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COMPETITIVE *VERSUS* COOPERATIVE PERFORMANCES OF A STACKELBERG GAME BETWEEN TWO SUPPLIERS

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Abstract. We investigate the competitive and cooperative performances of a supply chain with two capacitated suppliers solicited by a customer who offers a new product procurement suggestion. Suppliers have the option to accept or reject the new product offer according to its profitability. In addition, suppliers have to decide on their base stock levels. We classify suppliers as principal and secondary. The customer usually addresses demand to the principal supplier at first. We consider two schemes: in the first scheme, the principal supplier informs the customer about the demand ratio he wants to be allocated. The customer allocates the remaining quantity to the secondary supplier. In the second scheme, the principal supplier decides to respond to the entire demand and to subcontract a part of it to the secondary supplier. In the competitive situation, we give conditions that allow principal supplier to select the best scheme. We show that the new product offer can be refused while it is accepted when suppliers cooperate. We present a profit allocation policy under which collaboration is beneficial for the two suppliers.

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1. Introduction

Several works highlighted the inefficiency resulting from decentralization of decisions within a supply chain. In fact, the obvious behavior of each agent is to optimize his individual profit. Yet, agents' objectives are often antagonistic and can lead to irrational choices with regard to the whole system. In order to succeed in overcoming this loss, companies have to network by coordinating their actions.

In this paper, we underline the wastefulness due to decentralization within a supply chain. Thus, we prove that the system can significantly perform better if agents decide to cooperate. In particular, we consider a supply chain formed with two different suppliers and a customer. The first supplier is considered as a principal supplier for the customer and often receives suggestions of new products deals. We suppose that he faces the offer of a new product contract and we consider two plausible schemes: in the first scheme, the principal supplier

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allocates himself the demand amount that he wishes to get. The customer allocates the remaining quantity to the secondary supplier. In the second scheme, the principal supplier gets the total demand volume, and subcontracts a part of it to the second supplier. Suppliers operate in a make to stock manner and choose base stock control policy for their inventories replenishment. In addition, each supplier has to decide on accepting or refusing the offer according to its profitability. We analyze both competitive and cooperative systems under each scheme. We derive analytical/numerical solutions for the different decision variables. Then we draw some comparisons between schemes from the principal supplier perspectives. We study the optimal centralized system performances and propose that suppliers cooperate by sharing the resulting profit among them according to the equal allocation policy [16]. We prove that thanks to this profit allocation rule, both suppliers improve their profits. Through numerical examples, we show that decentralization of decisions can lead to the loss of the offer, nevertheless, it is accepted if the system is managed in a cooperative manner.

Our model may be useful for studying many real situations in the B to B context. In this context, the decision-making process generally occurs in two stages: firstly, the customer selects a short list of suppliers from existing suppliers based on various factors such as price, capacity, quality and service level. It is a process of supplier selection (see for example [20]). The second stage is a demand allocation process where the customer splits the demand between selected suppliers. In this work, we assume that the suppliers-selection step was already established, so we focus on the demand allocation step.

The retail sector is a good example for the problem studied in this paper. In fact, it is well known that a leading retailer (like WALMART or CARREFOUR, *etc.*) has various suppliers that provide him with a very large number of products. It is true that for some products (namely the most important ones) such a company must use a suitable policy where demand allocation between the suppliers is optimized. But, given the huge number of products and suppliers to be managed, these companies usually handle the most of the products by just using simple policies. One of such policies is to classify suppliers as principal, secondary, *etc.* The leading retailer addresses its demand to the principal supplier at first. If the principal supplier has not enough capacity, the leading retailer purchases the shortfall from the other suppliers. Moreover, it often happens that a supplier subcontracts a part of the demand from other suppliers. This does not matter for the leading retailer. In fact, the principle supplier is responsible to meeting the total demand by providing the total volume of the same product and with the same quality.

The remaining of this paper is structured as follows: in Section 2, we present related literature review. In Section 3, we introduce the studied model and give notations. In Section 4, we focus on the competitive system. Section 5 is devoted to the analysis of the cooperative system. In Section 6, we expose numerical examples. Finally, in Section 7, we give conclusions and research perspectives.

2. Literature review

Literature which is related to the problem considered here can be divided into four categories.

The first stream of papers dealt with centralized performances optimization in supply chains. Example are [2, 4, 5]. In [2], the authors analyzed a model where a producer allocates demand to several facilities where make to order policy is employed. In [4], the authors gave a procedure solution to obtain optimal demand allocation parameters and optimal base stock levels in a multiple manufacturing facilities system. [5] treated the case of multiple demand sources and multiple distribution centers and analyzes how to allocate each demand source volume to the distribution centers.

The second stream of papers dealt with the interaction between one supplier and a downstream customer. See for example [11, 12, 18, 19]. These papers investigated the effect of decentralization of decisions on the performances of a supply chain and provide coordination mechanisms to improve them.

The third stream of papers treated competition between actors in the same stage of the supply chain. In [17] the authors presented a model with two competing suppliers serving one buyer. This latter allocates demand according to suppliers' delivery mean time in deterministic environment. [10] investigated a model in which a buyer has to split demand volume between two make to order suppliers who are competing on their capacities.

They exposed some demand allocation policies that exist in literature and studied suppliers responses for each one. In [3] the authors presented a way of stressing competition between identical suppliers by means of the proportional demand allocation policy. According to this policy each supplier gets more demand volume if he offers better service level. [6] studied a system which includes two suppliers and two buyers. Each buyer offers a new item contract and allocates all its demand volume to the supplier who offers the higher backorder penalty. Later, [14] generalized the model presented in [3] by including penalties for the suppliers who don't respect promised sojourn time. For identical supply chain structure, [15] compares service level and inventory level competitions and shows that the latter creates a higher service level for the buyer.

The last stream of these works dealt with cooperation between actors in the same stage and its benefits for the whole supply chain. These benefits result from decisions centralization and from pooling cooperating actors' efforts. Particularly in [22], the authors highlighted the benefit of cooperation among suppliers that decide to pool a single facility instead of operating standalone. They showed that cooperation leads to interesting savings that can be divided between suppliers. [1] dealt with a model where a number of suppliers pool their service capacities to serve the totality of customer orders.

Our paper is in the field of the two last categories and treats competition and cooperation between two suppliers who faced a downstream customer demand. Our contributions consist of two parts:

- Presented papers focus either on the entire supply chain performances or on downstream customers' ones. Nash equilibrium is generally used to model suppliers' competition. We consider one supplier perspectives (supplier-1 without loss of generality), who acts as a Stackelberg leader compared to supplier-2. In order to respond to the customer demand, we investigate two schemes. Under the first scheme supplier-1 decides on demand ratio he wants to be allocated knowing that supplier-2 accepts the remaining part of demand. Under the second scheme supplier-1 accepts the entire demand and subcontracts a part of it to supplier-2.
- Earlier mentioned papers consider that suppliers accept the demand volume they are allocated and don't discuss about the refusal possibility. We introduce the acceptation variable as a decision parameter in our model. Literature that studied the problem of admitting or rejecting a new item is limited. We note the model of [13] that gives the supplier the possibility to accept or to reject orders. Their problem is rather a scheduling one. Our paper is different in that we treat the possibilities of acceptation or rejection of a stationary stochastic demand.

3. Model and notations

We consider a supply chain with two capacitated suppliers faced with the suggestion of a new product procurement contract. Demand is stochastic according to a Poisson process with rate λ. We denote by $\propto_i \lambda$ the demand rate allocated to supplier-i, where $\alpha_i \in [0, 1]$.

Processing time for supplier-i is stochastic, independent and exponentially distributed with average $\dfrac{1}{\mu_i} \forall_i \in \{1, 2\}$. Supplier-$i$ utilization rate is denoted by ρ_i where $\rho_i = \dfrac{\alpha_i \lambda}{\mu_i}, \forall i \in \{1, 2\}$.

