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A GREEN SUPPLY CHAIN MODEL OF VENDOR AND BUYER FOR REMANUFACTURING

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Abstract. Due to environmental protection and limited resource utilization at optimum level, remanufacturing of defective products or reused products is an essential strategy in green supply chain inventory control system. This paper presents a green supply chain inventory model for integrated production of new items and remanufacturing of redeemable returned items under a situation in which the vendor provides the buyer with a permissible delay of payments and supplies the serviceable items to the buyer on lot-for-lot basis. It is assumed that every member of the chain is ready to collaborate for the maximum benefit of the supply chain. A mathematical formulation is developed to derive the optimal number of deliveries and the replenishment cycle time for the integrated vendor-buyer inventory model. Theoretical results show the global optimality of the solution and it is concluded that the collaboration is profitable for the chain and participating members reach the optimal level under co-operative decision making. Numerical analysis is given to illustrate the theoretical results. Finally, a sensitivity analysis is reported to study the effect of various parameters.

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1. Introduction

Due to consumers' concern and government regulations towards environmental issues, firms are enforced to take green initiatives. Hence, the green issues in supply chain inventory management have attracted a great interest from practitioners and supply chain managers. Green supply chain is an addition of green issues into supply chain management that involves from suppliers to manufacturers, customers and reverse logistics throughout the product life cycle. Srivastava [32] showed that the green supply chain not only reduces environmental burden but also brings economic benefit to the manufacturers. Reverse logistics (RL) plays an important role in greening process. It provides a platform to the customers to return the warranty and defective products or to resell them for the purpose of remanufacturing. According to Dekker *et al.* [5], the uncertainties of the returned amount and quality complicates the process of reverse logistics and greatly affects the collection and inventory decisions in an attempt to achieve greater economic benefits. Consequently, it becomes a challenge of the decision maker to incorporate the reverse channel into their forward logistics activities. However, in the last decades, a lot of work has been done in the field of reverse logistics. Schrady [27] was the first researcher who considered the

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joint determination of production and recovery. He analysed the problem within the traditional EOQ (economic order quantity) framework with instantaneous production and recovery rates. Nahmias and Rivera [13] generalized the Schrady's [27] model for a finite recovery rate. In the same line of research, Richter [18] and Richter and Dobos [19] investigated the reverse logistics model. In these problems, the return rate had been taken as a decision variable. They examined a problem with pure or bang-bang policy (total repair or total wastage disposal). Richter [18] concluded that the bang-bang policy was optimal on the case of mixture of production and remanufacturing policy. Other achievements in the area of coordinated production and remanufacturing process, the works done by Chung and Wee [3], Singh and Saxena [29], Singh *et al.* [28], Yang *et al.* [33], Singh and Saxena [30], Kim *et al.* [11] and Singh and Saxena [31] are worth mentioning with different assumptions.

There is a myriad of literature that studied the effect of vendor buyer integration on inventory control system. However, the joint optimization for supplier and buyer was introduced by Goyal [6]. He suggested a joint economic lot size model where the objective was to minimize the total relevant costs for both the vendor and the buyer who shared the necessary information dealt with the uncertainty of demand and supply. They collaborated together in order to maximize their profit. Banerjee [1] considered a joint economic lot size model for a single vendor, single buyer system with the finite vendor's production rate considering lot-splitting. Recently, Ouyang *et al.* [14], Huang [8], Panda [16, 17] and Lou and Wang [12] developed the models with vendor-buyer integration. Sarkar [24] suggested the optimal production and inventory quantities of probabilistic deteriorating items in two-echelon supply chain system. Sarkar [21] studied a supply chain coordination mechanism considering variable backorder, inspections, and discount policy for fixed lifetime products.

The traditional EOQ model assumes that the retailer must be paid for the items as soon as the items are received. In practice, supplier may offer a credit period to the retailer to settle his account within a certain period to promote his commodities and to attract specific group of customers. Before the end of trade credit period, the retailer accumulates revenue from his sold products and earns interest. If the payment is not settled by the end of trade credit period, a higher interest is charged by the supplier. On the other hand supplier lost some opportunity cost during the period between product shipped and paid for.

There are numerous researches on trade credit policy. The earliest approach in the field of trade credit was made by Goyal [7]. He developed an EOQ model with permissible delay in payments. Ouyang *et al.* [14] presented a production model under trade credit with the condition that trade credit and freight rate were simultaneously linked to the ordering quantity. Huang [8] investigated an integrated inventory model with ordering cost reduction and permissible delay in payment. Lou and Wang [12] extended Huang's [8] model for the condition where the buyer deposited sales revenue in an interest baring account. In this article, they determined the optimum solution treating the total number of shipments as a decision variable. Sarkar [25, 26] developed an EOQ model with finite replenishment rate of perishable and imperfect items allowing permissible delay time for payment of purchased items where demand rates were time varying and stock-dependent respectively. Chung and Cárdenas−Barrón [4] provided a simplified solution procedure for perishable products under stock-dependent demand and two-level trade credit in the supply chain management. Chen *et al.* [2] revisited an EOQ model under conditionally permissible delay in payments, in which the supplier offers the retailer a fully permissible delay periods. The noteable works of Lashgari *et al.*[9], Khanra *et al.* [10], Pal *et al.* [15], Sana and Chaudhuri [20] and Sarkar *et al.* [22, 23] should be mentioned in this line of works, among others.

A long period EOQ model is widely used by investigators as a decision-making tool for the trade credit. Lots of literature is also available on production quantity but there are a few research in the area of green supply chain inventory model with trade credit. In the recent age, concerns of environment are becoming critical issues. Especially, companies are enforced to implement environmental practices to enhance green image with increasing awareness of environment protection. In this proposed article, we establish an integrated model with the green components under the permissible delay in payment. This model determines both the forward and reverse channel in a supply chain where vendor and buyer are two members of the chain. Both members determine the delivery quantities during the contract agreement period. Here, the vendor fulfils the buyer's demand in small shipment of frequent deliveries with the produced/remanufactured items. Besides a production and a remanufacturing facility, the system consists of one inventory for collected returned items and one for

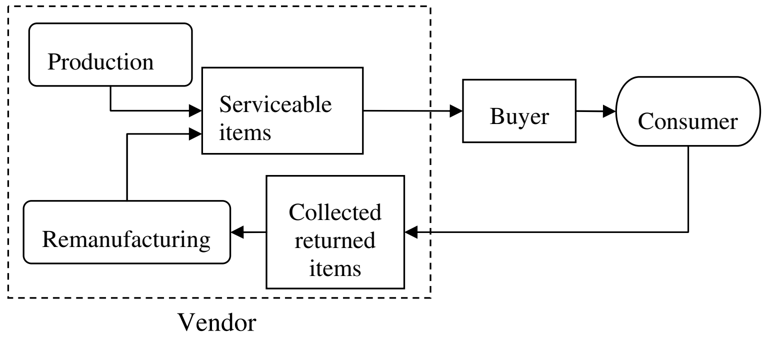

FIGURE 1. Flow of inventory in the green supply chain.

serviceable items. Demands are also satisfied from serviceable inventory, which can be replenished by newly produced or remanufactured of the returned items. The quality standard of the remanufactured ones is assumed to be "as good as those of new items". The transportation cost is also considered and it depends on the lot-size. A general framework of such a system is depicted in Figure 1.

