{Y}_1 q}$-module of finite type induced from $(\mathscr{E}_{\mathrm{X}}\otimes_{\mathscr{D}_{\mathrm{X}}}\mathscr{M})_p$. Let $\mathrm{N}_1^-=\mathrm{N}_1\cap\mathrm{K}$. Then we have (*cf*. the proof of Proposition 5.4)

$$\mathrm{H}^0\,\mathbf{R}\,\mathscr{H}om_{\mathscr{E}_{\mathrm{X}}}(\mathscr{M},\mathscr{C}_{\Omega\mid\mathrm{X}})_p\cong\mathrm{Ext}^1_{\mathscr{E}_{\mathrm{X}\,p}}(\mathscr{M}_p,\mathscr{C}_{\mathrm{N}_1^-\mid\mathrm{X}\,p})\cong\mathrm{Hom}_{\mathscr{E}_{\mathrm{Y}_1 q}}(\mathscr{M}_{\mathrm{Y}_1}^{\{p\}},\mathscr{C}_{\mathrm{N}_1^-\mid\mathrm{Y}_1 q}).$$

Since Y_0 is non characteristic for $\mathscr{M}$, $\mathrm{Y}_0\subsetneq\mathrm{Y}_1$ is non characteristic for $\mathscr{M}_{\mathrm{Y}_1}^{\{p\}}$ at q. Hence, by the unique continuation property of the sheaf $\mathscr{C}_{\mathrm{N}_1^-\mid\mathrm{Y}_1}$ (*cf*. [7], Prop. 1.8), we have $\mathrm{Hom}_{\mathscr{E}_{\mathrm{Y}_1 q}}(\mathscr{M}_{\mathrm{Y}_1}^{\{p\}},\mathscr{C}_{\mathrm{N}_1^-\mid\mathrm{Y}_1 q})=0$. This completes the proof.

In particular, when $\mathscr{M}$ is an elliptic system, we have the following theorem of Bochner type as an immediate corollary of Proposition 5.5 and Corollary 5.3.

COROLLARY 5.6. — *Let* $\Omega=\Omega_1\cup\Omega_2$ *be as in* 5.1. *Let* Y_0 *denote the complexification of the corner* N_0 *of* $\mathrm{M}\setminus\Omega$. *Assume that* $\mathscr{M}$ *is elliptic and that* Y_0 *is non characteristic for* $\mathscr{M}$. *Then the restriction homomorphism*

$$\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M},\mathscr{A}_{\mathrm{M}})\,|_{\mathrm{N}_0}\to\Gamma_\Omega\,\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M},\mathscr{A}_{\mathrm{M}})\,|_{\mathrm{N}_0}$$

is an isomorphism.

Remark. — The boundary value problems for elliptic systems are systematically studied by Kashiwara-Kawai [3], and Corollary 5.6 is just one of the conclusions from their main theorem. But here we are stressing the fact that the noncharacteristic condition of Y_0 makes the problem trivial outside $\mathrm{T}^*_{\mathrm{M}}\mathrm{X}$.

6. Microlocal analysis at the corner of an obstacle

Let $\Omega=\Omega_1\cup\Omega_2$ be a domain given as the union of two open subsets with transversal C^ω-boundaries (*cf*. 5.1). We follow the notations of 5.1. For $p\in\mathrm{N}_0\times_{\mathrm{M}}\mathrm{T}^*_{\mathrm{M}}\mathrm{X}$, we set $\mathrm{E}_p=\rho_0^{-1}\rho_0(p)\cap\mathrm{T}^*_{\mathrm{M}}\mathrm{X}$, with ρ_0 being the natural projection $\mathrm{Y}_0\times_{\mathrm{X}}\mathrm{T}^*\mathrm{X}\to\mathrm{T}^*\mathrm{Y}_0$.

6.1. A KEY LEMMA. — Let $p\in\mathrm{N}_0\times_{\mathrm{M}}\mathrm{T}^*_{\mathrm{M}}\mathrm{X}$. Let f be a homogeneous holomorphic function on $\mathrm{T}^*\mathrm{X}$ defined in a neighborhood of p with $f(p)=0$.

Let $\mathscr{M}$ be a coherent $\mathscr{D}_{\mathrm{X}}$-module. Assume the following:

(6.1) each of Y_1 and Y_2 is non characteristic for $\mathscr{M}$;

(6.2) $\mathrm{Char}(\mathscr{M})\subset\{f=0\}$ in a neighborhood of p;

(6.3) $\mathrm{Im}f\,|\,\mathrm{T}^*_{\mathrm{M}}\mathrm{X}\equiv 0$;

(6.4) $\{f,\varphi_1\}(p)\neq 0,\qquad\{f,\varphi_2\}(p)\neq 0,$

where φ_1, φ_2 are defining functions of Y_1, Y_2 respectively with $d\varphi_1\wedge d\varphi_2\neq 0$.

Set $\mathrm{C}_p=\mathrm{E}_p\cap\{f=0\}$; then C_p is a nonsingular curve in E_p passing through p, in which curve the sheaf $\mathrm{H}^0\,\mathbf{R}\,\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M},\mathscr{C}_{\Omega\mid\mathrm{X}})\,|_{\mathrm{E}_p}$ is supported.

LEMMA 6.1. — *Assume* (6.1)-(6.4). *Let* U *be a small connected open neighborhood of* p *in* C_p, Z *a closed proper subset of* U. *Then the homomorphism* [*cf*. (2.5)]

$$\Gamma_Z(U, H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega\,|\,X})\,|\,C_p) \to \Gamma_Z(U, H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega_j\,|\,X})\,|\,C_p)$$

is injective ($j=1, 2$).

Proof. — We shall prove the injectivity for $j=1$. Set $\Omega_{12}=\Omega_1\cap\Omega_2$, and define the complexes of sheaves: $F=\mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega\,|\,X})$, $F_k=\mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega_k\,|\,X})$ ($k=1, 2, 12$). Since F_1, F_2, and F are concentrated in degrees$\geqq 0$ in virtue of (6.1), $H^0\mathbf{R}\Gamma_Z(U, F_k)\to\Gamma_Z(U, H^0 F_k)$ is an isomorphism for $k=1,2,\varnothing$. Thus we have the commutative diagram

$$\begin{array}{ccc} \Gamma_Z(U, H^0 F) & \xrightarrow{\beta_1} & \Gamma_Z(U, H^0 F_1) \\ {\scriptstyle\beta_2}\downarrow & & \downarrow{\scriptstyle\beta_{12/1}} \\ \Gamma_Z(U, H^0 F_2) & \xrightarrow[\beta_{12/2}]{} & H^0\mathbf{R}\Gamma_Z(U, F_{12}). \end{array}$$

Let $u\in\Gamma_Z(U, H^0 F)$, $\beta_1(u)=0$. Then $\beta_{12/2}\circ\beta_2(u)=\beta_{12/1}\circ\beta_1(u)=0$. Since $\beta_{12/2}$ is injective by Theorem 4.1, we have $\beta_2(u)=0$. Together with $\beta_1(u)=0$, this implies $u=0$ on account of Proposition 5.1.

Remark. — The first (resp. second) condition of (6.4) on f is not necessary to prove injectivity of the restriction homomorphism from Ω to Ω_1 (resp. Ω_2).

In particular we have

THEOREM 6.2. — *Assume* (6.1)-(6.4). *Then*

$$\Gamma_{\{p\}}(H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega\,|\,X})\,|\,E_p) \to \Gamma_{\{p\}}(H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega_j\,|\,X})\,|\,E_p)$$

is injective ($j=1, 2$).