Supplier-i incurs a unit production cost c_i, a holding cost h_i per item per unit time and a backorder penalty b_i per item per unit time. Supplier-i earns a sale price p_i for each product unit sold.

Suppliers adopt base stock control policy for their inventories replenishment, we denote by s_i the new product base stock level of supplier-i. Each supplier decides to accept or refuse the new product offer according to its profitability. We assume that supplier-i accepts the offer if the resulting profit is higher than or equal to a fixed threshold π_i^0. We denote by A_i a binary variable which expresses supplier-i acceptance $(A_i = 1)$ or refusal $(A_i = 0)$ of the new product offer. Supplier-i gets no demand volume if he refuses the offer, consequently $a_i \leqslant A_i \forall i \in \{1, 2\}$. In addition, all demand volume must be produced in case of acceptance, so $\alpha_1 + \alpha_2 = 1$ if $A_1 + A_2 \geqslant 1$.

As the arrival of demand is a Poisson process and service time has exponential distribution, the production system can be modeled as an $M/M/1$ queue. The average inventory and backorder levels were established by [7] in such case. Let's denote by $\overline{X_i}$ and by $\overline{Y_i}$ the average inventory and the average backorder at supplier-i, then:

$$\overline{X_i}\left(s_{i,}\rho_i\right) = s_i - \frac{\rho_i}{1-\rho_i}\left(1-\rho_i^{s_i}\right), \tag{3.1}$$

$$\overline{Y}_i\left(s_{i,}\rho_i\right) = \frac{\rho_i^{s_{i+1}}}{1-\rho_i}\cdot \tag{3.2}$$

Stability condition of each supplier production system requires that $\rho_i < 1$. In addition, the whole system stability requires that system load $\rho = \dfrac{\lambda}{\mu_1+\mu_2}$ verifies $\rho < 1$. We can resume all these constraints as follows: $\alpha_1 \in [0,\,1] \cap \left]1-\frac{\mu_2}{\lambda}, \dfrac{\mu_1}{\lambda}\right]$, we denote this interval by J.

Without loss of generality we suppose that supplier-1 is the principal supplier, and that supplier-2 is the secondary one. We assume that both suppliers are risk neutral. As stated in [3], actors are assumed to be risk neutral when some system parameters are random variables. This assumption is widely adopted in the literature related to our paper (see for example [3, 9, 15]). Furthermore, we suppose that suppliers have full information. In fact suppliers belong to the same manufacturing sector so they are able to obtain reliable information about different operational costs of their competitors and subcontractors. This assumption is used for example in [3,8]. In Addition, we assume that suppliers' prices do not affect the volume of the customer demand. Indeed, we are interested in the B to B context where the dependence between demands and prices is generally low and not prevalent compared to the B to C environment.

Supplier-1 seeks to get the demand amount that maximizes his own profit. We investigate two schemes that suppliers could use: in the first scheme, supplier-1 decides on demand ratio he wants to be allocated knowing that supplier-2 accepts the remaining portion of demand. In this case, both suppliers are charged the backorder penalty from the customer. In the second scheme, supplier-1 gets the entire demand, and subcontracts a part of it to supplier-2. Thus, supplier-1 pays the totality of suppliers' backorder penalties to the customer. Supplier-2 pays its backorder penalty to supplier-1.

4. Competitive system performances

Since supplier-1 is principal, we define a Stackelberg game between the two suppliers where supplier-1 is the leader. In the case of scheme-j, $j \in \{1,\,2\}$, supplier-1 decides first by choosing $\left(A_1^j\,, s_1^j,\, \alpha_1^j\right)$. Afterward, supplier-2 decides on $\left(A_2^j, s_2^j\right)$.

Let's denote by π_i^1 supplier-i profit in the case of scheme-1 then:

$$\pi_1^1\left(A_1^1, s_1^1, \alpha_1^1, A_2^1\right) = \begin{cases} A_1^1A_2^1\big((p_1-c_1)\alpha_1^1\lambda - b_1\overline{Y}_1^1\big) - h_1\overline{X}_1^1 & \text{if} \quad \alpha_1^1 < 1 \\ A_1^1\big((p_1-c_1)\lambda - b_1\overline{Y}_1^1\big) - h_1\overline{X}_1^1 & \text{if} \quad \alpha_1^1 = 1 \end{cases} \tag{4.1}$$

$$\pi_2^1\left(A_2^1, s_2^1, \alpha_1^1, A_1^1\right) = \begin{cases} A_1^1A_2^1\big((p_2-c_1)\big(1-\alpha_1^1\big)\lambda - b_2\overline{Y}_2^1\big) - h_2\overline{X}_2^1 & \text{if} \quad \alpha_1^1 > 0. \\ A_2^1\big((p_2-c_2)\lambda - b_2\overline{Y}_2^1\big) - h_2\overline{X}_2^1 & \text{if} \quad \alpha_1^1 = 0 \end{cases} \tag{4.2}$$

By the same way, let's denote by π_i^2 supplier-i profit in the case of scheme-2 then:

$$\pi_1^2\left(A_1^2, s_1^2, \alpha_1^2, A_2^2\right) = \begin{cases} A_1^2A_2^2\big(p_1\lambda - p_2\big(1-\alpha_1^2\big)\lambda - c_1\alpha_1^2\lambda - b_1\big(\overline{Y}_1^2+\overline{Y}_2^2\big) + b_2\overline{Y}_2^2\big) - h_1\overline{X}_1^2 & \text{if} \quad \alpha_1^2<1 \\ A_1^2\big((p_1-c_1)\lambda - b_1\overline{Y}_1^2\big) - h_1\overline{X}_1^2 & \text{if} \quad \alpha_1^2=1 \end{cases} \tag{4.3}$$

$$\pi_2^2\left(A_2^2, s_2^2, \alpha_1^2, A_1^2\right) = A_1^2A_2^2\left((p_2-c_2)\left(1-\alpha_1^2\right)\lambda - b_2\overline{Y}_2^2\right) - h_2\overline{X}_2^2, \tag{4.4}$$

where $\forall i \in \{1,2\}$ and $\forall j \in \{1,2\}$, $\overline{X}_i^j = \overline{X}_i\left(s_i^j, \rho_i^j\right)$ and $\overline{Y}_i^j = \overline{Y}_i\left(s_i^j, \rho_i^j\right)$, where $\overline{X}_i$ and $\overline{Y}_i$ are given by expressions (3.1), (3.2); $\rho_1^j = \frac{\alpha_1^j \lambda}{\mu_1}$ and $\rho_2^j = \frac{(1-\alpha_1^j)\lambda}{\mu_2}$.

Expressions (4.1)−(4.4) show that suppliers' profits are interdependent. In fact, under scheme-j ($j \in \{1,2\}$), supplier-1 profit depends on supplier-2 acceptation parameter A_2^j. On the other hand, supplier-2 profit depends on the acceptation parameter A_1^j and the demand allocation decision α_1^j that are chosen by supplier-1.

Note that, under scheme-2, supplier-2 gets no demand volume if supplier-1 refuses the offer.