The paper is organized as follows: In the next section, we describe the assumptions and notation. Then, we develop a mathematical model for the problem in Section 3. In Section 4, we derive theoretical results and obtain a solution procedure to obtain an optimal solution and a numerical example is given to illustrate the theoretical results in Section 5. In Section 6, the sensitivity analysis is provided to study the effect of the parameters. Section 7 concludes the proposed work.

2. Assumptions and Notations

The following notations and assumptions are used to develop the model.

2.1. Notation

P_r	Remanufacturing rate
P_m	Production rate
D	Demand rate
R	Returned rate
u_m	Procurement cost for the vendor
u_R	Accusation cost (item cost of returned products)
v	Procurement cost for the buyer (selling price for the vendor)
M	Trade credit period
p	Selling price for the buyer
h_b	Holding cost per unit item per unit time for the buyer
h_v	Holding cost per unit item per unit time for the vendor
h_R	Holding cost per unit item per unit time for the returned items
c_r	Remanufacturing cost
c_m	Production cost
S	Set up cost for the vendor
A	Ordering cost for the buyer
F	Transportation cost per unit
I_{be}	Rate of interest earned by the buyer
I_{bp}	Rate of interest paid by the buyer
I_{vp}	Rate of interest paid by the vendor

$N = m + n$	Total number of deliveries per cycle time
m	Fraction of total number of deliveries of the re-manufacturing items.
n	Fraction of total number of deliveries delivered of the newly produced items.
T	Time interval between successive deliveries for the buyer
$Q = DT$	Delivery size per delivery
$TP_b = \begin{cases} TP_{b1} \ \ when\ T \leqslant M \\ TP_{b2}\ when\ T > M \end{cases}$	The total profit function of the buyer
TP_S	The total profit function of the vendor
$TP = \begin{cases} TP_1 \ \ when\ T \leqslant M \\ TP_2\ when\ T > M \end{cases}$	The total profit function for the whole system.

2.2. Assumptions

(1) The supply chain consists of a vendor and a buyer for a single product where vendor fulfils the buyer's demand with the newly produced and remanufactured items.
(2) The model is developed with single setup and multiple deliveries.
(3) The remanufactured products are considered as good as those of new products.
(4) The transportation cost is also considered and it depends on the lot-size.
(5) At each setup, the vendor supplies NQ units in N (a positive integer) small shipment to buyer where the size of each shipment to the vendor is Q. For each cycle, m number of shipments from the total number deliveries is delivered by remanufactured material while n shipments are delivered by newly produced items.

3. Model formulation

The items are ordered by buyer in N equal lots of size Q, until the inventory level at vendor falls to zero and a new lot of size Q is delivered to the vendor. The model assumes that the successive delivering batch arrives at the store as soon as the previous batch is depleted. The changes in the inventory levels of buyer is shown in Figures 2 and 4. On the other hand, the remanufacturing process starts at the beginning of the production system. Vendor's remanufacturing proceeds the first batch of the buyer's need and is continuous until the remanufacturing lot satisfies the remanufacturing-cycle-time demand in m small shipments. Similarly, for each production cycle, manufacturer produced nQ units and fulfils the buyer's need. Figure 2 depicts the behaviour of the vendor's inventory level. In the corresponding returned cycle, as the remanufacturing system starts operating, the inventory level (collected returns) is depleted until the time $\frac{mDT}{P_r}$ by which the remanufacturing process ceased and the inventory level becomes zero. Then the inventory level starts to go up with a constant returned rate (Fig. 3).

We derive the total profit of an integrated inventory model considering the relevant costs with trade credit. Some of these costs are developed in a manner similar to the work of Ouyang *et al.* [14] and Lou and Wang [12].The annual integrated total profit consists of the vendor's annual total profit and the buyer's total profit. We discuss them separately as follows:

3.1. Net profit of Buyer's per unit time

Since replenishment in each cycle is done at the starting of each delivery, the ordering cost per unit time during one cycle is $\frac{A}{T}$

For each ordering quantity Q, the buyer pays vQ to the vendor and gets pQ from the customer. Thus the sales revenue per unit time is $\frac{(p-v)Q}{T} = (p - v)D$.

Transportation cost is $\frac{FQ}{T} = FD$.

As depicted in Figure 4, the inventory holding cost per unit time is $v\frac{h_b}{T}\left(\frac{QT}{2}\right) = h_b v \left(\frac{DT}{2}\right)$.

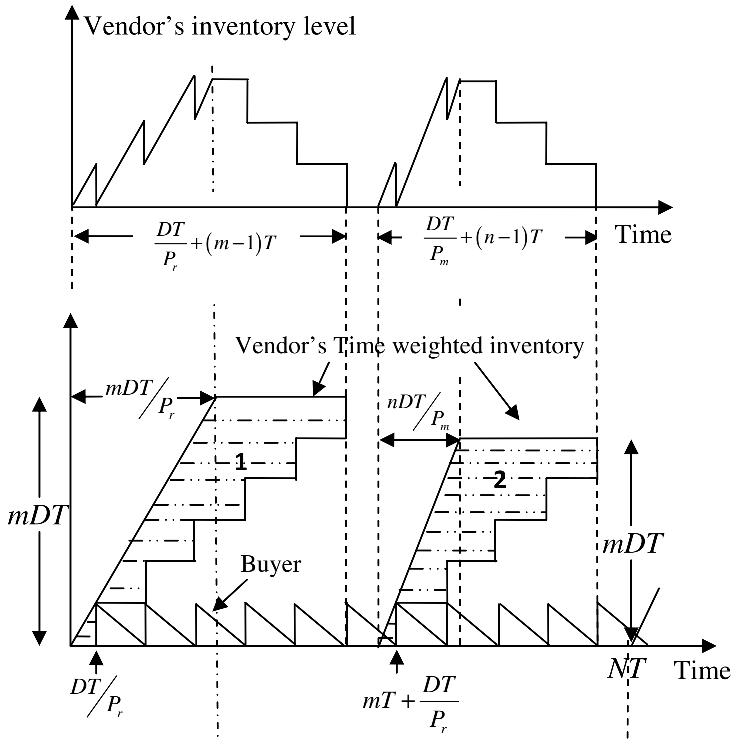

FIGURE 2. The time weighted inventory (TWI) level of the vendor.

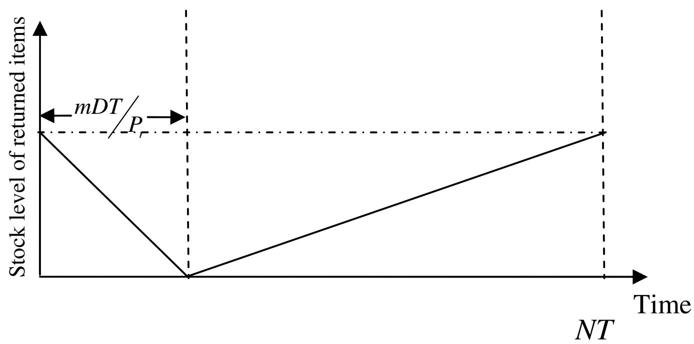

FIGURE 3. Behaviour of the collected returns.