Proof. — Since p is not an isolated point of $E_p\cap\{f=0\}$, we can apply Theorem 6.1 for a small connected open neighborhood U of p and $Z=\{p\}$.

NOTATIONS (*cf*. Sect. 2.6). — Let u be section of $H^0\mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega\,|\,X})$. In this paper, we use the following notations for simplicity: $SS_\Omega(u)=\operatorname{supp}(u)\cap T^*_M X$ and $SS_{\Omega_i}(u)=SS^{\mathscr{M}}_{\Omega_i}(u)\cap T^*_M X$ ($i=1, 2$). (*Cf*. also Sect. 2.4 and 2.5.)

COROLLARY 6.3. — *Assume* (6.1)-(6.4).

Let u *be a section of* $H^0\mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega\,|\,X})$ *in a neighborhood of* p. *If* $SS_{\Omega_1}(u)\cap E_p=\{p\}$ *in a neighborhood of* p, *then* $p\in SS_{\Omega_2}(u)$.

Proof. — Suppose that $p\notin SS_{\Omega_2}(u)$, and we shall see the contradiction. Since $q\notin SS_{\Omega_1}(u)$ for every $q\in E_p$ close to p with $q\neq p$, it follows from Proposition 5.1 that $q\notin SS_\Omega(u)$. This implies that u belongs to the stalk of the subsheaf $\Gamma_{\{p\}}(H^0\mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega\,|\,X})\,|_{E_p})$. Then, by Theorem 6.2, we have $u=0$, which is a contradiction.

6.2. PROPAGATION BEYOND A CORNER OF ANALYTIC SINGULARITIES. — We apply Corollary 6.3 to the case where the bicharacteristic curve of f passing through p is transversal to a corner.

Let $p \in N_0 \times_M T^*_M X$. Let f be a homogeneous holomorphic function defined in a neighborhood of p with $f(p)=0$. Let $\mathscr{M}$ be a coherent $\mathscr{D}_X$-module and assume (6.1)-(6.4). Denote by $H_g^{\mathbb{R}}$ the real Hamiltonian vector field of $g = \operatorname{Im} f$ on $(T^*X, \operatorname{Re}\omega)$. We assume moreover

$$\pm d\pi\, H_g^{\mathbb{R}}(p) \notin C_{\pi(p)}(K; N_0), \tag{6.5}$$

where $d\pi : T_p T^* X \to T_{\pi(p)} X$ and $C(K; N_0)$ is the normal cone of K along N_0 (*cf.* 1.1). Let $b^{\pm}(p)$ denote the positive (resp. negative) integral curve of $H_g^{\mathbb{R}}$ issued from p. Condition (6.5) implies that, roughly speaking, $b^{\pm}(p)$ issues into Ω transversally to the corner K (cf. *Fig.* 6.1).

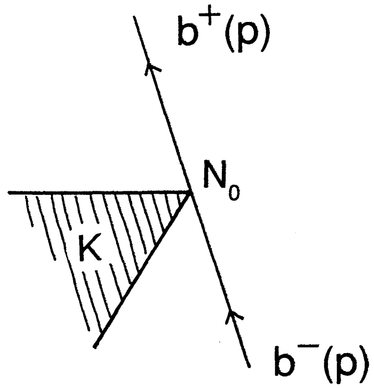

Fig. 6.1.

NOTATION. — For a section u of $H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega|X})$, let

$$SS(u|\Omega) = SS_\Omega(u) \cap \pi^{-1}(\Omega).$$

Remark. — When u is a hyperfunction solution to $\mathscr{M}$ defined on Ω, $SS(u|\Omega)$ is nothing but the singular spectrum (*i.e.* the analytic wavefront set) of u over Ω.

THEOREM 6.4. — *Assume* (6.5) *in addition to* (6.1)-(6.4). *Let u be a section of* $H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega|X})$ *in a neighborhood of p. If $b^-(p) \subset SS(u|\Omega)$ and if $b^-(q) \cap SS(u|\Omega) = \varnothing$ for neighborhood $C_p(\varepsilon)$ of p in C_p and for every $q \in C_p(\varepsilon) \setminus \{p\}$, then $p \in SS_\Omega(u)$ and $b^+(p) \subset SS(u|\Omega)$.*

Proof. — This is an interpretation of Corollary 6.3 by using the theorem of propagation of regularity up to the smooth boundaries N_1, N_2 (*cf.* Kataoka [7], Schapira [16], [17]).

This theorem asserts that an "isolated" singularity p of u propagates up to the corner and goes beyond the corner along the bicharacteristic curve $b(p)$.

6.3. Diffraction by a corner. — Let $p \in \mathrm{N}_0 \times_{\mathrm{M}} \mathrm{T}^*_{\mathrm{M}}\mathrm{X}$. Let f, $\mathscr{M}$ and $b^{\pm}(p)$ be as in 6.2. Note that $\pi(b^+(p)) \subset \Omega_1$ (resp. Ω_2) if and only if $\pi(b^-(p)) \subset \Omega_2$ (resp. Ω_1).

As a corollary of Lemma 6.1, we have the following theorem which proves the appearance of diffracted rays at the corner of K (cf. *Fig.* 6.2).

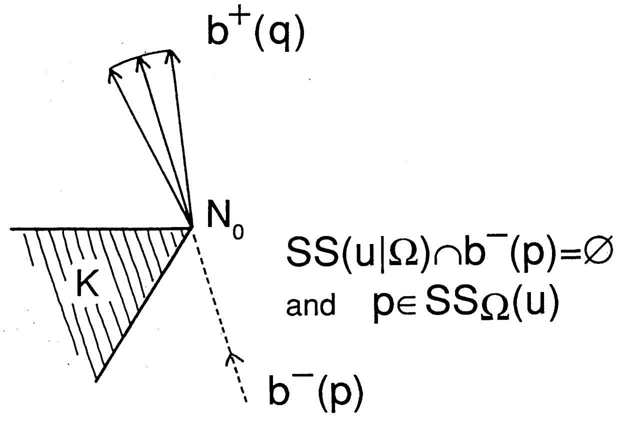

Fig. 6.2.

Set again $\mathrm{C}_p = \mathrm{E}_p \cap \{ f = 0 \}$.

Theorem 6.5. — *Let $\mathscr{M}$ be a coherent $\mathscr{D}_{\mathrm{X}}$-module satisfying the conditions of* 6.2 *at p. Let u be a section of $\mathbf{R}\mathscr{H}om_{\mathscr{D}_{\mathrm{X}}}(\mathscr{M}, \mathscr{C}_{\Omega \mid \mathrm{X}})$ in a neighborhood of p. If $p \in \mathrm{SS}_{\Omega}(u) \setminus \overline{\mathrm{SS}(u \mid \Omega) \cap b^-(p)}$, then there is a neighborhood $\mathrm{C}_p(\varepsilon)$ of p in C_p* such that *$\mathrm{C}_p(\varepsilon) \subset \mathrm{SS}_{\Omega}(u)$ and $b^+(q) \subset \mathrm{SS}(u \mid \Omega)$ for all $q \in \mathrm{C}_q(\varepsilon)$.*

Proof. — We may assume that $\pi(b^-(p)) \subset \Omega_1$. Assume that there exists a sequence $\{q_\nu\}$ of C_p converging to p such that $b^+(q_\nu) \not\subset \mathrm{SS}(u \mid \Omega)$. Then, by propagation of regularity up to the boundary N_2, $q_\nu \notin \mathrm{SS}_{\Omega_2}(u)$. Since on the other hand $p \notin \mathrm{SS}_{\Omega_1}(u)$, it follows from Proposition 5.1 that $q_\nu \notin \mathrm{SS}_{\Omega}(u)$ for $\nu \gg 1$. Thus, by Lemma 6.1, $p \notin \mathrm{SS}_{\Omega_1}(u)$ implies that $p \notin \mathrm{SS}_{\Omega}(u)$, which is a contradiction.