Let $\alpha_1^j \in J$. We denote by $\widetilde{s}_i^j\left(\alpha_1^j\right)$ the following expressions where $j \in \{1,2\}$ and $i \in \{1,2\}$:

$$\widetilde{s}_1^j\left(\alpha_1^j\right) = \left\lfloor \frac{\operatorname{Log}\left(\frac{h_1}{h_1+b_1}\right)}{\operatorname{Log}\left(\rho_1^j\right)} \right\rfloor \quad \text{if} \quad \alpha_1^j \neq 0 \quad \text{and} \quad \widetilde{s}_1^j(0) = 0,$$

$$\widetilde{s}_2^j\left(\alpha_1^j\right) = \left\lfloor \frac{\operatorname{Log}\left(\frac{h_2}{h_2+b_2}\right)}{\operatorname{Log}\left(\rho_2^j\right)} \right\rfloor \quad \text{if} \quad \alpha_1^j \neq 1 \quad \text{and} \quad \widetilde{s}_2^j(1) = 0, \tag{4.5}$$

where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

In the case of scheme-j, $j \in \{1,2\}$, we define $\widetilde{A}_1^j\left(\alpha_1^j\right)$ as follows:

$$- \text{ If } \pi_2^j\left(1, \tilde{s}_2^j\left(\alpha_1^j\right), \alpha_1^j, 1\right) \geqslant \pi_2^0 \text{ then } \widetilde{A}_2^j\left(\alpha_1^j\right) = 1, \text{ otherwise, } \widetilde{A}_2^j\left(\alpha_1^j\right) = 0. \tag{4.6}$$

$$- \text{ If } \pi_1^j\left(1, \tilde{s}_1^j\left(\alpha_1^j\right), \alpha_1^j, \widetilde{A}_2^j\left(\alpha_1^j\right)\right) \geqslant \pi_1^0 \text{ then } \widetilde{A}_1^j\left(\alpha_1^j\right) = 1, \text{ otherwise, } \widetilde{A}_1^j\left(\alpha_1^j\right) = 0. \tag{4.7}$$

$$\text{Let } \Omega^j = \left\{\widetilde{\alpha}_1^j / \widetilde{\alpha}_1^j \in \operatorname{ArgMax}_{\alpha_1^j \in J} \pi_1^j\left(\widetilde{A}_1^j\left(\alpha_1^j\right), \widetilde{s}_1^j\left(\alpha_1^j\right), \alpha_1^j,\ \widetilde{A}_2^j\left(\alpha_1^j\right)\right)\right\}, j \in \{1,\ 2\}. \tag{4.8}$$

Let $\mathbb{E}^j$ be the set of Stackelberg equilibriums when scheme-j is adopted $\forall j \in \{1,2\}$.

Proposition 4.1. $\mathbb{E}^j = \left\{\xi^j\left(\widetilde{\alpha}_1^j\right) / \widetilde{\alpha}_1^j \in \Omega^j\right\}$, *where:*

- $\xi^j\left(\widetilde{\alpha}_1^j\right) = \left(A_1^{*j}, \alpha_1^{*j}, s_1^{*j}, A_2^{*j}, s_2^{*j}\right)$,
- $A_1^{*j} = \widetilde{A}_1^j\left(\widetilde{\alpha}_1^j\right)$,
- $\alpha_1^{*j} = A_1^{*j}\widetilde{\alpha}_1^j$,
- $s_1^{*j} = A_1^{*j}\widetilde{s}_1^j\left(\widetilde{\alpha}_1^j\right)$,
- $A_2^{*j} = A_1^{*j}\widetilde{A}_2^j\left(\widetilde{\alpha}_1^j\right)$ *if* $\widetilde{\alpha}_1^j \neq 0$, *otherwise* $\left\{\begin{smallmatrix} A_2^{*1} = \widetilde{A}_2^1(0) \\ A_2^{*2} = A_1^{*2} \end{smallmatrix}\right.$,
- $s_2^{*j} = A_2^{*j}\widetilde{s}_2^j\left(\widetilde{\alpha}_1^j\right)$.

Proof. According to expressions (4.1)−(4.4), each supplier-i profit is concave with respect to s_i^j if $A_i^j = 1$. When $0 < \alpha_1^j < 1$, expressions (4.5) represent the optimal base stock levels which are obtained *via* the classical method presented by [7]. If $\alpha_1^j = 0$ then according to (4.1) and (4.3) supplier-1 would rather not to set inventory, so $\widetilde{s}_1^j = 0$. Analogously, if $\alpha_1^j = 1$ then according to (4.2) and (4.4) supplier-2 would rather not to set inventory, so $\widetilde{s}_1^j(1) = 0$.

According to (4.6), $\widetilde{A}_2^j$ is the best supplier-2 choice if we suppose that supplier-1 accepted the offer. According to (4.7), $\widetilde{A}_1^j$ is the best supplier-1 choice knowing that such that supplier-2 has chosen $\widetilde{A}_2^j$.

- Supplier-1 decides on $(A_1^j,\ \alpha_1^j,\ s_1^j)$. As $\widetilde{\alpha}_1^j \in \Omega^j$, then it corresponds to the best supplier-1 choice in the case of scheme-j when he accepts the offer. In addition, since $\alpha_1^{*j} \leq A_1^{*j}$ then $\alpha_1^{*j} = 0$ if $A_1^{*j} = 0$, otherwise $\alpha_1^{*j} = \widetilde{\alpha}_1^j$. We resume these results in the expression: $\alpha_1^{*j} = A_1^{*j}\widetilde{\alpha}_1^j$.
  According to (4.7), $\widetilde{A}_1^j$ takes into consideration the value of $\widetilde{A}_2^j$ which is given by (4.5), then $A_1^{*j} = \widetilde{A}_1^j\left(\widetilde{\alpha}_1^j\right)$ corresponds to supplier-1 best acceptance decision.
  According to expressions (4.1) and (4.3), if $A_1^{*j} = 0$ then $\pi_1^j = -h_1\overline{X_1}$ so $s_1^{*j} = 0$. If $A_1^{*j} = 1$, then s_1^{*j} corresponds to its optimal expression provided by (4.5). We resume these results by letting: $s_1^{*j} = A_1^{*j}\widetilde{S}_1^j\left(\widetilde{\alpha}_1^j\right)$.
  Therefore $\left(A_1^{*j}, S_1^{*j}, \alpha_1^{*j}\right)$ is the best supplier-1 strategy.
- Supplier-2 decides on $\left(A_2^j, s_2^j\right)$. Consider the case where $\widetilde{\alpha}_1^j \neq 0$. If $A_1^{*j} = 0$, then according to expressions (4.2) and (4.4), $\pi_2^j \leq 0$. It leads to $A_2^{*j} = 0$ by definition of the acceptation parameter. If $A_1^{*j} = 1$, then $\widetilde{A}_2^j\left(\widetilde{\alpha}_1^j\right)$ is the best supplier-2 choice as it is given by (4.6). To resume these constraints we let $A_2^{*j} = \widetilde{A}_1^j\left(\widetilde{\alpha}_1^j\right)\widetilde{A}_2^j\left(\widetilde{\alpha}_1^j\right)$. When $\widetilde{\alpha}_1^j = 0$, this corresponds to allocating all demand volume to supplier-2. In the first scheme, supplier-2 can get the whole demand volume regardless of supplier-1 acceptance decision as shown in (4.2). So $A_2^{*1} = \widetilde{A}_2^1(0)$. In the second scheme, as shown by expression (4.4), if A_1^{*2} then $\pi_2^2 \leq 0$. It turns out that $A_2^{*2} = 0$. Suppose that A_1^{*2}, then $\widetilde{A}_1^2(0) = 1$ a s$A_1^{*2} = \widetilde{A}_1^2(0)$. According to (4.6) and (4.7), $\widetilde{A}_1^2(0) = 1$ means that $\widetilde{A}_2^{*2}(0) = 1$. So $A_2^{*2} = 1$. We let $A_2^{*2} = A_1^{*2}$ in an attempt to resume these results.
  According to expressions (4.2) and (4.4), if $A_2^{*j} = 0$ then $\pi_2^1 = -h_2\overline{X_2}$ so $s_2^{*j} = 0$. If $A_2^{*j} = 1$, then s_2^{*j} corresponds to its optimal expression provided by (4.5). We resume these results by letting: $s_3^{*j} = A_2^{*j}\widetilde{S}_2^j\left(\widetilde{\alpha}_1^j\right)$.
  Therefore $\left(A_2^{*j}, s_2^{*j}\right)$ is the best supplier-2 strategy. □

$\widetilde{\alpha}_1^j$ is computed numerically according to an iterative algorithm for each scheme-$j \in \{1,2\}$. We know that $\tilde{\alpha}_1^j$ belongs to the interval $J = [0,1] \cap]1 - \frac{\mu_2}{\lambda}, \frac{\mu_1}{\lambda}[$ which is bounded. So we constructed an iterative algorithm which tests profit values by means of a small increment and saves the optimal one. As we look for demand allocation scheme between suppliers we need not to be accurate to more than three decimals. A dichotomy search algorithm will also work.