For the possible values of M and T, the buyer faces the following possible cases:

Case 1. $T \leqslant M$

When the cycle time T is less than or equal to the buyer's credit period M, the buyer settles the account at M. Therefore, the retailer does not have to pay any interest charges. At the same time before settling the account, the buyer takes the benefit of trade credit and sells the product and earns the interest by putting generated revenue in an interest bearing account at the rate of I_{be} per unit per

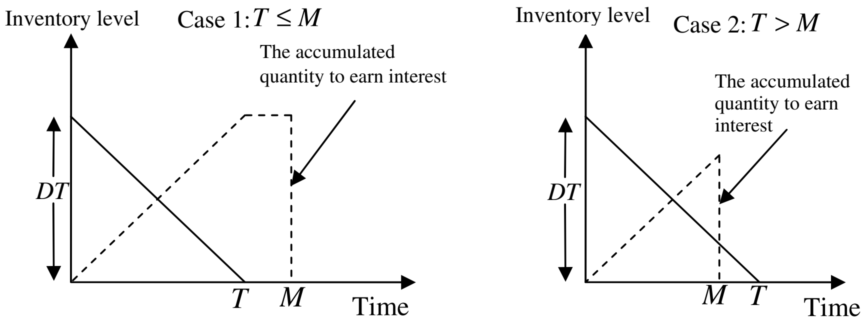

FIGURE 4. The inventory level and interest earned for the buyer.

unit time during the period [0, M], which is

$$p\frac{I_{be}}{T}\left\{\frac{QT}{2}+Q\left(M-T\right)\right\}=pDI_{be}\left(M-\frac{T}{2}\right)$$

Case 2. $T > M$

When the cycle time T is greater than buyer's credit period M, offered by the vendor, the buyer settles the account at$T = M$. At the end of the credit period, the buyer pays for the product sold and keeps his profit and starts paying interest on unsold items at the rate I_{bp}. Therefore, the interest paid by the buyer per unit time is

$$vI_{bp}\int_{M}^{T}D\left(T-t\right)\mathrm{d}t=\frac{vI_{bp}}{2T}D\left(T-M\right)^2$$

The retailer earns the interest by putting generated revenue in an interest bearing account at the rate of I_{be} per unit time during the period [0, M] is $pI_{be}\frac{DM^2}{2T}$.

The buyer's total profit per unit time consists of ordering cost, transportation cost, holding cost, interest earned, interest paid and sales revenue. Therefore, the buyer's total profit per unit time can be expressed as

$$TP_b=\begin{cases}TP_{b1}, & when \quad T\leqslant M\\ TP_{b2}, & when \quad T>M\end{cases}$$

where

$$TP_{b1}=(p-v)D-\frac{A}{T}-FD-h_bv\left(\frac{DT}{2}\right)+pDI_{be}\left(M-\frac{T}{2}\right) \tag{3.1}$$

and

$$TP_{b2}=(p-v)D-\frac{A}{T}-FD-h_bv\left(\frac{DT}{2}\right)-\frac{vI_{bp}}{2T}D\left(T-M\right)^2+pI_{be}\frac{DM^2}{2T} \tag{3.2}$$

3.2. Vendor's profit per unit time

Set up cost per unit time is $\frac{S}{NT}$.

For the remanufacturing period, vendor's TWI (time weighted inventory: area of the shaded region 1, Fig. 2) is

$$\left[\left\{\frac{DT}{P_r}+(m-1)\,T\right\}mDT-\frac{mDT}{P_r}\left(\frac{mDT}{2}\right)-\{T+2T+\ldots(m-1)\,T\}\,DT\right]$$
$$=\frac{mDT^2}{2}\left[(m-2)\left(1-\frac{D}{P_r}\right)+1\right]$$

And, for the production period, vendor's TWI (area of the shaded region 2, Fig. 2) is

$$\left[\left\{\frac{DT}{P_m}+(n-1)\,T\right\}nDT-\frac{nDT}{P_m}\left(\frac{nDT}{2}\right)-\{T+2T+\ldots(n-1)\,T\}\,DT\right]$$
$$=\frac{nDT^2}{2}\left[(n-2)\left(1-\frac{D}{P_m}\right)+1\right]$$

Hence, vendor's holding cost per unit time is

$$\frac{(h_v+I_{vp})}{NT}\left[\frac{c_r mDT^2}{2}\left\{(m-2)\left(1-\frac{D}{P_r}\right)+1\right\}+\frac{c_m nDT^2}{2}\left\{(n-2)\left(1-\frac{D}{P_m}\right)+1\right\}\right]$$
$$=(h_v+I_{vp})\,\frac{DT}{2N}\left[c_r m\left\{(m-2)\left(1-\frac{D}{P_r}\right)+1\right\}+c_m n\left\{(n-2)\left(1-\frac{D}{P_m}\right)+1\right\}\right]$$

In offering trade credit to the buyer, the vendor endures a capital opportunity cost at rate I_{vp} during the period between product shipped and paid for. The lost opportunity cost per unit time is

$$\frac{1}{NT}\,(vI_{vp}NDTM)=vI_{vp}DM$$

In each ordering quantity NQ, the vendor spends $c_r mQ$ for remanufacturing and $c_m nQ$ for production and receives $vNQ=v\,(m+n)\,Q$ from the buyer. Thus the sales revenue per unit time is

$$\frac{1}{NT}\left\{(v-c_r)\,mDT+(v-c_m)\,nDT\right\}=\frac{D}{N}\left\{(v-c_r)\,m+(v-c_m)\,n\right\}.$$

As the stock level of returned items which is depicted in Figure 3, the holding cost of the collected returned items is

$$\frac{u_R h_R}{NT}\left[\left(\frac{nDT}{P_r}\right)^2\frac{(P_r-R)}{2}+\left\{NT-\frac{mDT}{P_r}\right\}^2\frac{R}{2}\right]$$

Here, the vendor spends u_RQ on the procurement of the raw material in order to produce the new items and spends u_mQ on the accusation of the buyback products collected from the consumers. Hence, the procurement and accusation cost is

$$\frac{(u_R mDT+u_m nDT)}{NT}$$

Therefore, the total profit per unit time of the vendor comprising of sales revenue, set up cost, procurement and accusation cost, holding cost and opportunity cost is

$$\begin{aligned} TP_v =& \frac{D}{N}\left\{\left(v-c_r\right)m+\left(v-c_m\right)n\right\}-\left(h_v+I_{vp}\right)\frac{DT}{2N} \\ &\times\left[c_r m\left\{(m-2)\left(1-\frac{D}{P_r}\right)+1\right\}+c_m n\left\{(n-2)\left(1-\frac{D}{P_m}\right)+1\right\}\right] \\ &-\frac{S}{NT}-vI_{vp}DM-\frac{u_R h_R}{NT}\left[\left(\frac{nDT}{P_r}\right)^2\frac{\left(P_r-R\right)}{2}+\left\{NT-\frac{mDT}{P_r}\right\}^2\frac{R}{2}\right]-\frac{\left(u_R mDT+u_m nDT\right)}{NT} \end{aligned} \tag{3.3}$$

In our model, we consider that total collected returned items are remanufactured. Therefore, the total returns in the cycle time T is equal to the total remanufacturing in the cycle time $\frac{mDT}{P_r}$ that results in

$$RNT = P_r\frac{mDT}{P_r}$$

Hence, $m=\frac{RN}{D}$ and $n=N-m=N\left(1-\frac{R}{D}\right)$.