Corollary 6.6. — *Let u be as in Theorem 6.5. If $b^-(p) \not\subset \mathrm{SS}(u \mid \Omega)$ and if there exists a sequence $\{p_\nu\}$ of $\mathrm{SS}_{\Omega}(u)$ converging to p, then $b^+(q) \subset \mathrm{SS}(u \mid \Omega)$ for all $q \in \mathrm{C}_p(\varepsilon)$, with $\mathrm{C}_p(\varepsilon)$ being a neighborhood of p.*

Remark. — $b^+(q)$ $(q \in \mathrm{C}_p(\varepsilon))$ are the so-called diffracted rays produced by $p \in \mathrm{SS}_{\Omega}(u) \setminus \overline{\mathrm{SS}(u \mid \Omega) \cap b^-(p)}$ at the corner (*cf.* Keller [11]).

6.4. Boundary analytic singularities in the complex region.

Let $\Omega = \Omega_1 \cup \Omega_2$. Let $\Omega_3 = \{ \varphi_3 > 0 \}$ be an open subset of M with C^∞-boundary $\mathrm{N}_3 = \{ \varphi_3 = 0 \}$ $(d\varphi_3 \neq 0)$ and contained in Ω. Let $\mathrm{Y}_3 = \{ z \in \mathrm{X} \mid \varphi_3(z) = 0 \}$.

Let $x \in \mathrm{N}_0 \cap \mathrm{N}_3$, $p \in \mathrm{T}^*_{\mathrm{N}_3}\mathrm{X} \cap \pi^{-1}(x)$. Let f be a homogeneous holomorphic function on $\mathrm{T}^*\mathrm{X}$ defined in a neighborhood of p with $f(p) = 0$. Assume

$$\{ f, \varphi_3 \}(p) \neq 0. \tag{6.6}$$

Let $\rho_0 : T^*X \times_X Y_0 \to T^*Y_0$ be the natural projection. We then set

$$C_p^{(3)} = \{ q \in T^*_{N_3}X \cap \pi^{-1}(x) \mid f(q) = 0,\ \rho_0(q) = \rho_0(p) \}; \tag{6.7}$$

$C_p^{(3)}$ is a nonsingular curve in a neighborhood of p passing through p, since $\{ f, \varphi_3 \} \neq 0$.

Let $\mathscr{M}$ be a coherent $\mathscr{D}_X$-module defined in a neighorhood of x and assume that Y_3 is non characteristic for $\mathscr{M}$. Then we have:

THEOREM 6.7. — *Let $p \notin T^*_M X$. Let f be as above. Let $\mathscr{M}$ be a coherent $\mathscr{D}_X$-module for which Y_3 is noncharacteristic. Assume that* $\operatorname{Char}(\mathscr{M}) \subset \{ f = 0 \}$ *in a neighborhood of p. Let u be a section of* $H^0 \mathbf{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{C}_{\Omega \mid X})$ *in a neighborhood of p. Then*

(i) $\operatorname{Supp}(u) \cap \rho_0^{-1} \rho_0(p) \cap T^*_{N_3}X \subset C_p^{(3)}$.

(ii) $C_p^{(3)} \subset \operatorname{Supp}(u)$ *in a neighborhood of p if $u \not\equiv 0$.*

Proof. — The first assertion is trivial. To prove the second assertion, by replacing $\Omega = \Omega_1 \cup \Omega_2$ by $\Omega_3 \cup \Omega_2$ (or $\Omega_1 \cup \Omega_3$), we may assume from the beginning that $\Omega_3 = \Omega_1$. Define $C (= C_p^{(3)})$ by (6.7).

Let us define the complexes of sheaves F_1, F_2, F_{12}, and F on T^*X as in the proof of Lemma 6.1. Let Z be a closed proper subset of a small connected neighborhood of p in C and consider the commutative diagram

$$\begin{array}{ccc} H^0 \mathbf{R}\Gamma_Z(C, F) & \xrightarrow{\beta_1} & H^0 \mathbf{R}\Gamma_Z(C, F_1) \\ {\scriptstyle\beta_2}\downarrow & & \downarrow{\scriptstyle\beta_{12/1}} \\ H^0 \mathbf{R}\Gamma_Z(C, F_2) & \xrightarrow[\beta_{12/2}]{} & H^0 \mathbf{R}\Gamma_Z(C, F_{12}). \end{array}$$

Since $T^*_{\Omega_1}X \cap T^*_{\Omega_2}X \subset T^*_M X$, $F_2 = 0$ on C in a neighborhood of p; thus $\beta_{12/1} \circ \beta_1 = \beta_{12/2} \circ \beta_2$ is the zero map. Then, since $\beta_{12/1}$ is injective by Theorem 4.1, β_1 is still the zero map. The complexes F and F_1 being concentrated in degrees $\geqq 0$ by Proposition 3.2, we have

$$H^0 \mathbf{R}\Gamma_Z(C, F) = \Gamma_Z(C, H^0 F) \qquad \text{and} \qquad H^0 \mathbf{R}\Gamma_Z(C, F_1) = \Gamma_Z(C, H^0 F_1).$$

Thus the map

$$\Gamma_Z(C, H^0 F) \to \Gamma_Z(C, H^0 F_1)$$

is trivial. On the other hand, this map is injective by Proposition 5.4. Hence we have $\Gamma_Z(C, H^0 F) = 0$, which completes the proof.

Remark. — The above theorem will be applied in the forthcoming paper to the problem of continuation of analytic solutions to differential equations.

7. Application to Dirichlet problems

Let $\Omega_1 = \{ \varphi_1 > 0 \}$, $\Omega_2 = \{ \varphi_2 > 0 \}$ be a pair of open subsets of M given by C^ω-functions φ_1 and φ_2 with $d\varphi_1 \wedge d\varphi_2 \neq 0$; let $\Omega = \Omega_1 \cup \Omega_2$ (*cf.* 5.1). We follow the notations of 5.1.

7.1. ASSUMPTIONS AND NOTATIONS. — Let $P = P(x, D)$ be a second order differential operator with analytic coefficients with principal symbol f. We shall always assume that each of Y_1 and Y_2 is non characteristic for P and

$$\operatorname{Im} f \,|\, T_M^* X = 0. \tag{7.1}$$

Let $\rho_j : T^*X \times_X Y_j \to T^*Y_j$ ($j = 0, 1, 2$) be the natural projection.

Let $q_0 \in T_{N_0}^* Y_0$; set $E(q_0) = \rho_0^{-1}(q_0) \cap T_M^* X$, which is an affine plane in $T_M^* X \cap \pi^{-1}(\pi(q_0))$. Considering that $f \,|\, E(q_0)$ is a polynomial of degree two, we assume moreover that

$$C = \{ p \in E(q_0) \,|\, f(p) = 0 \} \quad \text{is an ellipse in } E(q_0). \tag{7.2}$$

Henceforth we fix q_0 throughout.