A closer examination of (4.8) shows that several Stackelberg equilibriums may exist. This is the case when function $\pi_1^j\left(\widetilde{A}_1^j\left(\alpha_1^j\right), \widetilde{s}_1^j\left(\alpha_1^j\right), \alpha_1^j, \widetilde{A}_2^j\left(\alpha_1^j\right)\right)$achieves the same maximum for different α_1^j values. In this case, all Stackelberg equilibriums are identical from supplier-1 view point. In fact, they contribute to the same supplier-1 equilibrium profit. So supplier-1 has the liberty to choose one among them.

In addition, we note that, at least, one Stackelberg equilibrium exists. If no demand allocation policy leads to the acceptance of the offer $\left(A_1^{*j} = A_2^{*j} = 0\right)$, then, according to Proposition 4.1, the resulting Stackelberg equilibrium is $\xi^j = (0,0,0,0,0)$.

In the next two properties, we draw some comparisons between schemes-1 and 2.

Property 4.2. *If* $p_1 = p_2$ *and* $b_1 = b_2$ *then:*

- *If* $\mathbb{E}^1 = \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$ *then* $\mathbb{E}^2 = \{(0,0,0,0,0)\}$ *otherwise* $\mathbb{E}^2 = \mathbb{E}^1$.
- *If* $\mathbb{E}^2 = \{(0,0,0,0,0)\}$ *then* $\mathbb{E}^1 = \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$ *or* $\mathbb{E}^1 = \{(0,0,0,0,0)\}$ *otherwise* $\mathbb{E}^1 = \mathbb{E}^2$.

This property shows that, when $p_1 = p_2$ and $b_1 = b_2$, then scheme-1 and scheme-2 lead to the same set of Stackelberg equilibriums, except if $\mathbb{E}^1 = \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$ or if $\mathbb{E}^2 = \{(0,0,0,0,0)\}$. In fact,

$\mathbb{E}^1 = \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$ means that supplier-2 accepts the totality of demand volume despite supplier-1 refusal of the offer. This cannot occur under scheme-2 as supplier-2 cannot get any part of demand volume if supplier-1 refuses the offer. Analogously, $\mathbb{E}^2 = \{(0,0,0,0,0)\}$ doesn't imply necessarily that $\mathbb{E}^1 = \{(0,0,0,0,0)\}$ because under scheme-1, supplier-2 can accept the totality of demand volume provided that the offer is profitable (despite supplier-1 refusal).

Proof.

- $\mathbb{E}^1 = \{(0,0,0,0,0)\}$ or $\mathbb{E}^1 = \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$ mean that $\nexists\, \widetilde{\alpha}_1^1 \in \Omega^1$ leading to $\pi_1^1 \geqslant \pi_1^0$. If $p_1 = p_2$ and $b_1 = b_2$ then we notice from (4.1)–(4.4) that $\pi_2^1\left(A_2, s_2, \alpha_1, 1\right) = \pi_2^2\left(A_2, s_2, \alpha_1, 1\right) \forall \alpha_1 \in J, A_2, s_2$. Thus according to (4.6) we have $\widetilde{A}_2^1\left(\alpha_1\right) = \widetilde{A}_2^2\left(\alpha_1\right) \forall \alpha_1 \in J$. In particular $\pi_1^1\left(1, s_1, \alpha_1, \widetilde{A}_2^1(\alpha_1)\right) = \pi_1^2\left(1, s_1, \alpha_1, \widetilde{A}_2^2(\alpha_1)\right) \forall \alpha_1 \in J, s_1$. So $\nexists\, \widetilde{\alpha}_1^2 \in \Omega^2$ such that $\pi_1^2 \geqslant \pi_1^0$. Consequently $A_1^{*2} = 0$. According to Proposition 4.1, it leads to $\mathbb{E}^2 = \{(0,0,0,0,0)\}$. Suppose that $\mathbb{E}^1 \neq \{(0,0,0,0,0)\}$ and $\mathbb{E}^1 \neq \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$. By looking at Proposition 4.1, it is clear that if $A_1^{*1} = 0$ then $\mathbb{E}^1 = \{(0,0,0,0,0)\}$ or $\mathbb{E}^1 = \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$. Consequently, we have necessarily $A_1^{*1} = 1$. If $p_1 = p_2$ and $b_1 = b_2$ then we notice from (4.1)–(4.4) that $\pi_1^1\left(1, s_1, \alpha_1, A_2\right) = \pi_1^2\left(1, s_1, \alpha_1, A_2\right)$ and that $\pi_2^1\left(A_2, s_2, \alpha_1, 1\right) = \pi_2^2\left(A_2, s_2, \alpha_1, 1\right) \forall \alpha_1 \in J, s_1, A_2, s_2$. As $A_1^{*1} = 1$ then $\exists\, \widetilde{\alpha}_1^1 \in \Omega^1$ leading to $\pi_1^1 \geqslant \pi_1^0$ so $\exists\, \widetilde{\alpha}_1^2 \in \Omega^2$ leading to $\pi_1^2 \geqslant \pi_1^0$ and as a result $A_1^{*2} = 1$. It results also in $\widetilde{\alpha}_1^2 = \widetilde{\alpha}_1^1$. In addition, if $p_1 = p_2$ and $b_1 = b_2$ then $\forall\, \alpha_1 \in J \widetilde{s}_1^1\left(\alpha_1\right) = \widetilde{s}_1^2\left(\alpha_1\right), \widetilde{A}_2^1\left(\alpha_1\right) = \widetilde{A}_2^2\left(\alpha_1\right)$ and $\widetilde{s}_2^1\left(\alpha_1\right) = \widetilde{s}_2^2\left(\alpha_1\right)$. So by looking at Proposition 4.1, $\forall \xi^1 \in \mathbb{E}^1$ we have $\xi^1 \in \mathbb{E}^2$ and $\forall \xi^2 \in \mathbb{E}^2$ we have $\xi^2 \in \mathbb{E}^1$. Consequently, if $\mathbb{E}^1 \neq \{(0,0,0,0,0)\}$ and $\mathbb{E}^1 \neq \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$ then $\mathbb{E}^1 = \mathbb{E}^2$.
- $\mathbb{E}^2 = \{(0,0,0,0,0)\}$ means that $\nexists\, \widetilde{\alpha}_1^2 \in \Omega^2$ leading to $\pi_1^2 \geqslant \pi_1^0$. If $p_1 = p_2$ and $b_1 = b_2$ then we notice from (4.1) and (4.3) that $\pi_2^1\left(A_2, s_2, \alpha_1, 1\right) = \pi_2^2\left(A_2, s_2, \alpha_1, 1\right) \forall \alpha_1 \in J, A_2, s_2$. Thus according to (4.6) we have $\widetilde{A}_2^1\left(\alpha_1\right) = \widetilde{A}_2^2\left(\alpha_1\right) \forall \alpha_1 \in J$. In particular $\pi_1^1\left(1, s_1, \alpha_1, \widetilde{A}_2^1\left(\alpha_1\right)\right) = \pi_1^2\left(1, s_1, \alpha_1, \widetilde{A}_2^2\left(\alpha_1\right)\right) \forall \alpha_1 \in J, s_1$. So $\nexists\, \widetilde{\alpha}_1^1 \in \Omega^1$ such that $\pi_1^1 \geqslant \pi_1^0$. Consequently $A_1^{*1} = 0$. According to Proposition 4.1, $\mathbb{E}^1 = \{(0,0,0,0,0)\}$ if $\widetilde{A}_2^1(0) = 0$ and $\mathbb{E}^1 = \left\{\left(0,0,0,1,s_2^{*1}\right)\right\}$ otherwise. Suppose that $\mathbb{E}^2 \neq \{(0,0,0,0,0)\}$. By looking at Proposition 4.1, it is clear that if $A_1^{*2} = 0$ then $\alpha_1^{*2} = s_1^{*2} = A_2^{*2} = s_2^{*2} = 0$. Consequently, $\mathbb{E}^2 \neq \{(0,0,0,0,0)\}$ requires necessarily that $A_1^{*2} = 1$. If $p_1 = p_2$ and $b_1 = b_2$ then we notice from (4.1)–(4.4) that $\pi_1^1\left(1, s_1, \alpha_1, A_2\right) = \pi_1^2\left(1, s_1, \alpha_1, A_2\right)$ and that $\pi_2^1\left(A_2, s_2, \alpha_1, 1\right) = \pi_2^2\left(A_2, s_2, \alpha_1, 1\right) \forall \alpha_1 \in J, s_1, A_2, s_2$. As $A_1^{*2} = 1$ then $\exists\, \widetilde{\alpha}_1^2 \in \Omega^2$ leading to $\pi_1^2 \geqslant \pi_1^0$ so $\exists\, \widetilde{\alpha}_1^1 \in \Omega^1$ leading to $\pi_1^1 \geqslant \pi_1^0$ and as a result $A_1^{*1} = 1$. It results also to $\widetilde{\alpha}_1^2 = \widetilde{\alpha}_1^1$. In addition, if $p_1 = p_2$ and $b_1 = b_2$ then $\forall\, \alpha_1 \in J \widetilde{s}_1^1\left(\alpha_1\right) = \widetilde{s}_1^2\left(\alpha_1\right), \widetilde{A}_2^1\left(\alpha_1\right) = \widetilde{A}_2^2\left(\alpha_1\right)$ and $\widetilde{s}_2^1\left(\alpha_1\right) = \widetilde{s}_2^2\left(\alpha_1\right)$. So by looking at Proposition 4.1, $\forall \xi^1 \in \mathbb{E}^1$ we have $\xi^1 \in \mathbb{E}^2$ and $\forall \xi^2 \in \mathbb{E}^2$ we have $\xi^2 \in \mathbb{E}^1$. Consequently, if $\mathbb{E}^2 \neq \{(0,0,0,0,0)\}$ then $\mathbb{E}^1 = \mathbb{E}^2$.