Putting the value of m and n in the equation (3.2) and (3.3) and adding them, we have the total profit of the system. Therefore, the total profit per unit time of the whole system can be expressed as

$$TP=\begin{cases} TP_1, & \text{when} \quad T\leqslant M \\ TP_2, & \text{when} \quad T>M \end{cases}$$

where

$$\begin{aligned} TP_1 =& (p-v)D-\frac{A}{T}-FD-h_b v\left(\frac{DT}{2}\right)+pDI_{be}\left(M-\frac{T}{2}\right)+D\left\{\left(v-c_r\right)\frac{R}{D}+\left(v-c_m\right)\left(1-\frac{R}{D}\right)\right\} \\ &-D\left\{u_R\frac{R}{D}+u_m\left(1-\frac{R}{D}\right)\right\}-\frac{S}{NT}-vI_{vp}DM-\frac{u_R h_R}{2}NT\left[\left(\frac{R}{P_r}\right)^2\left(P_r-R\right)+\left(1-\frac{R}{P_r}\right)^2R\right] \\ &-\left(h_v+I_{vp}\right)\frac{DT}{2}\left[c_r\frac{R}{D}\left\{\left(\frac{NR}{D}-2\right)\left(1-\frac{D}{P_r}\right)+1\right\}\right. \\ &\left.+c_m\left(1-\frac{R}{D}\right)\left\{\left(N\left(1-\frac{R}{D}\right)-2\right)\left(1-\frac{D}{P_m}\right)+1\right\}\right] \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} TP_2 =& (p-v)D-\frac{A}{T}-FD-h_b v\left(\frac{DT}{2}\right)-\frac{vI_{bp}}{2T}D\left(T-M\right)^2 \\ &+pI_{be}\frac{DM^2}{2T}+D\left\{\left(v-c_r\right)\frac{R}{D}+\left(v-c_m\right)\left(1-\frac{R}{D}\right)\right\}-D\left\{u_R\frac{R}{D}+u_m\left(1-\frac{R}{D}\right)\right\} \\ &-\frac{S}{NT}-vI_{vp}DM-\frac{u_R h_R}{2}NT\left[\left(\frac{R}{P_r}\right)^2\left(P_r-R\right)+\left(1-\frac{R}{P_r}\right)^2R\right] \\ &-\left(h_v+I_{vp}\right)\frac{DT}{2}\left[c_r\frac{R}{D}\left\{\left(\frac{NR}{D}-2\right)\left(1-\frac{D}{P_r}\right)+1\right\}\right. \\ &\left.+c_m\left(1-\frac{R}{D}\right)\left\{\left(N\left(1-\frac{R}{D}\right)-2\right)\left(1-\frac{D}{P_m}\right)+1\right\}\right] \end{aligned} \tag{3.5}$$

4. Theoretical results and Solution procedure

The purpose of this study is to derive the optimal number of deliveries and the replenishment cycle time by determining the optimal values of N and T that maximize the total profit. In order to find the optimal solution, we have to differentiate the profit function with respect to the decision variables T and N as follows:

Case 1. $T \leqslant M$

Taking the first order partial derivative of the profit function TP_1, with respect to T and N, we have

$$\begin{aligned}\frac{\partial TP_1}{\partial T} =& \frac{A}{T^2} + \frac{S}{NT^2} - \frac{Dh_b v}{2} - \frac{pDI_{be}}{2} - \frac{u_R h_R}{2} N\left[\left(\frac{R}{P_r}\right)^2 (P_r - R) + \left(1 - \frac{R}{P_r}\right)^2 R\right] \\ &- (h_v + I_{vp})\frac{D}{2}\left[c_r \frac{R}{D}\left\{\left(\frac{NR}{D} - 2\right)\left(1 - \frac{D}{P_r}\right) + 1\right\}\right. \\ &\left. + c_m\left(1 - \frac{R}{D}\right)\left\{\left(N\left(1 - \frac{R}{D}\right) - 2\right)\left(1 - \frac{D}{P_m}\right) + 1\right\}\right]\end{aligned} \tag{4.1}$$

$$\begin{aligned}\frac{\partial TP_1}{\partial N} =& \frac{S}{N^2 T} - \frac{u_R h_R}{2} T\left[\left(\frac{R}{P_r}\right)^2 (P_r - R) + \left(1 - \frac{R}{P_r}\right)^2 R\right] \\ &- (h_v + I_{vp})\frac{DT}{2}\left[c_r\left(\frac{R}{D}\right)^2\left(1 - \frac{D}{P_r}\right) + c_m\left(1 - \frac{R}{D}\right)^2\left(1 - \frac{D}{P_m}\right)\right]\end{aligned} \tag{4.2}$$

Equating the above equations to zero and solving them simultaneously, we obtain the optimal solution at (T_1^*, N_1^*),

$$T_1^* = \sqrt{\frac{N_1^* A + S}{N_1^*\left\{\varphi + \frac{D}{2}(h_b v + pI_{be}) + \frac{N_1^*}{2}\gamma\right\}}} \tag{4.3}$$

$$N_1^* = \sqrt{\frac{2SX_2 + DS(h_b v + pI_{be})}{X_1 A}} \tag{4.4}$$

where

$$\varphi = (h_v + I_{vp})\frac{D}{2}\left[c_r \frac{R}{D}\left\{\left(\frac{N_1^* R}{D} - 2\right)\left(1 - \frac{D}{P_r}\right) + 1\right\} + c_m\left(1 - \frac{R}{D}\right)\left\{(N_1^*\left(1 - \frac{R}{D}\right) - 2)\left(1 - \frac{D}{P_m}\right) + 1\right\}\right],$$

$$\gamma = u_R h_R\left[\left(\frac{R}{P_r}\right)^2 (P_r - R) + \left(1 - \frac{R}{P_r}\right)^2 R\right],$$

$$X_1 = \gamma + (h_v + I_{vp})D\left\{C_r\left(\frac{R}{D}\right)^2\left(1 - \frac{D}{P_r}\right) + C_m\left(1 - \frac{R}{D}\right)^2\left(1 - \frac{D}{P_m}\right)\right\}$$

and

$$X_2 = (h_v + I_{vp})\frac{D}{2}\left\{-C_r\left(\frac{R}{D}\right)\left(1 - \frac{2D}{P_r}\right) + C_m\left(1 - \frac{R}{D}\right)\left(-1 + \frac{2D}{P_m}\right)\right\}$$

For feasibility of N_1^* , $-C_r\left(\frac{R}{D}\right)\left(1 - \frac{2D}{P_r}\right) + C_m\left(1 - \frac{R}{D}\right)\left(-1 + \frac{2D}{P_m}\right) + \left(\frac{h_b v + pI_{be}}{h_v + I_{vp}}\right) > 0$ must be satisfied.