Let $H_g^{\mathbb{R}}$ denote the real Hamiltonian vector field of $g = \operatorname{Im} f$ on $(T^*X, \operatorname{Re}\omega)$, and let $b^{\pm}(p)$ denote the positive (resp. negative) integral curve of $H_g^{\mathbb{R}}$ issued from $p \in C$. Set

$$C_k^{\pm} = \{ p \in C \,|\, \pm \langle H_g^{\mathbb{R}}(p), d\varphi_k \rangle > 0 \}, \qquad D_k = \{ p \in C \,|\, \langle H_g^{\mathbb{R}}(p), d\varphi_k \rangle = 0 \}$$

for $k = 1, 2$. C is divided into four regions $C_1^{\alpha} \cap C_2^{\beta}$ ($\alpha = \pm, \beta = \pm$) by four points of $D_1 \cup D_2$ (cf. *Fig.* 7.1). Every point of $C_1^- \cup C_2^-$ (resp. $C_1^+ \cup C_2^+$) corresponds to an incoming (resp. outgoing) singularity at the corner on $E(q_0)$; *i.e.*, if $p \in C_1^{\alpha} \cup C_2^{\alpha}$ ($\alpha = \pm$), then $b^{\alpha}(p)$ is transversal to K and $\pi(b^{\alpha}(p)) \subset \Omega$.

For $p \in C$, we denote by $p_{\hat{(k)}}$ ($k = 1,2$) the reflected point of p with respect to N_k, *i.e.*, the point of C satisfying $\rho_k^{-1}\rho_k(p) \cap C = \{p, p_{\hat{(k)}}\}$ (cf. *Fig.* 7.2). Observe that $D_k = \{ p \in C \,|\, p = p_{\hat{(k)}} \}$ and $p_{\hat{(k)}} \in C_k^+$ if and only if $p \in C_k^-$ ($k = 1, 2$).

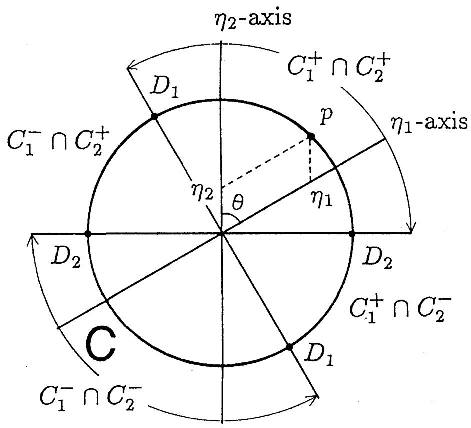

Fig. 7.1. — $C = E(q_0) \cap \{ f = 0 \}$.

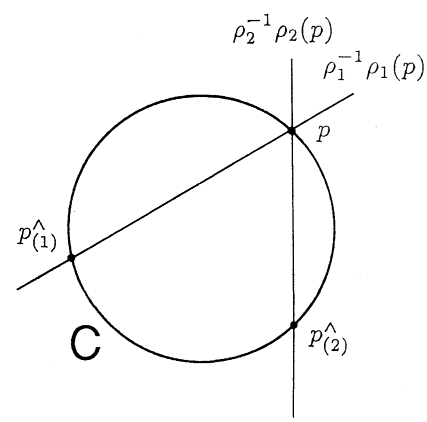

Fig. 7.2.

7.2. BOUNDARY CONDITIONS. — Set $\partial_j \Omega = \partial\Omega \cap N_j \setminus N_0$ ($j = 1, 2$); $\partial_j \Omega$ is an open subset of N_j lying on one side of N_0 ($\subset N_j$), and $\partial\Omega = \partial_1 \Omega \cup \partial_2 \Omega \cup N_0$ (disjoint).

7.2.1. *Boundary values on* $\partial_1 \Omega$ *and* $\partial_2 \Omega$. — Let P be a differential operator of 7.1. Let $\mathcal{M} = \mathcal{D}_X / \mathcal{D}_X P = \mathcal{D}_X u$ with $u = 1 \bmod \mathcal{D}_X P$.

The tangential system $\mathcal{M}_{Y_1}$ of $\mathcal{M}$ on Y_1 is given by definition as

$$\begin{aligned}\mathcal{M}_{Y_1} &= \mathcal{D}_{Y_1 \to X} \otimes_{\mathcal{D}_X} \mathcal{M} \\ &\cong \mathcal{D}_{Y_1}(1_{Y_1 \to X} \otimes u) \oplus \mathcal{D}_{Y_1}(1_{Y_1 \to X} \otimes \partial_{\nu_1} u),\end{aligned}$$

where $\partial_{\nu_1} = \sum \partial \varphi_1 / \partial x_i . \partial / \partial x_i$ for a choice of local coordinates $(x_1, \ldots, x_n)$. Define a coherent $\mathcal{D}_{Y_1}$-submodule $\mathcal{N}_1$ of $\mathcal{M}_{Y_1}$ by

$$\mathcal{N}_1 = \mathcal{D}_{Y_1}(1_{Y_1 \to X} \otimes u).$$

We have the following chain of $\mathbb{C}$-linear maps:

$$\begin{aligned}(7.3) \quad \Gamma(E(q_0); H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega \mid X})) &\to \Gamma(E(q_0); H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega_1 \mid X})) \\ &\overset{\mathrm{bv}}{\to} \Gamma(\rho_1(E(q_0)); \mathcal{H}om_{\mathcal{D}_{Y_1}}(\mathcal{M}_{Y_1}, \mathcal{C}_{N_1})) \\ &\to \Gamma(\rho_1(E(q_0)); \mathcal{H}om_{\mathcal{D}_{Y_1}}(\mathcal{N}_1, \mathcal{C}_{N_1})) \\ &\to \Gamma(\rho_1(E(q_0)); H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_{Y_1}}(\mathcal{N}_1, \mathcal{C}_{\partial_1 \Omega \mid Y_1}))\end{aligned}$$

[*cf.* (2.3) for the first and the fourth maps; *cf.* (2.4) for the second map]. For a section u of $H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega \mid X})$ on $E(q_0)$, we denote by $u|_{\partial_1 \Omega}^{(\mu)}$ the image of u in $\Gamma(\rho_1(E(q_0)); H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_{Y_1}}(\mathcal{N}_1, \mathcal{C}_{\partial_1 \Omega \mid Y_1}))$ by the composite of (7.3).

Remark. — Let u be a hyperfunction solution to P defined on Ω. Denote by $u|_{N_1}$ the first trace of $u|_{\Omega_1}$ on N_1 as a hyperfunction (*cf.* [8], [10], [12], [16], [17]); set $u|_{\partial_1 \Omega} = (u|_{N_1})|_{\partial_1 \Omega}$, the first trace of u on $\partial_1 \Omega$. Then $u|_{\partial_1 \Omega}^{(\mu)} = \alpha(u|_{\partial_1 \Omega})$, where

$$\alpha: \quad \Gamma(\partial_1 \Omega; H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_{Y_1}}(\mathcal{N}_1, \mathcal{B}_{N_1})) \to \Gamma(\rho_1(E(q_0)); H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_{Y_1}}(\mathcal{N}_1, \mathcal{C}_{\partial_1 \Omega \mid Y_1}))$$

(*cf.* Sect. 2.5). In particular, $SS_{\partial_1 \Omega}(u|_{\partial_1 \Omega}) \cap \rho_1(E(q_0)) = \varnothing$ if and only if $u|_{\partial_1 \Omega}^{(\mu)} \equiv 0$ on $\rho_1(E(q_0))$.

We also define a section $u|_{\partial_2 \Omega}^{(\mu)}$ of $H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_{Y_2}}(\mathcal{N}_2, \mathcal{C}_{\partial_2 \Omega \mid Y_2})$ through the same procedure as above for $\mathcal{N}_2 = \mathcal{D}_{Y_2}(1_{Y_2 \to X} \otimes u)$.