Property 4.3 *Let* $\left(A_1^{*1}, \alpha_1^{*1}, s_1^{*1}, A_2^{*1}, s_2^{*1}\right)$ *be a Stackelberg equilibrium in the case of scheme-*1*, and* $\left(A_1^{*2}, \alpha_1^{*2}, s_1^{*2}, A_2^{*2}, s_2^{*2}\right)$ *be a Stackelberg equilibrium for scheme-*2*, then:*

- *If* $p_1 \geqslant p_2$ *and* $b_1 \leqslant b_2$ *then* $\pi_1^1\left(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}\right) \leqslant \pi_1^2\left(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}\right)$.
- *If* $p_1 \leqslant p_2$ *and* $b_1 \geqslant b_2$ *then* $\pi_1^1\left(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}\right) \geqslant \pi_1^2\left(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}\right)$.

This property shows that, if $p_1 \geqslant p_2$ and $b_1 \leqslant b_2$, then supplier-1 would rather adopt scheme-2. In fact, he takes advantage of supplier-2 lower sale price and higher unit backorder penalty. If however, $p_1 \leqslant p_2$ and $b_1 \geqslant b_2$it is better for supplier-1 to choose scheme-1. □

Proof. Let's note that $\pi_1^2(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}) - \pi_1^1(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}) = A_1^{*1} A_2^{*1}((p_1 - p_2)(1 - \alpha_1^{*1})\lambda + (b_2 - b_1)\overline{Y}_2)$. If $p_1 \geqslant p_2$ and $b_1 \leqslant b_2$ then $\pi_1^2(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}) \geqslant \pi_1^1(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1})$.

We know that $(A_1^{*2}, \alpha_1^{*2}, s_1^{*2}, A_2^{*2})$ corresponds to the best supplier-1 profit in the case of scheme-2. So, $\pi_1^2(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}) \leqslant \pi_1^2(A_1^{*2}, s_1^{*2}, \alpha_1^{*2}, A_2^{*2})$. This implies that $\pi_1^1(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}) \leqslant \pi_1^2(A_1^{*2}, s_1^{*2}, \alpha_1^{*2}, A_2^{*2})$. By the same way, we can show that $\pi_1^1(A_1^{*1}, s_1^{*1}, \alpha_1^{*1}, A_2^{*1}) \geqslant \pi_1^2(A_1^{*2}, s_1^{*2}, \alpha_1^{*2}, A_2^{*2})$ if $p_1 \leqslant p_2$ and $b_1 \geqslant b_2$. □

5. Cooperative system performances

To achieve the centralized performances with independent decision-making suppliers, there are two streams that are proposed in the literature. In the first stream, authors study competitive games (see for example [2,3,6]). To coordinate the supply chain authors design contract arrangement between actors, nevertheless, each player manage its system in a local manner. The second stream treated in literature concerns cooperative games. According to this perspective, players who decide to cooperate no longer have a competitive behavior. They act as if they belong to a same parent company (see for example [1, 16, 21]). Our paper belongs to the second category of literature.

In order to introduce the cooperation procedure that we suggest, let's consider first a fictive sub-problem where suppliers look for maximizing the sum of their profits without considering their acceptation thresholds. This sub-problem can then be formulated as follows:

$$\max_{\alpha_1, s_1, s_2} \widetilde{\pi}_c^j = \pi_1^j \left(1, s_1, \alpha_1, 1\right) + \pi_2^j \left(1, s_2, \alpha_1, 1\right) \tag{5.1}$$

$\widetilde{\pi}_c^j$ is the total profit corresponding to the fictive sub-problem described above when scheme-j is used $\forall j \in \{1,2\}$.

Let's denote by $\widetilde{s}_{ic}^j$, $i \in \{1,2\}$and $j \in \{1,2\}$, the following expressions:

$$\widetilde{s}_{1c}^j \left(\alpha_1\right) = \left\lfloor \frac{\operatorname{Log}\left(\frac{h_1}{h_1+b_1}\right)}{\operatorname{Log}\left(\rho_1^j\right)} \right\rfloor \quad \text{if } \alpha_1 \neq 0 \quad \text{and } \widetilde{s}_{1c}^j \left(0\right) = 0$$

$$\begin{cases} \widetilde{s}_{2c}^1 \left(\alpha_1\right) = \left\lfloor \dfrac{\operatorname{Log}\left(\frac{h_2}{h_2+b_2}\right)}{\operatorname{Log}\left(\rho_2\right)} \right\rfloor \quad \text{if } \alpha_1 \neq 1 \quad \text{and } \widetilde{s}_{2c}^1 \left(1\right) = 0. \\ \\ \widetilde{s}_{2c}^2 \left(\alpha_1\right) = \left\lfloor \dfrac{\operatorname{Log}\left(\frac{h_2}{h_2+b_1}\right)}{\operatorname{Log}\left(\rho_2\right)} \right\rfloor \quad \text{if } \alpha_1 \neq 1 \quad \text{and } \widetilde{s}_{2c}^2 \left(1\right) = 0. \end{cases} \tag{5.2}$$

Let $\Omega_c^j = \left\{ \widetilde{\alpha}_{1c}^j / \widetilde{\alpha}_{1c}^j \in \operatorname{Arg\,max}_{\alpha_1 \in J} \widetilde{\pi}_c^j \left(\alpha_1, \widetilde{s}_{1c}^j \left(\alpha_1\right), \widetilde{s}_{2c}^j \left(\alpha_1\right)\right) \right\}$.