From equation (4.3) and (4.4), we have to ensure that $T \leqslant M$, we substitute (4.3) into inequality $T \leqslant M$ and it holds if and only if

$$NA + S \leqslant M^2 N\left\{\varphi + \frac{D}{2}(h_b v + pI_{be}) + \frac{N}{2}\gamma\right\} \tag{4.5}$$

is satisfied.

Case 2. $T > M$

Taking the first order partial derivative of the profit function TP_2 with respect to T and N, we get

$$\begin{aligned}\frac{\partial TP_2}{\partial T} = & \frac{A}{T^2} + \frac{S}{NT^2} - \frac{Dh_b v}{2} - \frac{vI_{bp}D}{2} + (vI_{bp} - pI_{be})\frac{DM^2}{2T^2} \\ & - \frac{u_R h_R}{2} N \left[\left(\frac{R}{P_r}\right)^2 (P_r - R) + \left(1 - \frac{R}{P_r}\right)^2 R \right] \\ & - (h_v + I_{vp}) \frac{D}{2} \left[c_r \frac{R}{D} \left\{ \left(\frac{NR}{D} - 2\right)\left(1 - \frac{D}{P_r}\right) + 1 \right\} \right. \\ & \left. + c_m \left(1 - \frac{R}{D}\right) \left\{ \left(N \left(1 - \frac{R}{D}\right) - 2 \right) \left(1 - \frac{D}{P_m}\right) + 1 \right\} \right] \end{aligned} \tag{4.6}$$

and

$$\begin{aligned}\frac{\partial TP_2}{\partial N} = & \frac{S}{N^2 T} - \frac{u_R h_R}{2} T \left[\left(\frac{R}{P_r}\right)^2 (P_r - R) + \left(1 - \frac{R}{P_r}\right)^2 R \right] \\ & - (h_v + I_{vp}) \frac{DT}{2} \left[c_r \left(\frac{R}{D}\right)^2 \left(1 - \frac{D}{P_r}\right) + c_m \left(1 - \frac{R}{D}\right)^2 \left(1 - \frac{D}{P_m}\right) \right] \end{aligned} \tag{4.7}$$

To find the optimum solution, setting $\frac{\partial TP_2}{\partial T} = 0$ and $\frac{\partial TP_2}{\partial N} = 0$ and solving the equations, we have the optimal solution at (T_2^*, N_2^*),

$$T_2^* = \sqrt{\frac{N_2^* A + S + M^2 D \frac{N_2^*}{2} (vI_{bp} - pI_{be})}{N_2^* \left\{ \varphi + \frac{D}{2} (h_b v + vI_{bp}) + \frac{N_2^*}{2} \gamma \right\}}} \tag{4.8}$$

$$N_2^* = \sqrt{\frac{X_3}{X_4}} \tag{4.9}$$

where

$$\varphi = (h_v + I_{vp}) \frac{D}{2} \left[c_r \frac{R}{D} \left\{ \left(\frac{N_2^* R}{D} - 2 \right) \left(1 - \frac{D}{P_r}\right) + 1 \right\} + c_m \left(1 - \frac{R}{D}\right) \left\{ \left(N_2^* \left(1 - \frac{R}{D}\right) - 2 \right) \left(1 - \frac{D}{P_m}\right) + 1 \right\} \right],$$

$$\gamma = u_R h_R \left[\left(\frac{R}{P_r}\right)^2 (P_r - R) + \left(1 - \frac{R}{P_r}\right)^2 R \right],$$

$$X_3 = SD(h_v + I_{bp}) \left\{ \frac{C_r R}{D} \left(\frac{2D}{P_r} - 1 \right) + C_m \left(1 - \frac{R}{D}\right) \left(\frac{2D}{P_m} - 1 \right) \right\} + DS(h_b v + vI_{bp})$$

and

$$X_4 = AX_1 + \frac{M^2 D}{2} (vI_{bp} - pI_{be}).$$

For feasibility of N_2^*, the inequality $X_3 X_4 > 0$ must be satisfied.

To ensure that $T > M$ (*i.e.*, case 2), we substitute (4.8) into inequality $T > M$ and obtain the following inequality

$$NA + S + M^2 D \frac{N}{2} (vI_{bp} - pI_{be}) > M^2 N \left\{ \varphi + \frac{D}{2} (h_b v + vI_{bp}) + \frac{N}{2} \gamma \right\}$$

or

$$NA + S > M^2 N \left\{ \varphi + \frac{D}{2} (h_b v + pI_{be}) + \frac{N}{2} \gamma \right\}, \quad \text{then} \quad T > M \tag{4.10}$$

Proposition 4.1. *The total profit per unit time TP_1 has a unique global maximum value at the point (T_1^*, N_1^*), which is given by inequalities (4.3) and (4.4).*

Proof.
The Hessian matrix associated with TP_1 is given below:

$$H = \begin{pmatrix} \frac{\partial^2 TP_1}{\partial T^2} & \frac{\partial^2 TP_1}{\partial T \partial N} \\ \frac{\partial^2 TP_1}{\partial N \partial T} & \frac{\partial^2 TP_1}{\partial N^2} \end{pmatrix}$$
$$= \begin{pmatrix} -\frac{2A}{T^3} - \frac{2S}{NT^3} - (h_v + I_{vp}) \frac{D}{2} \left\{ c_r \left(\frac{R}{D}\right)^2 \left(1 - \frac{D}{P_r}\right) + c_m \left(1 - \frac{R}{D}\right)^2 \left(1 - \frac{D}{P_m}\right) \right\} - \frac{S}{N^2T^2} - \frac{\gamma}{2} \\ -(h_v + I_{vp}) \frac{D}{2} \left\{ c_r \left(\frac{R}{D}\right)^2 \left(1 - \frac{D}{P_r}\right) + c_m \left(1 - \frac{R}{D}\right)^2 \left(1 - \frac{D}{P_m}\right) \right\} - \frac{S}{N^2T^2} - \frac{\gamma}{2} - \frac{2S}{N^3T} \end{pmatrix}$$

Now, we examine the corresponding second order sufficient conditions for the optimal solutions.

$$-\frac{2A}{T^3} - \frac{2S}{NT^3} < 0, \quad -\frac{2S}{N^3T} < 0$$

and

$$|H| = \left(\frac{2A}{T^3} + \frac{2S}{NT^3}\right)\left(\frac{2S}{N^3T}\right) - \left\{ (h_v + I_{vp}) \frac{D}{2} \left\{ c_r \left(\frac{R}{D}\right)^2 \left(1 - \frac{D}{P_r}\right) + c_m \left(1 - \frac{R}{D}\right)^2 \left(1 - \frac{D}{P_m}\right) \right\} + \frac{S}{N^2T^2} + \frac{\gamma}{2} \right\}^2$$

Using the equation (4.4), we get

$$|H| = \left(\frac{2A}{T^3} + \frac{2S}{NT^3}\right)\left(\frac{2S}{N^3T}\right) - \left\{\frac{S}{N^2T^2} + \frac{S}{N^2T^2}\right\}^2 = \left(\frac{4S}{N^4T^4}\right)(NA + S) - \left(\frac{4S^2}{N^4T^4}\right) = \left(\frac{4AS}{N^3T^4}\right) > 0$$

Hence, the Hessian matrix of H is negative definite. Consequently, we can conclude that the profit function of the system TP_1 is concave function of $N \, and \, T$ and the stationary point for our optimization problem exists and is also unique.