7.2.2. *A lemma.* — We follow the notations of 7.1 and 7.2.1. For a section u of $\mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega \mid X})$ on $E(q_0)$, we denote by $u|_{\Omega_1}$ the image of u by the first map of (7.3); we set $SS_{\Omega_1}(u) = \mathrm{supp}(u|_{\Omega_1})$ (*cf.* Sect. 2.6).

LEMMA 7.1. — *Let* $u \in \Gamma(E(q_0); H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\Omega \mid X}))$ *with* $u|_{\partial_1 \Omega}^{(\mu)} = 0$.

(i) *Let* U^+ *be an open subset of* C_1^+; *set* $U^- = \rho_1^{-1} \rho_1(U^+) \cap C_1^-$. *Assume* $u|_{\Omega_1} \equiv 0$ *on* $U^{\mp}$. *If* $u|_{\Omega_1} = 0$ *at a point* $p_0^{\pm} \in U^{\pm}$, *then* $u|_{\Omega_1} \equiv 0$ *on* $U^{\pm}$. (U^+ *is not necessarily connected.*)

In particular we have the following:

(ii) *Let* $p \in C$. *If* $p \in SS_{\Omega_1}(u)$ *and if there is a sequence* $\{p_\nu\}$ *of* C *converging to* p *with* $p_\nu \notin SS_{\Omega_1}(u)$, *then* $p_{\hat{(1)}} \in SS_{\Omega_1}(u)$.

Proof. — We shall prove (i). Let $v=\mathrm{bv}(u|_{\Omega_1})$ be the image of $u|_{\Omega_1}$ by the second map of (7.3), w the image of v by the third map; then $w\in\Gamma(\rho_1(\mathrm{E}(q_0));\ \mathscr{H}om_{\mathscr{D}_{\mathrm{Y}_1}}(\mathscr{N}_1, \mathscr{C}_{\mathrm{N}_1}))$, and $u|^{(\mu)}_{\partial_1\Omega}$ is the image of w by the fourth map.

Assume that $u|_{\Omega_1}=0$ at a point p_0^+ of U^+. Since $u|_{\Omega_1}=0$ at $(p_0^+)^{\wedge}_{(1)}\in\mathrm{U}^-$, we have $w=0$ at $\rho_1(p_0^+)$. Let $\mathrm{G}_1=\mathrm{N}_1\diagdown\partial_1\Omega$; which is a closed subset of N_1 with C^ω-boundary N_0. We then have the short exact sequence

$$0\to\mathscr{H}om_{\mathscr{D}_{\mathrm{Y}_1}}(\mathscr{N}_1, \mathscr{C}_{\mathrm{G}_1|\mathrm{Y}_1})\to\mathscr{H}om_{\mathscr{D}_{\mathrm{Y}_1}}(\mathscr{N}_1, \mathscr{C}_{\mathrm{N}_1})\to\mathrm{H}^0\,\mathbf{R}\,\mathscr{H}om_{\mathscr{D}_{\mathrm{Y}_1}}(\mathscr{N}_1, \mathscr{C}_{\partial_1\Omega|\mathrm{Y}_1})$$

[*cf.* (2.2)]. Hence it follows from the assumption $(u|^{(\mu)}_{\partial_1\Omega}\equiv 0)$ that w is in fact a section of the subsheaf $\mathscr{H}om_{\mathscr{D}_{\mathrm{Y}_1}}(\mathscr{N}_1, \mathscr{C}_{\mathrm{G}_1|\mathrm{Y}_1})$. The sheaf $\mathscr{C}_{\mathrm{G}_1|\mathrm{Y}_1}$ having the unique continuation property on $\rho_1(\mathrm{E}(q_0))$ (*cf.* [7], Prop. 1.8; [9], Prop. 4.1.11), we have $w\equiv 0$ on $\rho_1(\mathrm{E}(q_0))$.

Now let $p^\pm\in\mathrm{U}^\pm$ with $\rho_1(p^\pm)=q_1$. Since $\mathscr{M}_{\mathrm{Y}_1 q_1}(=\mathscr{E}_{\mathrm{Y}_1 q_1}\otimes_{\mathscr{D}_{\mathrm{Y}_1}}\mathscr{M}_{\mathrm{Y}_1})$ is isomorphic to $\mathscr{N}_{1\,q_1}\oplus(\mathscr{E}_{\mathrm{Y}_1\to\mathrm{X}}\otimes_{\mathscr{E}_\mathrm{X}}\mathscr{M}_{p^-})$ as an $\mathscr{E}_{\mathrm{Y}_1 q_1}$-module,

$$(7.4)\quad \bigoplus\nolimits_\pm \mathrm{H}^0\,\mathbf{R}\,\mathrm{Hom}_{\mathscr{E}_{\mathrm{X}\,p^\pm}}(\mathscr{M}_{p^\pm}, \mathscr{C}_{\Omega_1|\mathrm{X},\,p^\pm})$$
$$\to\mathrm{Hom}_{\mathscr{E}_{\mathrm{Y}_1 q_1}}(\mathscr{N}_{1\,q_1}, \mathscr{C}_{\mathrm{N}_1 q_1})\oplus\mathrm{H}^0\,\mathbf{R}\,\mathrm{Hom}_{\mathscr{E}_{\mathrm{X}\,p^-}}(\mathscr{M}_{p^-}, \mathscr{C}_{\Omega_1|\mathrm{X},\,p^-}),$$
$$((u|_{\Omega_1})_{p^+}, (u|_{\Omega_1})_{p^-})\mapsto(w_{q_1}, (u|_{\Omega_1})_{p^-})$$

is injective. Since $u|_{\Omega_1}\equiv 0$ on U^- and $w\equiv 0$ on $\rho_1(\mathrm{E}(q_0))$, the injectivity of (7.4) implies that $u|_{\Omega_1}=0$ at any point $p^+\in\mathrm{U}^+$. This completes the proof.

7.3. Diffraction by a corner of incident rays. — As an application of Theorems 6.4 (or 6.3) and 6.5, we prove that a cone of diffracted rays is produced when a single ray hits the corner or when finitely many rays hit the corner simultaneously.

7.3.1. Let $p_0\in\mathrm{E}(q_0)$. Let $(\eta_1, \eta_2)\in\mathbb{R}^2$ be affine coordinates on $\mathrm{E}(q_0)$ such that (η_1, η_2) corresponds to the point $p=p_0+i\eta_1\,d\varphi_1+i\eta_2\,d\varphi_2$. Then each fibre of $\rho_k|\mathrm{E}(q_0)\,(k=1, 2)$ is given by

$$\rho_1|\mathrm{E}(q_0):(\eta_1, \eta_2)\mapsto\eta_2,\qquad \rho_2|\mathrm{E}(q_0):(\eta_1, \eta_2)\mapsto\eta_1$$

and C is given by

$$(7.5)\quad \mathrm{C}=\left\{(\eta_1, \eta_2)\left|\left(\frac{\eta_1-\mathring{\eta}_1}{a}\right)^2+2\left(\frac{\eta_1-\mathring{\eta}_1}{a}\right)\left(\frac{\eta_2-\mathring{\eta}_2}{b}\right)\cos\theta+\left(\frac{\eta_2-\mathring{\eta}_2}{b}\right)^2=1\right.\right\}$$

for some $a>0$, $b>0$, $0<\theta<\pi$, $(\mathring{\eta}_1, \mathring{\eta}_2)\in\mathbf{R}^2$ (cf. *Fig.* 7.1). Note that θ is determined independently of the choice of φ_1, φ_2 and p_0.