Let $\tilde{E}_c^j$ be the set of optimal solutions of the sub-problem (5.1).

Proposition 5.1. $\forall j \in \{1, 2\} \tilde{E}_c^j = \left\{ \xi_c^j \left(\widetilde{\alpha}_{1c}^j\right) / \widetilde{\alpha}_{1c}^j \in \Omega_c^j \right\}$ *where*

- $\widetilde{\xi}_c^j \left(\widetilde{\alpha}_{1c}^j\right) = \left(1, \widetilde{\alpha}_{1c}^{*j}, \widetilde{s}_{1c}^{*j}, 1, \widetilde{s}_{2c}^{*j}\right)$
- $\widetilde{\alpha}_{1c}^{*j} = \widetilde{\alpha}_{1c}^j$
- $\widetilde{s}_{1c}^{*j} = \widetilde{s}_{1c}^j \left(\widetilde{\alpha}_{1c}^j\right)$
- $\widetilde{s}_{2c}^{*j} = \widetilde{s}_{2c}^j \left(\widetilde{\alpha}_{1c}^j\right)$.

Proof. For each scheme-j, expression (5.1) is formulated as follows:

$$\max \tilde{\pi}_c^1 = \left(p_1 - c_1\right) \alpha_1 \lambda + \left(p_2 - c_2\right) \left(1 - \alpha_1\right) \lambda - h_1 \overline{X}_1 - h_2 \overline{X}_2 - b_1 \overline{Y}_1 - b_2 \overline{Y}_2$$

$$\max \tilde{\pi}_c^2 = p_1 \lambda - c_1 \alpha_1 \lambda - c_2 \left(1 - \alpha_1\right) \lambda - h_1 \overline{X}_1 - h_2 \overline{X}_2 - b_1 \left(\overline{Y}_1 + \overline{Y}_2\right)$$

where $\overline{X}_i$ and $\overline{Y}_i$ are given respectively by expressions (3.1) and (3.2), $\forall i \in \{1,2\}$.

A closer examination of $\widetilde{\pi}_c^j$ $\forall j \in \{1,2\}$shows that it is concave with respect to base stock levels. When $0 < \alpha_1 < 1$, (5.2) represent the optimal base stock expressions which are obtained *via* the classical method presented by [7]. If $\alpha_1 = 0$ then it is clear that $\widetilde{s}_{1c}^j \left(0\right) = 0$. Analogously, if $\alpha_1 =$1 then $\widetilde{s}_{2c}^j \left(1\right) = 0$. As $\widetilde{\alpha}_{1c}^j \in \Omega_c^j$, it corresponds to the optimal demand allocation policy. ☐

Let's note that the optimal strategies' set is not necessarily a singleton as function $\widetilde{\pi}_c^j$ may achieve the same optimum for several different α_{1c}^j values. The different $\widetilde{\alpha}_{1c}^{*j}$ values are computed numerically according to an iterative algorithm. We denote by $\widetilde{\pi}_c^{*j}$ the optimal $\widetilde{\pi}_c^j$ value resulting from solving problem (5.1).

Remark 5.2. *Let π_i^{*j} be supplier-i profit corresponding to the Stackelberg equilibrium ξ^j then $\forall j \in \{1,2\}$ and $\forall \xi^j \in E^j$ we have: $\widetilde{\pi}_c^{*j} \geqslant \pi_1^{*j} + \pi_2^{*j}$. In fact given that $\widetilde{\pi}_c^{*j}$ is the optimal solution of (5.1), then $\widetilde{\pi}_c^{*j} \geqslant \pi_1^j(1, s_1, \alpha_1, 1) + \pi_2^j(1, s_2, \alpha_1, 1) \forall (\alpha_1, s_1, s_2)$, particularly $\widetilde{\pi}_c^{*j} \geqslant \pi_1^{*j} + \pi_2^{*j}$.*

According to Remark 5.2, if the two suppliers adopt the optimal sub-problem parameters, they will generate a total profit which is higher than the sum of their Stackeleberg equilibrium profits. However, these optimal sub-problem parameters will not be optimal for each supplier if he is considered alone. For example, supplier-1 profit corresponding to the sub-problem parameters can be less than his profit resulting from the Stackelberg game as he is the leader in this latter case. On the other hand, as we don't consider the acceptation constraint in the maximization problem (5.1), suppliers can be involved in the new product offer while their resulting profits are less than their acceptation thresholds.

In order to take advantage of the sub-problem results and to cope with the aforementioned problems, we propose a cooperation procedure. We suggest that suppliers' profits are allocated according to the equal allocation rule that divides the benefit of the collaboration equally between the two suppliers (see [16]). Other allocations are proposed in the literature (see [16]). Since in this paper our aim is not to discuss allocation rules, we considered the equal allocation rule because it is a simple and practical rule for suppliers. It is possible, however, to use other allocation rules in our model.

Let $\xi_c^j = \left(A_{1c}^j, \alpha_{1c}^j, s_{1c}^j, A_{2c}^j, s_{2c}^j\right)$ the parameters vector that suppliers have to adopt under cooperation and π_{ic}^j supplier-i corresponding profit where $i \in \{1,2\}$and $j \in \{1,2\}$. We denote by $\mathbb{E}_{\mathrm{c}}^{\mathrm{j}}$ the set of cooperative system vectors when scheme-j is adopted $j \in \{1,2\}$.

Then, $\forall \xi^j \in \mathbb{E}^{\mathrm{j}}$, we suggest that:

– If $\exists \ \widetilde{\xi}_c^j \in \ \tilde{E}_{\mathrm{c}}^{\mathrm{j}}$ such that $\widetilde{\pi}_c^{*j} \geqslant \sum_{i=1}^{2} \max\left(\pi_i^{*j}; \pi_i^0\right)$ then

$$\xi_c^j = \widetilde{\xi}_c^j \text{ and } \pi_{ic}^j\left(\xi_c^j\right) = \max\left(\pi_i^{*j}; \pi_i^0\right) + \frac{\widetilde{\pi}_c^{*j} - \sum_{i=1}^{2} \max\left(\pi_i^{*j}; \pi_i^0\right)}{2} \forall i \in \{1,\,2\}, \forall \quad j \in \{1,\,2\}. \tag{5.3}$$

– Otherwise, $\xi_c^j = \xi^j$ and $\pi_{ic}^j\left(\xi_c^j\right) = \pi_i^{*j} \forall i \in \{1,2\}, \forall \quad j \in \{1,2\}$.

As shown in the following property, this cooperation procedure allows each supplier to improve his profit and to satisfy his acceptation threshold constraint. In addition, we show through the following property that no other cooperation procedure could more improve the suppliers' profits.

Property 5.3. *Let π_i^{*j} be supplier-i profit corresponding to the Stackelberg equilibrium ξ^j under scheme-j, then $\forall i \in \{1, 2\}$, $\forall j \in \{1, 2\}$ and $\forall \xi^j \in E^j$ we have:*

– $\pi_{ic}^j\left(\xi^j\right) \geqslant \pi_i^{*j}$.
– *If $A_{ic}^j = 1$ then $\pi_{ic}^j\left(\xi^j\right) \geqslant \pi_i^0$.*

In addition, $\forall j \in \{1, 2\}$, there is no other cooperation procedure satisfying the acceptation thresholds constraint and leading to a profit $\pi_{1c}^{'j}$ for supplier-1 and $\pi_{2c}^{'j}$ for supplier-2 such that ($\pi_{1c}^{'j} > \pi_{1c}^j$ and $\pi_{2c}^{'j} > \pi_{2c}^j$) or ($\pi_{1c}^{'j} = \pi_{1c}^j$ and $\pi_{2c}^{'j} > \pi_{2c}^j$) or ($\pi_{1c}^{'j} > \pi_{1c}^j$ and $\pi_{2c}^{'j} = \pi_{2c}^j$).

Proof.