Proposition 4.2. *If $(vI_{bp} - pI_{be}) > 0$, a globally optimal solution for the total profit TP_2 is not only exist but is also unique, at the point (T_2^*, N_2^*), which is given by inequalities (4.8) and (4.9).*

Proof. The Hessian matrix associated with TP_2 is given below:

$$H = \begin{pmatrix} \frac{\partial^2 TP_2}{\partial T^2} & \frac{\partial^2 TP_2}{\partial T \partial N} \\ \frac{\partial^2 TP_2}{\partial N \partial T} & \frac{\partial^2 TP_2}{\partial N^2} \end{pmatrix}$$
$$= \begin{pmatrix} -\frac{2A}{T^3} - \frac{2S}{NT^3} - DM^2 \frac{(vI_{bp} - pI_{be})}{T^3} - (h_v + I_{vp}) \frac{D}{2} \left\{ c_r \left(\frac{R}{D}\right)^2 \left(1 - \frac{D}{P_r}\right) + c_m \left(1 - \frac{R}{D}\right)^2 \left(1 - \frac{D}{P_m}\right) \right\} - \frac{S}{N^2T^2} - \frac{\gamma}{2} \\ -(h_v + I_{vp}) \frac{D}{2} \left\{ c_r \left(\frac{R}{D}\right)^2 \left(1 - \frac{D}{P_r}\right) + c_m \left(1 - \frac{R}{D}\right)^2 \left(1 - \frac{D}{P_m}\right) \right\} - \frac{S}{N^2T^2} - \frac{\gamma}{2} - \frac{2S}{N^3T} \end{pmatrix}$$

Now, we examine the corresponding second order sufficient conditions for the optimal solutions.

Now, $(vI_{bp} - pI_{be}) > 0$, using the equation (4.9), it can be easily verified that

$$-\frac{2A}{T^3} - \frac{2S}{NT^3} - DM^2\frac{(vI_{bp} - pI_{be})}{T^3} < 0, -\frac{2S}{N^3T} < 0$$

and

$$|H| = \left(\frac{2A}{T^3} + \frac{2S}{NT^3} + DM^2\frac{(vI_{bp} - pI_{be})}{T^3}\right)\left(\frac{2S}{N^3T}\right)$$
$$- \left\{(h_v + I_{vp})\frac{D}{2}\left\{c_r\left(\frac{R}{D}\right)^2\left(1 - \frac{D}{P_r}\right) + c_m\left(1 - \frac{R}{D}\right)^2\left(1 - \frac{D}{P_m}\right)\right\} + \frac{S}{N^2T^2} + \frac{\gamma}{2}\right\}^2$$
$$= \left(\frac{4S}{N^3T^4}\right)\left(A + DM^2\frac{(vI_{bp} - pI_{be})}{2}\right) > 0$$

Hence, the Hessian matrix of H is negative definite. Therefore, the profit function of the system TP_2 is concave function of $N \, and \, T$ and the stationary point for our optimization problem exists and is also unique.

Theorem 4.3. *For any given N and T, let $\Delta = M^2N\left\{\varphi + \frac{D}{2}(h_bv + pI_{be}) + \frac{N}{2}\gamma\right\} - NA + S$, we have*

(1) If $\Delta > 0$, then the buyer's optimal replenishment cycle length is

$$T = \sqrt{\frac{NA + S}{N\left\{\varphi + \frac{D}{2}(h_bv + pI_{be}) + \frac{N}{2}\gamma\right\}}} < M$$

(2) If $\Delta < 0$, then the buyer's optimal replenishment cycle length is

$$T = \sqrt{\frac{NA + S + M^2D\frac{N}{2}(vI_{bp} - pI_{be})}{N\left\{\varphi + \frac{D}{2}(h_bv + vI_{bp}) + \frac{N}{2}\gamma\right\}}} > M$$

(3) If $\Delta = 0$, then

$$T = M$$

Proof. It is immediately followed from (4.5) and (15).

5. Numerical analysis

Example 5.1. The above theoretical results are illustrated through an exact numerical example for a single product by considering mutually the given values from the numerical problems in Ouyang *et al.* [14], Huang [8] and Lou & Wang [12]. Other required information for the revere supply system is taken from Singh & Saxena [29]. Since many models have considered forward supply chain model. But, a reasonable estimation can be done for reverse logistics. As far as the authors' knowledge goes, no previous study has been done in this area incorporating trade credit consideration in reverse supply system. We consider the values of the following input parameters in appropriate units.

$$u_R = 1, u_m = 3, c_r = 2.5, c_p = 4, v = 20, p = 38, I_{bp} = 0.15, I_{be} = 0.05, I_{vp} = 0.10, P_r = 1800,$$
$$D = 1000, P_m = 2000, R = 400, A = 200, S = 1000, F = 1.2, M = 0.3, h_v = 0.05, h_b = 0.1, \; h_R = 0.1.$$

Applying the solution procedure mentioned above, we derive the optimal solution and results are presented in the Table 1.

TABLE 1. The optimal results of the inventory model under the above parametric values.

T	N	m	n	Δ	Q	TP_b	TP_v	TP
0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0*

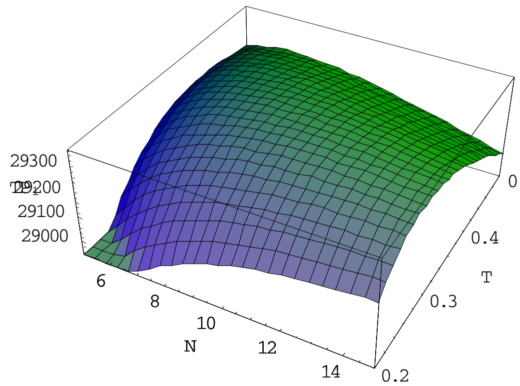

FIGURE 5. Concavity for the case 1, when $M = 0.4$; keeping the values of other parameters same as in Example 5.1.