7.3.2. *Statement of Theorems.* — Let u be a section of $\mathrm{H}^0\,\mathbf{R}\,\mathscr{H}om_{\mathscr{D}_\mathrm{X}}(\mathscr{D}_\mathrm{X}/\mathscr{D}_\mathrm{X}\mathrm{P}, \mathscr{C}_{\Omega|\mathrm{X}})$ in a neighborhood of C (or a hyperfunction solution to P defined on Ω). Assume that u satisfies the microlocal boundary condition

$$(7.6)\qquad u|^{(\mu)}_{\partial_1\Omega}\equiv 0\quad\text{on }\rho_1(\mathrm{C})\qquad\text{and}\qquad u|^{(\mu)}_{\partial_2\Omega}\equiv 0\quad\text{on }\rho_2(\mathrm{C}).$$

Remark. — In the case where u is a hyperfunction solution to P on Ω, (7.6) is fulfilled in particular if u satisfies the boundary condition

$$(7.7)\qquad u|_{\partial_1\Omega}=g_1 \quad \text{and} \quad u|_{\partial_2\Omega}=g_2$$

for $g_1\in\Gamma(\overline{\partial_1\Omega};\mathscr{A}_{N_1})$ and $g_2\in\Gamma(\overline{\partial_2\Omega};\mathscr{A}_{N_2})$.

Cf. the notations in Corollary 6.3 and Theorem 6.4 for $SS_{\Omega_1}(u)$, $SS_{\Omega_2}(u)$, $SS(u|\Omega)$. For $p\in C$, let $R_p=\{p, p_{\hat{(1)}}, p_{\hat{(2)}}, p_{\hat{(1)}\hat{(2)}}, p_{\hat{(2)}\hat{(1)}}, p_{\hat{(1)}\hat{(2)}\hat{(1)}}, p_{\hat{(2)}\hat{(1)}\hat{(2)}}, \ldots\}$.

THEOREM 7.2. — *Let u be as above. Let* $p\in C_1^-\cup C_2^-$. *Assume*

$$R_p\cap(C_1^-\cup C_2^-)\neq\{p\}.$$

Then we have:

(i) *If* $SS_{\Omega_k}(u)\cap C_k^-=\{p\}\cap C_k^-$ *for* $k=1, 2$, *then* $C_1^+\subset SS_{\Omega_2}(u)$ *and* $C_2^+\subset SS_{\Omega_2}(u)$. *In other words, we have:*

(ii) *If* $b^-(p)\subset SS(u|\Omega)$ *and if* $b^-(p')\not\subset SS(u|\Omega)$ $(\forall p'\in C_1^-\cup C_2^-$ *with* $p'\neq p)$, *then* $b^+(q)\subset SS(u|\Omega)$ *for all* $q\in C_1^+\cup C_2^+$.

Remark 1. — Let C be given by (7.5). Assume $R_p\cap(C_1^-\cup C_2^-)=\{p\}$. Then θ and p satisfy one of the following (cf. *Fig.* 7.3):

(*a*) $\theta=\pi/3$, $p=(\mathring{\eta}_1\pm a/\sqrt{3}, \mathring{\eta}_2\pm b/\sqrt{3})$.

(*b*) $\theta=\pi/2$, $p=(\mathring{\eta}_1\pm a, \mathring{\eta}_2)$, $(\mathring{\eta}_1, \mathring{\eta}_2\pm b)$.

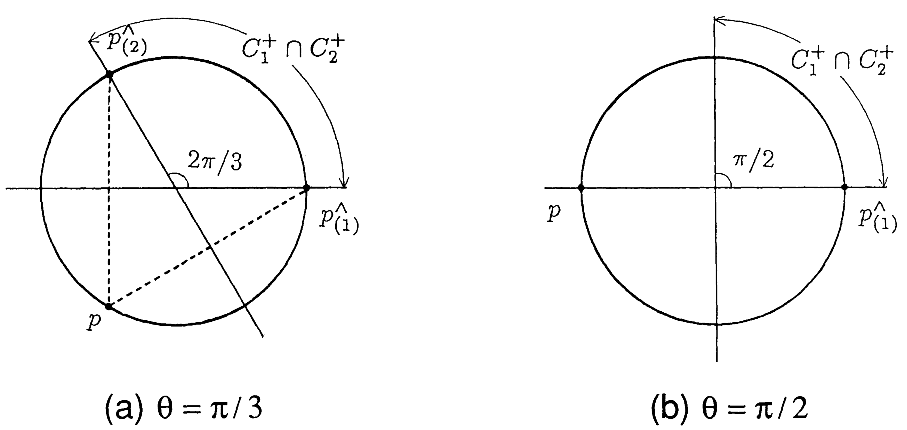

Fig. 7.3.

Remark 2. — The positive bicharacteristic curves $b^+(q)$ $(q\in C_1^+\cup C_2^+)$ are called the diffracted rays produced by a single incident ray $b^-(p)$, and they form the surface of a cone (cf. *Fig.* 7.4; *cf.* Keller [11], Fig. 5).

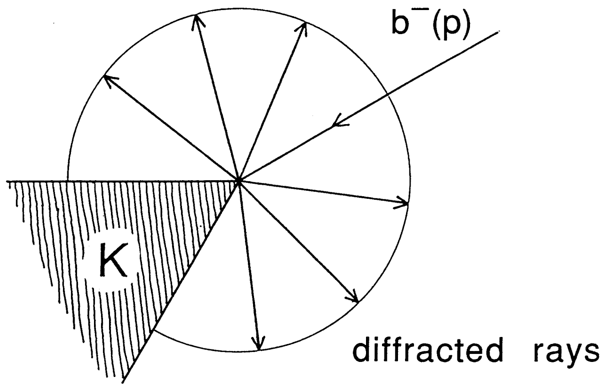

Fig. 7.4. – Diffraction of a single incident ray.

THEOREM 7.3. — *Let u be as above. Let* Z *be a finite subset of* $C_1^- \cup C_2^-$. *Assume that there exists* $p \in Z$ *such that* $R_p \cap (C_1^- \cup C_2^-) \not\subset Z$. *Then we have:*

(i) *If* $SS_{\Omega_k}(u) \cap C_k^- = Z \cap C_k^-$ *for* $k=1, 2$, *then* $C_1^+ \subset SS_{\Omega_1}(u)$ *and* $C_2^+ \subset SS_{\Omega_2}(u)$.

(ii) *If* $b^-(p) \subset SS(u \mid \Omega)\,(\forall p \in Z)$ *and if* $b^-(p') \not\subset SS(u \mid \Omega)$ $(\forall p' \in C_1^- \cup C_2^- \setminus Z)$, *then* $b^+(q) \subset SS(u \mid \Omega)$ *for all* $q \in C_1^+ \cup C_2^+$.

Remark. – Let C be given by (7.5) with $\theta \notin \mathbb{Q}\pi$. Then, for any nonempty finite subset Z of $C_1^- \cup C_2^-$, there exists $p \in Z$ such that $R_p \cap (C_1^- \cup C_2^-) \not\subset Z$.

7.3.3. *Proof of Theorems.* – To prove Theorems 7.2 and 7.3, we first prepare the following general lemma:

LEMMA 7.4. — *Let u be a section of* $H^0 \mathbf{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{D}_X/\mathcal{D}_X P, \mathcal{C}_{\Omega \mid X})$ *in a neighborhood of* $p \in C_1^+ \cap C_2^+$. *Then* $SS_{\Omega_1}(u) = SS_{\Omega_2}(u)$ *in a neighborhood of p.*

Proof. – Let $p' \in C_1^+ \cap C_2^+$. Since $b^+(p')$ is transversal to each of N_1 and N_2, and $\pi(b^+(p')) \subset \Omega_1 \cap \Omega_2$, it follows from the theorem of propagation of regularity up to smooth boundary (*cf.*, e.g., [7], [16], [17]) that, for $k=1, 2$, $p' \in SS_{\Omega_k}(u)$ if and only if $b^+(p') \subset SS(u \mid \Omega)$. This completes the proof.