- If $\widetilde{\pi}_c^{*j} \geqslant \sum_{i=1}^{2} \max\left(\pi_i^{*j};\pi_i^0\right)$, according to expressions (5.3) we have: $\pi_{ic}^j\left(\xi^j\right) = \max\left(\pi_i^{*j};\pi_i^0\right) + \frac{\widetilde{\pi}_c^{*j} - \sum_{i=1}^{2}\max\left(\pi_i^{*j};\pi_i^0\right)}{2} \forall i \in \{1,2\}$. So $\pi_{ic}^j\left(\xi^j\right) \geqslant \pi_i^{*j} \forall i \in \{1,2\}$.
  If $\widetilde{\pi}_c^{*j} < \sum_{i=1}^{2} \max\left(\pi_i^{*j};\pi_i^0\right)$ then $\pi_{ic}^j\left(\xi^j\right) = \pi_i^{*j} \quad \forall i \in \{1,2\}$.
  Therefore, $\pi_{ic}^j\left(\xi^j\right) \geqslant \pi_i^{*j} \forall i \in \{1,2\}$, $\forall j \in \{1,2\}$.
- According to expressions (5.3) either $\pi_{ic}^j\left(\xi^j\right) = \max\left(\pi_i^{*j};\pi_i^0\right) + \frac{\widetilde{\pi}_c^{*j} - \sum_{i=1}^{2}\max\left(\pi_i^{*j};\pi_i^0\right)}{2}$ or $\pi_{ic}^j\left(\xi^j\right) = \pi_i^{*j}$. In the first case, it is clear that $\pi_{ic}^j\left(\xi^j\right) \geqslant \pi_i^0$. In the second case we have $\xi_c^j = \xi^j$, in this case $A_{ic}^j = A_i^{*j}$. So if $A_{ic}^j = 1$ then $A_i^{*j} = 1$. According to (4.6) and (4.7), $A_i^{*j} = 1$ occurs only if $\pi_i^{*j} \geqslant \pi_i^0$.
  Consequently, if $A_{ic}^j = 1$ then $\pi_{ic}^j\left(\xi^j\right) \geqslant \pi_i^0$
- If $\widetilde{\pi}_c^{*j} \geqslant \sum_{i=1}^{2} \max\left(\pi_i^{*j};\pi_i^0\right)$ then $\pi_{1c}^j + \pi_{2c}^j = \widetilde{\pi}_c^{*j}$. Yet, $\widetilde{\pi}_c^{*j}$ is the maximal value the sum of suppliers profit can ever achieve. So there is no other possible profit allocation rule that leads to better profit for supplier-1 and 2.

Let's consider now the case where $\widetilde{\pi}_c^{*j} < \sum_{i=1}^{2} \max\left(\pi_i^{*j};\pi_i^0\right)$. In this case $\pi_{1c}^j = \pi_1^{*j}$ and $\pi_{2c}^j = \pi_2^{*j}$. Suppose that there is another cooperation procedure leading to the profit $\pi_{1c}^{'j}$ for supplier-1 and $\pi_{2c}^{'j}$ for supplier-2 such that ($\pi_{1c}^{'j} > \pi_{1c}^j$ and $\pi_{2c}^{'j} > \pi_{2c}^j$). To be accepted by suppliers we need necessarily that $\pi_{1c}^{'j} \geqslant \pi_1^0$ and $\pi_{2c}^{'j} \geqslant \pi_2^0$. So $\pi_{1c}^{'j} \geqslant \max\left(\pi_1^{*j};\pi_1^0\right)$ and $\pi_{2c}^{'j} \geqslant \max\left(\pi_2^{*j};\pi_2^0\right)$. On the other hand, we know that $\pi_{1c}^{'j} + \pi_{2c}^{'j} \leqslant \widetilde{\pi}_c^{*j}$ as $\widetilde{\pi}_c^{*j}$ is the maximal profit suppliers can generate. Consequently, $\widetilde{\pi}_c^{*j} \geqslant \sum_{i=1}^{2} \max\left(\pi_i^{*j};\pi_i^0\right)$. This result is absurd as we supposed that $\widetilde{\pi}_c^{*j} < \sum_{i=1}^{2} \max\left(\pi_i^{*j};\pi_i^0\right)$.

Therefore, there is no other cooperation procedure satisfying the acceptation thresholds constraint and leading to suppliers profits satisfying ($\pi_{1c}^{'j} > \pi_{1c}^j$ and $\pi_{2c}^{'j} > \pi_{2c}^j$).

By the same way, we can easily prove that there is no other cooperation procedure satisfying the acceptation thresholds constraint and leading to suppliers' profits satisfying ($\pi_{1c}^{'j} = \pi_{1c}^j$ and $\pi_{2c}^{'j} > \pi_{2c}^j$) or ($\pi_{1c}^{'j} > \pi_{1c}^j$ and $\pi_{2c}^{'j} = \pi_{2c}^j$). □

6. Numerical study

In this section we give numerical examples to illustrate the obtained results. We consider the case of two suppliers who face the offer of a new product from a customer. Demand arrival rate λ is equal to 50 000 items/year. We suppose that demand occurs at units of pallets where a pallet contains 1000 items. We consider a production cost $c_1 = c_2 = 8$ €/item and a profit rate of 25% which leads to suppliers' sale prices $p_1 = p_2 = 10$ €/item. The unit inventory holding costs are $h_1 = h_2 = 38$ €/Pallet/week. The unit backorder costs are $b_1 = b_2 = 480$ €/Pallet/week. The acceptation profit thresholds are $\pi_1^0 = \pi_2^0 = 10$ thousand of euro (10 K€).

In Table 1 are presented suppliers' profits under both competition and cooperation settings as functions of the suppliers' production capacities. In this table, the base stocks are expressed in (Pallets), the profits are expressed in (K€).

Note for this example that schemes 1 and 2 have reached the same Stackelberg equilibrium. This confirms Property 4.2 since $p_1 = p_2$ and $b_1 = b_2$.

TABLE 1. Suppliers profits under competition and cooperation settings as functions of their production capacities.

		Case 1 $\mu_1 = 15\,000$ items/ year $\mu_2 = 52\,000$ items/ year	Case 2 $\mu_1 = \mu_2 =$ $52\,000$ items/ year	Case 3 $\mu_1 = 52\,000$ items/year $\mu_2 = 15\,000$ items/year
Scheme-j $\forall j \in \{1,2\}$	$\mathbb{E}^{\mathrm{j}} = \{(A_1^{*j}, \alpha_1^{*j}, s_1^{*j}, A_2^{*j}, s_2^{*j})\}$	$\{(0,0,0,0,0)\}$	$\{(1, 80\%, 9, 1, 1)\}$	$\{(0,0,0,0,0)\}$
	$\mathbb{E}_{\mathrm{c}}^{\mathrm{j}} = \{(A_{1c}^{j}, \alpha_{1c}^{j}, s_{1c}^{j}, A_{2c}^{j}, s_{2c}^{j})\}$	$\{(1, 18\%, 10, 1, 5)\}$	$\{(1, 50\%, 3, 1, 3)\}$	$\{(1, 18\%, 5, 1, 10)\}$
	π_1^{*j}	0	60	0
	π_{1c}^{j}	34	65	34
	π_2^{*j}	0	17	0
	π_{2c}^{j}	34	22	34

In Cases 1 and 3, the new product offer is lost under the competitive system however it is accepted if suppliers cooperate. The profit of each supplier will then increase from 0 to 34 K€. In Case 2, the new product offer will be accepted for both competition and cooperation schemes. We note that even in this case, the cooperation has improved both suppliers profits (from 60 to 65 K€ for supplier-1 and from 17 to 22 K€ for supplier-2). So, although supplier-1 is the leader, it is in his advantage to cooperate with supplier-2.

To explain this, let's analyze the curves (Fig. 1) which show the evolution of the suppliers' profits as functions of the demand allocation parameter α_1 in the three cases considered above. Note again that, since $p_1 = p_2$ and $b_1 = b_2$, the curves corresponding to schemes 1 and 2 are identical.