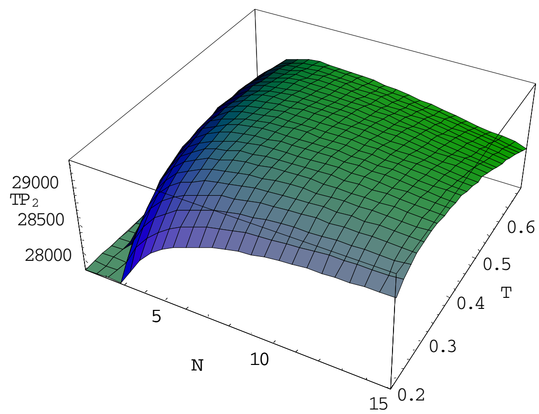

FIGURE 6. Concavity for the case 2, when $M = 0.2$; keeping the values of other parameters same as in Example 5.1.

From the above table, it is observed that vendor fulfils buyer's need in 11 shipments where four deliveries of buyer are fulfilled by the remanufactured material and six are fulfilled by the newly produced items, while for the one shipment a ratio (0.4) of the delivery size are fulfilled by the remanufactured products and the rest is fulfilled by the newly produced items.

We have already proved in preposition 4.1 and 4.2 that the functions are concave. Figures 5 and 6 show clear concavity of the respective objective functions.

6. SENSITIVITY ANALYSIS

In this section, sensitivity analysis is done to test the robustness of the proposed model.

TABLE 2. Optimal results for the same set of values as in Example 5.1 for different delay period.

M	T	N	m	n	Δ	Q	TP_b	TP_v	TP
0.1	0.287147	12	4.8	7.2	−3045.6	287.147	15666.5	13621.8	29288.3
0.2	0.2952345	12	4.8	7.2	−1982.6	295.234	15910.0	13421.6	29331.6
0.3	0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0
0.4	0.3183679	11	4.4	6.6	1851.4	318.368	16311.0	13021.5	29332.5
0.5	0.3183679	11	4.4	6.6	4692.81	318.368	16501.0	12821.5	29322.5
0.6	0.3183679	11	4.4	6.6	8165.64	318.368	16691.0	12621.5	29312.5
0.7	0.3183679	11	4.4	6.6	12269.9	318.368	16881.0	12421.5	29302.5

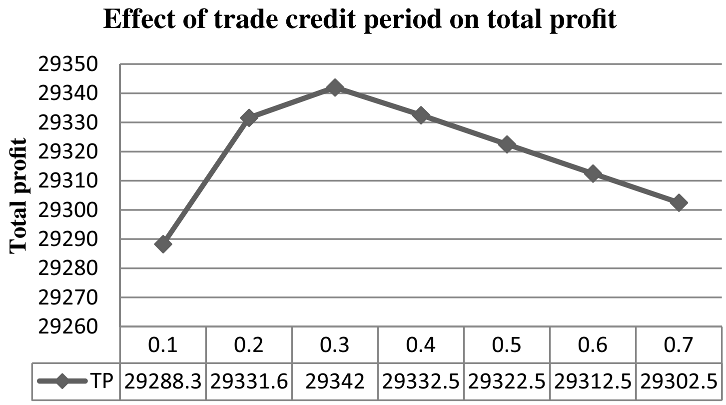

FIGURE 7. Effect of trade credit period on total profit.

Example 6.1. In this example, we study the effects of credit period M. Consider different $M \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7\}$, keeping the values of other parameters same as in Example 5.1. The solution procedure is applied to obtain the optimal solutions and the results are shown in Table 3. The graphical representation of the sensitivity of the profit function with varying values of M is provided in Figure 7.

Observations from example 2:

The main conclusions drawn from the Example 6.1 given above are as follows.

(1) Table 3 reveals that the time interval between successive deliveries decreases as the permissible delay period increases. Furthermore, when the credit period $M$is greater than the cycle length T and continuously increasing, the optimal solution (T, N) remains the same. The mathematical expressions of equation (4.3) and (4.4) are also verified for case 1 where $(T, N)$is independent of M.

(2) Most studies about supply chain management concern about how to speed up material, information, and cash flows but the available models who study the effect of permissible delay on supply chain concludes that the longer permissible delay period increases the profit of system. Therefore, the best policy in these models is to set the credit period as long as possible. This conclusion is apparently irrational. However, this model reveals that the total profit increases as the delay period increases up to a certain level when $T < M$. As the interest paid by the vender gets more than the interest earned by the buyer, profit starts to decrease with the increment in trade credit period. Hence, it is concluded that delay in payment is profitable for a certain limit which is reasonable.

(3) We have observed that Table 3 also verified the Theorem 4.3 that if $\Delta > 0$ then the time interval between successive deliveries $T < M$ and when $\Delta < 0$ then $T < M$.

TABLE 3. Sensitivity analysis with respect to vendor's set up cost.

S	T	N	m	n	Δ	Q	TP_b	TP_v	TP
600	0.311693	8	3.6	5.4	−209.49	311.693	16120.4	13351.0	29471.4
800	0.314221	9	3.6	5.4	−409.49	314.221	16120.4	13280.1	29400.5
1000	0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0
1200	0.315866	12	4.8	7.2	−410.76	315.866	16120.4	13166.6	29287.0
1400	0.315611	13	5.2	7.8	−448.01	315.611	16120.4	13116.1	29236.5

TABLE 4. Sensitivity analysis with respect to transportation cost.

F	T	N	m	n	Δ	Q	TP_b	TP_v	TP
0.72	0.315486	11	4.4	6.6	−358.59	315.486	16600.4	13221.6	29822.0
0.96	0.315486	11	4.4	6.6	−358.59	315.486	16360.4	13221.6	29582.0
1.20	0.315486	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0
1.44	0.315486	11	4.4	6.6	−358.59	315.486	15880.4	13221.6	29102.0
1.68	0.315486	11	4.4	6.6	−358.59	315.486	15640.4	13221.6	28862.0

TABLE 5. Sensitivity analysis with respect to buyer's ordering cost.

A	T	N	m	n	Δ	Q	TP_b	TP_v	TP
120	0.247749	14	5.6	8.4	1249.66	247.749	16402.5	13222.1	29624.7
160	0.287058	12	4.8	7.2	269.24	287.058	16252.9	13221.8	29474.6
200	0.315486	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0
240	0.341634	10	4	6	−891.5	341.634	15998.5	13221.3	29219.8
280	0.358748	10	4	6	−1291.5	358.748	15884.7	13220.9	29105.6

TABLE 6. Sensitivity analysis with respect to returned rate.

R	T	N	m	n	Δ	Q	TP_b	TP_v	TP
240	0.314951	10	2.4	7.6	−323.94	314.951	16120.4	12600.2	28720.6
320	0.3115	11	3.52	7.48	−271.38	311.5	16120.3	12914.1	29034.4
400	0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0*
480	0.31825	11	5.28	5.72	−417.15	318.25	16120.4	13520.3	29640.7
560	0.312638	12	6.72	5.28	−316.37	312.638	16120.4	13810.4	29930.8

Example 6.2. We now study the effects of changes in the values of the system parameters S, A, F, P_r, P_m, D, R, v and p on the optimal total profit and number of reorder. The sensitivity analysis is performed by changing each of the parameters by −40%, −20%, 20% and 40%, taking one parameter at a time and keeping the remaining parameters unchanged as in Example 1. The sensitivity analysis of the different parameters are shown in Tables 4−12.