Since Theorem 7.2 is a special case of Theorem 7.3, it is sufficient to prove Theorem 7.3. Assertions (i) and (ii) of Theorem 7.3 are equivalent by the theorem of propagation of microanalyticity up to smooth boundary (*cf.* e.g., [7], [16], [17]).

Proof of Theorem 7.3. – Let Z be a finite subset of $C_1^- \cup C_2^-$. Assume $C_k^- \cap SS_{\Omega_k}(u) = C_k^- \cap Z$ $(k=1, 2)$. Set $\tilde{Z} = Z \cup (C_1^+ \cap SS_{\Omega_1}(u)) \cup (C_2^+ \cap SS_{\Omega_2}(u))$.

Step 1. – Suppose

$$\text{(a-1)} \qquad \tilde{Z} \cap (C_1^- \cup C_2^-) = Z,$$

$$\text{(a-2)} \qquad \{p_{\hat{(1)}}, p_{\hat{(2)}}\} \subset \tilde{Z} \qquad (\forall p \in \tilde{Z}).$$

Then, for any $p \in Z$, $R_p \subset \tilde{Z}$ by (a-2). Hence $R_p \cap (C_1^- \cup C_2^-) \subset \tilde{Z} \cap (C_1^- \cup C_2^-) = Z$ by (a-1); this is a contradiction to the assumption of the theorem. Thus we may assume one of the following:

(b-1) $$\tilde{Z} \cap (C_1^- \cup C_2^-) \neq Z.$$

(b-2) there exists $p \in \tilde{Z}$ such that $\{p_{\hat{(1)}}, p_{\hat{(2)}}\} \not\subset \tilde{Z}$.

Step 2. — Assume (b-1). Let $p \in \tilde{Z} \cap C_1^- \setminus Z$; then $p \in (C_2^+ \cap SS_{\Omega_2}(u)) \setminus Z$. Since $p \in C_1^- \setminus Z$ [therefore $b^-(p) \cap SS(u|\Omega) = \varnothing$], it follows from Theorem 6.5 that a small neighborhood of p in C is contained in $SS_{\Omega_2}(u)$. Thus, applying Lemma 7.1 for $u|_{\Omega_2}$ on $U_2^+ = C_2^+ \setminus Z_{\hat{(2)}}$, we have $U_2^+ \subset SS_{\Omega_2}(u)$; therefore $C_2^+ (\subset \overline{U_2^+}) \subset SS_{\Omega_2}(u)$. In particular, we have $C_1^+ \cap C_2^+ \subset SS_{\Omega_2}(u)$; therefore $C_1^+ \cap C_2^+ \subset SS_{\Omega_1}(u)$ by Lemma 7.4. Applying again Lemma 7.1 for $u|_{\Omega_1}$ on $U_1^+ = C_1^+ \setminus Z_{\hat{(1)}}$, we have $C_1^+ \subset SS_{\Omega_1}(u)$. As a result, $C_1^+ \subset SS_{\Omega_1}(u)$ and $C_2^+ \subset SS_{\Omega_2}(u)$.

Step 3. — Assume (a-1). Then we may assume by (b-2) that there exists $p \in \tilde{Z}$ such that $p_{\hat{(1)}} \notin \tilde{Z}$. Suppose $p \in \tilde{Z} \cap C_1^-$. Then $p \in Z \cap C_1^-$ by (a-1). Since p is an isolated point of $Z \cap C_1^-$, we have $p_{\hat{(1)}} \in C_1^+ \cap SS_{\Omega_1}(u)$ by Lemma 7.1 (ii). Therefore $p_{\hat{(1)}} \in \tilde{Z}$, which is a contradiction. Suppose $p \in \tilde{Z} \cap D_1$. Then $p_{\hat{(1)}} (= p) \in \tilde{Z}$. Thus we have $p \in \tilde{Z} \cap C_1^+$.

(Case 1) Let $p \in C_1^+ \cap SS_{\Omega_1}(u)$ with $p_{\hat{(1)}} \notin \tilde{Z}$. Since $C_1^- \cap \tilde{Z} = C_1^- \cap Z$ by (a-1), $p_{\hat{(1)}} \notin Z$. Since $p \in C_1^+ \cap SS_{\Omega_1}(u)$, by applying Lemma 7.1 (i) for $u|_{\Omega_1}$ on $C_1^+ \setminus Z_{\hat{(1)}}$, we have $C_1^+ \subset SS_{\Omega_1}(u)$. Then, using the same argument as in Step 2 (with permutating Ω_1 and Ω_2), we have also $C_2^+ \subset SS_{\Omega_2}(u)$.

(*Case* 2) Let $p \in C_1^+ \cap C_2^+ \cap SS_{\Omega_2}(u)$ with $p_{\hat{(1)}} \notin \tilde{Z}$. By Lemma 7.4, we have: $p \in C_1^+ \cap C_2^+ \cap SS_{\Omega_1}(u)$. This is reduced to Case 1.

(*Case* 3) Let $p \in Z \cap C_1^+$ with $p_{\hat{(1)}} \notin \tilde{Z}$. Then $p \in Z \cap C_1^+ \cap C_2^+$; therefore $p \in SS_{\Omega_2}(u)$. Since p is an isolated point of $C_2^- \cap SS_{\Omega_2}(u)$, it follows from Corollary 6.3 that $p \in SS_{\Omega_1}(u)$. Thus this is reduced to Case 1.

This completes the proof.

Remark. — The argument used above is not restricted to the case where C is an ellipse nor to the case of second order differential equations. One can get an analogous result for higher order differential equations by imposing many enough boundary conditions.

7.3.4. *An example.* — Let P be the wave operator on a C^∞-Riemannian manifold (M', g) : $P = D_t^2 - \Delta_g$, where Δ_g is the Laplace operator on (M', g). Let $K' = \{\varphi_1 \leqq 0, \varphi_2 \leqq 0\}$, with φ_1, φ_2 being real-valued C^∞-functions on M' with $d\varphi_1 \wedge d\varphi_2 \neq 0$. Put $M = M' \times \mathbb{R}_t$, $K = K' \times \mathbb{R}_t$, $\Omega' = M' \setminus K'$. Syppose that u describes a simple processing curvilinear wave on Ω'; *i.e.*, u is the solution to the mixed problem

$$\begin{cases} Pu = 0 \quad \text{on } \Omega' \times \mathbb{R}_t, \\ u|_{\partial K} = 0, \\ u|_{t=0} = 0, \qquad D_t u|_{t=0} = \delta_{x_0}, \end{cases}$$