Observations from the sensitivity analysis:

The main conclusions drawn from the sensitivity analysis given above are as follows.

(1) In the Table 4, it is observed that as the vendor's set up cost increases, the optimal number of deliveries and cycle length increase while the aggregate profit of vendor decreases.
(2) In Table 5, it is noted that both the variable T and N are unaffected by the changes in transportation cost but the buyer's profit decreases as the transportation cost increases.
(3) Similarly from the Table 6, it is observed that as the buyer's ordering cost increases, the optimal number of deliveries decreases and the time interval between successive deliveries increases which results in the reduction of the buyer's aggregate profit.

TABLE 7. Sensitivity analysis with respect to production rate.

P_m	T	N	m	n	Δ	Q	TP_b	TP_v	TP
1200	0.3080543	15	6	9	−244.75	308.054	16119.9	13327.4	29447.3
1600	0.313523	12	4.8	7.2	−337.12	313.523	16120.4	13256.4	29376.8
2000	0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0*
2400	0.320746	10	4	6	-437.5	320.746	16120.3	13200.4	29320.7
2800	0.318742	10	4	6	−398.93	318.742	16120.4	13186.6	29307.0

TABLE 8. Sensitivity analysis with respect to remanufacturing rate.

P_r	T	N	m	n	Δ	Q	TP_b	TP_v	TP
1080	0.313354	12	4.8	7.2	−333.16	313.354	16120.4	13255.3	29375.7
1440	0.317217	11	4.4	6.6	−395.44	317.217	16120.4	13233.3	29353.8
1800	0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0*
2160	0.314348	11	4.4	6.6	−334.02	314.348	16120.4	13213.8	29334.2
2520	0.313542	11	4.4	6.6	−316.48	314.348	16120.4	13208.2	29328.6

TABLE 9. Sensitivity analysis with respect to demand.

D	T	N	m	n	Δ	Q	TP_b	TP_v	TP
600	0.3981433	10	6.67	3.33	−1425.1	238.886	9445.86	8357.96	17803.8
800	0.345802	11	5.5	5.5	−899.30	276.641	12775.5	10792.9	23568.4
1000	0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0*
1200	0.285845	12	4	8	345.08	343.014	19475.4	15666.1	35141.6
1400	0.263397	13	3.71	9.29	1070.06	368.756	22839.6	18135.7	40975.3

TABLE 10. Sensitivity analysis with respect to buyer's procurement cost.

v	T	N	m	n	Δ	Q	TP_b	TP_v	TP
12	0.355573	10	4	6	−851.5	355.573	24256.8	5461.07	29717.9
16	0.327609	11	4.4	6.6	-556.59	327.609	20185.6	9341.07	29526.7
20	0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0*
24	0.299769	12	4.8	7.2	5.24	299.769	12058.3	17101.3	29159.7
28	0.290691	12	4.8	7.2	221.24	290.691	7998.86	20981.8	28980.6

(4) It is clearly visible in the Tables 4, 5 and 6 that total profit TP is moderately negative sensitive to the changes in parameter S, A and F which is obvious. It is also noted from comparison of the behaviour of these three parameters that the ordering cost is more sensitive than the set up cost while less sensitive than the transportation cost. So, it is advised to the decision makers that they setup their system such that the transportation and ordering cost are lesser whatever the set up cost may be.

(5) Table 7 reveals that, as the returned rate increases, the number of deliveries fulfilled by the remanufactured items increases, *i.e.*, remanufacturing increases which results in the increment in total profit. Hence, the remanufacturing is profitable for the system.

(6) In Tables 8 and 9, it is observed that the vendor's profit $TP_v$is slightly negative sensitive to the changes in parameterP_m and P_r. Tables 8, 9 and 10 reveal that profit increases as the production/remanufacturing is close to the demand which is obvious because of the inventory in stock goes higher that increases holding cost and the profit decreases accordingly. Therefore, it is advised to the decision makers to determine the production/remanufacturing rate closed to the demand rate.

TABLE 11. Sensitivity analysis with respect to buyer's selling price.

p	T	N	m	n	Δ	Q	TP_b	TP_v	TP
22.8	0.3390809	10	4	6	−833.5	339.081	815.624	13221.2	14036.8
30.4	0.323313	11	4.4	6.6	–546.69	323.313	8467.13	13221.3	21688.5
38.0	0.3154863	11	4.4	6.6	−358.59	315.486	16120.4	13221.6	29342.0*
45.6	0.30746	11	4.4	6.6	−170.49	307.46	23775.5	13221.4	36996.9
53.2	0.291562	12	4.8	7.2	199.64	291.562	31432.7	13221.7	44654.4

(7) Table 10 reveals that, as the demand rate increases, the optimal number of deliveries increases while the time interval between successive deliveries decreases which results in the huge increment in the aggregate profit. This is a logical tendency since it allows the vendor and buyer to get more cost savings from accumulated revenue. Hence, demand rate should be estimated carefully to determine the average system profit correctly.

(8) In Table 11, we observe that optimal number of deliveries slightly increases while the time interval between successive deliveries decreases as the buyer's procurement cost increases. Hence the buyer's profit decreases and vendor's profit increases with the increasing buyer's procurement cost that results in the small reduction in total profit.

(9) Similarly, from the Table 12, it is noticed that optimal number of deliveries slightly increases and the time interval between successive deliveries decreases while the buyer's selling price increases. In this situation, the buyer's profit increases while vendor's profit remains unchanged and thus the total profit of the system increases.

7. Conclusion

In this article, a green supply chain control system with mixed strategy of production and remanufacturing is developed under the condition of permissible delay in payment. We formulate the theoretical results to find out the optimal number of deliveries and the time interval between successive deliveries. The theoretical results are illustrated through the numerical examples. Further, the effects of different parameters are compared and the results indicate that the total profit increases with increases in the demand rate, return rate and buyer's selling price and decreases as the production rate, remanufacturing rate, set up cost, ordering cost, transportation cost and buyer's procurement cost increase. The theoretical expression shows that whenever the replenishment cycle time is greater than the delay period, the system is globally optimal if and only if interest paid by the buyer is greater than the earned interest. This model suggests to the vender and buyer how to maximize their joint profit by determining optimal number of deliveries and time of successive deliveries under trade-credit financing for remanufacturing items. The proposed model has some limitations like as deterministic cost and profit parameters, fixed demand and production rate and zero lead times of deliveries. As the demand and production rates are fixed, the supply disruption at each state is neglected here. This model might be extended considering supply disruptions at vender and buyer. Moreover, our model considers remanufacturing of used products due to environmental consciousness and economical benefits. In such cases, advertising and promotional efforts such as discounts, gifts and warranty are required to attract the customers to buy more. This model may be studied further in the light of advertising for environmental consciousness, promotional efforts and variable cost and profit parameters of the systems.

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