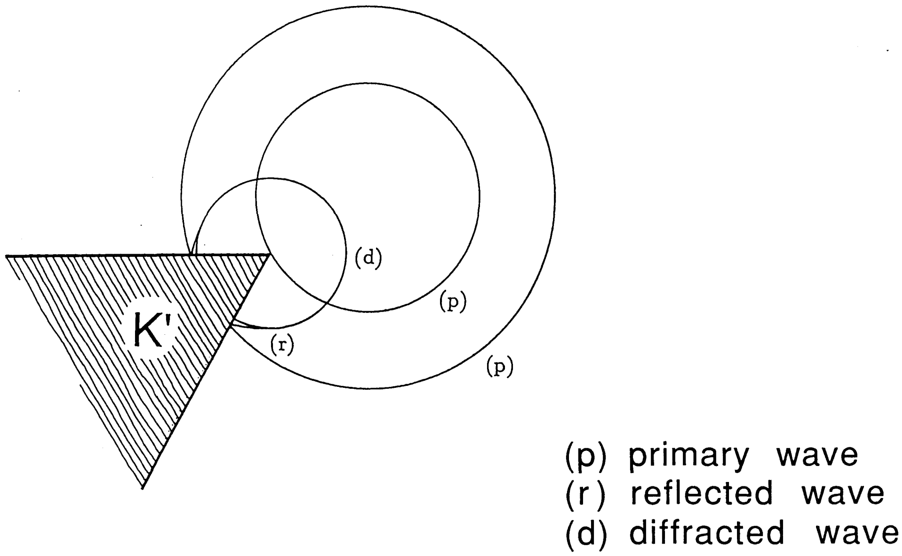

Fig. 7.5. — Diffraction of a simple progressing wave by a corner.

where δ_{x_0} is Dirac's δ-function on M′ supported at $x_0 \in \Omega'$. This is one of the typical cases to which the result of 7.3.2 is applicable (cf. *Fig.* 7.5 for diffraction by a corner of a simple progressing curvilinear wave). *Cf.* Cheeger-Taylor [1], Rouleux [14], Varrenne [27]; their results (determination of the locus of singularities of the fundamental solution to the considered mixed problem) give an estimate from above to the singularities of a general solution for any initial data, but they do not seem to treat the diffraction of a single incident ray by a corner.

Remark. — It is not proved in our situation that there are no other singularities than the incident rays and the diffracted rays (*cf.* the results of [1], [14], [27], etc.); it is in fact possible in a certain case to construct a solution having singularities on a "diffracted cone" with no incident rays. (The author is grateful to G. Lebeau for kindly suggesting this construction.)

REFERENCES

[1] J. CHEEGER and M. TAYLOR, *On the Diffraction of the Waves by Conical Singularities I-II* (*Comm. P.A.M.*, Vol. 35, 1982, pp. 275-331 et 487-529).
[2] R. HARTSHORNE, *Residues and Duality* (*Lect. Notes Math.*, No. 20, Springer-Verlag, 1966).
[3] M. KASHIWARA and T. KAWAI, *On the Boundary Value Problem for Elliptic System of Linear Partial Differential Equations I-II* (*Proc. Jpn Acad.*, Vol. 48, 1972, pp. 712-715; *ibid.*, Vol. 49, 1973, pp. 164-168).
[4] M. KASHIWARA, T. KAWAI and T. KIMURA, *Foundations of Algebraic Analysis*, Princeton University Press, 1986.
[5] M. KASHIWARA and P. SCHAPIRA, *Micro-Hyperbolic Systems* (*Acta Math.*, Vol. 142, 1979, pp. 1-55).
[6] M. KASHIWARA and P. SCHAPIRA, *Microlocal Study of Sheaves* (*Astérisque*, Vol. 128, Soc. Math. de France, 1985).

[7] K. KATAOKA, *A Microlocal Approach to General Boundary Value Problems* (*Publ. R.I.M.S., Kyoto Univ.*, Vol. 12, Suppl., 1977, pp. 147-153).
[8] K. KATAOKA, *Microlocal Theory of Boundary Value Problems I-II* (*J. Fac. Sci. Univ. Tokyo*, Vol. 27, 1980, pp. 355-399; *ibid.*, Vol. 28, 1981, pp. 31-56).
[9] K. KATAOKA, *On the Theory of Radon Transformations of Hyperfunctions* (*J. Fac. Sci. Univ. Tokyo*, Vol. 28, 1981, pp. 331-443).
[10] K. KATAOKA, *Some Properties of Mild Hyperfunctions and an Application to Propagation of Micro-Analyticity in Boundary Value Problems* (*J. Fac. Sci. Univ. Tokyo*, Vol. 30, 1983, pp. 279-296).
[11] J. B. KELLER, *A Geometrical Theory of Diffraction* (*J. Opt. Soc. Am.*, Vol. 52, 1962, pp. 116-130).
[12] H. KOMATSU and T. KAWAI, *Boundary Values of Hyperfunction Solution of Linear Partial Differential Equations* (*Publ. R.I.M.S., Kyoto Univ.*, Vol. 7, 1971, pp. 95-104).
[13] G. LEBEAU, *Une propriété d'invariance pour le spectre des traces* (*C.R. Acad. Sci. Paris*, T. 294, Série I, 1982, pp. 723-725).
[14] M. ROULEUX, *Diffraction analytique sur une variété à singularité conique* (*Comm. P.D.E.*, Vol. 11 (9), 1986, pp. 947-988).
[15] M. SATO, T. KAWAI and M. KASHIWARA, *Microfunctions and Pseudo-Differential Equations* (*Lect. Notes Math.*, No. 287, Springer-Verlag, 1973, pp. 265-529).
[16] P. SCHAPIRA, *Hyperfonctions et problème aux limites elliptiques* (*Bull. Soc. Math. Fr.*, T. 99, 1971, pp. 113-141).
[17] P. SCHAPIRA, *Propagation at the Boundary and Reflection of Analytic Singularities of Solutions of Linear Partial Differential Equations I-II* (*Publ. R.I.M.S., Kyoto Univ.*, Vol. 12, Suppl., 1977, pp. 441-453; *Séminaire Goulaouic-Schwartz*, 1976-1977, Exp. 9).
[18] P. SCHAPIRA, *Propagation at the Boundary of Analytic Singularities*. In *Singularities of Boundary Value Problems*, Reidel Publ. Co., 1981, pp. 185-212.
[19] P. SCHAPIRA, *Front d'onde analytique au bord II* (*Séminaire E.D.P.*, Ecole Polyt., 1985-1986, Exp. 13).
[20] P. SCHAPIRA, *Microfunctions for Boundary Value Problems*. In *Prospect in Algebraic Analysis*, Papers dedicated to M. Sato, Academic Press, 1989, pp. 809-819.
[21] P. SCHAPIRA, *Propagation of Regularity Up to Non Smooth Boundary* (*Journée E.D.P.*, Saint-Jean-de-Monts, Publ. École Polyt., 1987).
[22] P. SCHAPIRA, *Microdifferential Systems in the Complex Domain* (*Grundlehren der math. Wissenschaften*, Vol. 269, Springer-Verlag, 1985).
[23] J. SJÖSTRAND, *Propagation of Analytic Singularities for Second Order Dirichlet Problems I-II-III* (*Comm. P.D.E.*, Vol. 5, 1980, pp. 41-94; *ibid.*, pp. 187-207; *ibid.*, Vol. 6, 1981, pp.499-567).
[24] M. UCHIDA, *On the Complex* $\mathscr{C}_\Lambda$ *Attached to a Certain Class of Lagrangian Set* (*Proc. Jpn Acad.*, Vol. 64, 1988, pp. 126-129).
[25] M. UCHIDA, *Un théorème d'injectivité du morphisme de restriction pour les micro-fonctions* (*C.R. Acad. Sci. Paris.*, T. 308, Série I, 1989, pp. 83-85).
[26] M. UCHIDA, *Étude microlocale de la diffraction par un coin* (*Séminaire E.D.P.*, École Polyt. 1989-1990, Exp. 3).
[27] J.-P. VARRENNE, *Diffraction par un angle ou un dièdre* (*C.R. Acad. Sci. Paris*, T. 290, Série A, 1980, pp. 175-178).

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