Let's consider first $\mu_1 = \mu_2 = 52\,000$ items/year. Under the competition case, the deal is accepted and $\alpha_1^{*j} = 80\% \; \forall \; j \in \{1, 2\}$. Note that although supplier-1 is the leader, and has enough capacity to be allocated the totality of demand volume ($\mu_1 > \lambda$), he would rather be allocated only 80% of the total demand. In fact when the production system is extremely charged the resulting inventory/backorder costs are excessive leading to a decrease in the profit value. Figure 1 shows that when cooperating, the optimal proportion of demand allocated to supplier-1 is reduced to $\alpha_{1c}^{*j} = 50\%$. In fact it leads to a decrease in the inventory/backorder costs compared to the Stackelberg equilibrium. Suppliers earn then the cooperative system profit which they share among them according to expressions (5.3).

Let's consider now $\mu_1 = 15\,000$ items/year and $\mu_2 = 52\,000$ items/year. In this case, supplier-1 production capacity is small. Consequently his profit is lower than the acceptation threshold for all possible demand allocations values α_1. On the other hand, it is unprofitable for supplier-2 to accept the totality of demand amount although he has enough capacity (see Fig. 1 where $\alpha_1 = 0$). Thus, the new product offer is lost. However, when suppliers cooperate, supplier-1 is allocated 18% of de demand volume, in return, supplier-2 shares with him the total profit. The deal is then accepted and suppliers earn the cooperative system profit that they share.

The same thing happens when $\mu_1 = 52\,000$ items/year and $\mu_2 = 15\,000$ items/year. The deal is lost under competition. Indeed, it is not profitable for supplier-1 to get the entire offer. On the other hand, supplier-2 production capacity is so small that, he refuses to be allocated any amount of the total demand volume. When suppliers cooperate, supplier-2 is allocated 18% of the demand volume; in return, supplier-1 shares with him the total profit. The deal is then accepted.

In order to highlight some comparisons between schemes 1 and 2, let's now consider some cases where $p_1 \neq p_2$. We consider the same normative example seen above ($\lambda = 50\,000$ items/year; $c_1 = c_2 8$ €/item; $h_1 = h_2 = 38$ €/Pallet/week; $b_1 = b_2 = 480$ €/Pallet/week; $\pi_1^0 = \pi_2^0 = 10$ K€ and $p_1 = 10$ €/item). We determine the profits of both suppliers 1 and 2 as functions of supplier-2 sale price p_2, before and after cooperation (see Fig. 2). In this study, we will vary $\frac{p_2}{p_1}$ in the range [0.8, 1.2]. We assume, in fact, that for higher values of p_2 ($p_2 > 1.2\, p_1$) the new product offer is no longer interesting for the customer.

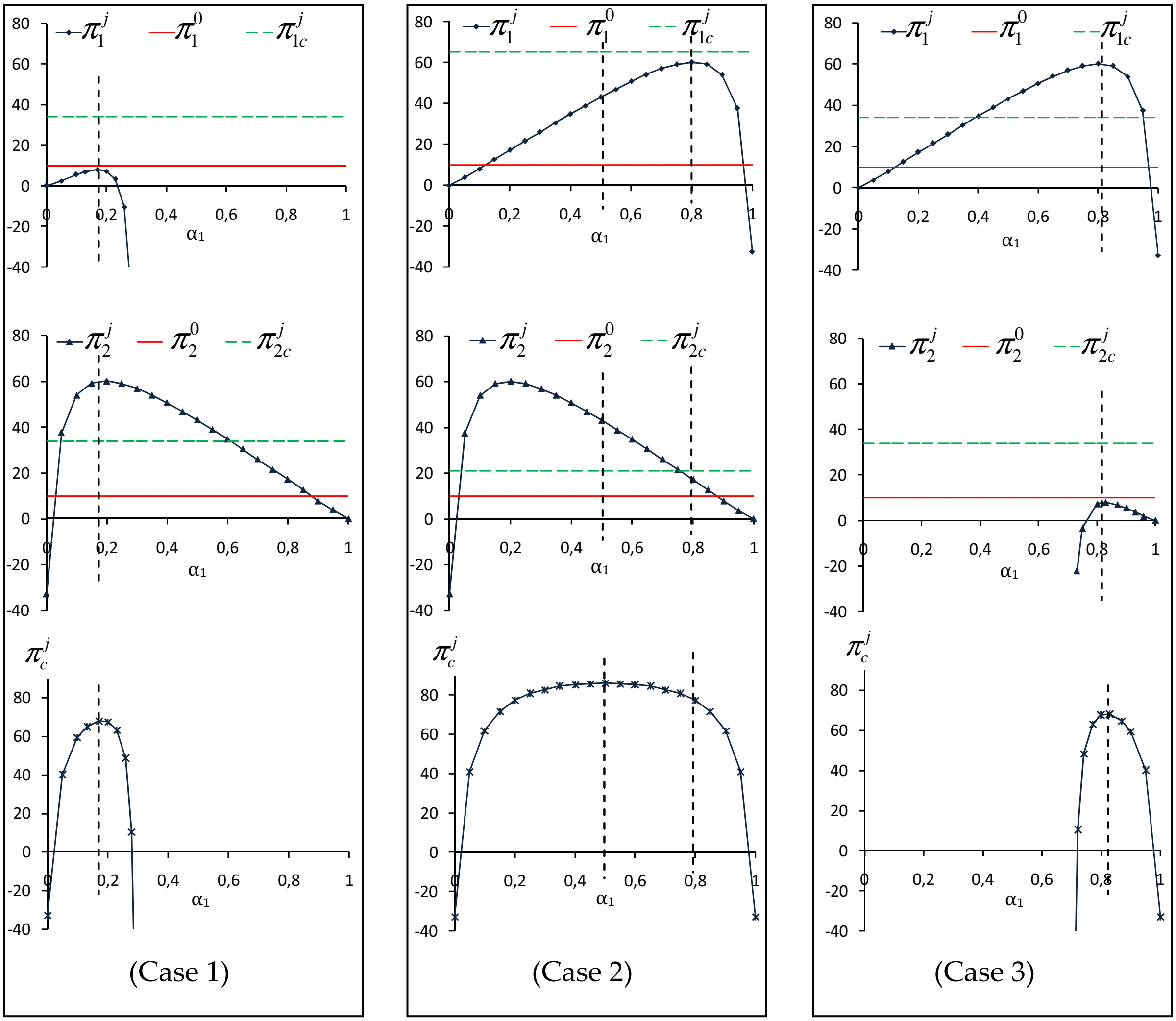

FIGURE 1. Suppliers profits and cooperative system profit (in K€) as functions of α_1: (Case 1) $\mu_1 = 15\,000$ items/year and $\mu_2 = 52\,000$ items/year, (Case 2) $\mu_1 = \mu_2 = 52\,000$ items/year, (Case 3) $\mu_1 = 52\,000$ items/year and $\mu_2 = 15\,000$ items/year.

In case 1, supplier-1 production capacity is small. On the other hand his profit does not depend on p_2 if scheme-1 (without cooperation) is used. Under this scheme, the deal is lost for all p_2 values. However, in scheme-2 supplier-1 purchases supplier-2 products and sells them at a different price. If supplier-2 chooses p_2 a little bit smaller then p_1 the deal becomes profitable for supplier-1. The new product offer is then accepted. If p_2 is very small, the deal is more profitable for supplier-1 but becomes unprofitable for supplier-2 and then again it will be lost.

Note that if supplier-2 has to choose p_2 value, he would rather choose $p_2 = 0.9p_1$. In this situation, supplier-1 will be obliged to choose scheme-2 otherwise the deal will be lost. In this case, supplier-1 profit will be 25 K€ and supplier-2 profit will be 40 K€. If suppliers decide to cooperate, their profits will be always better regardless

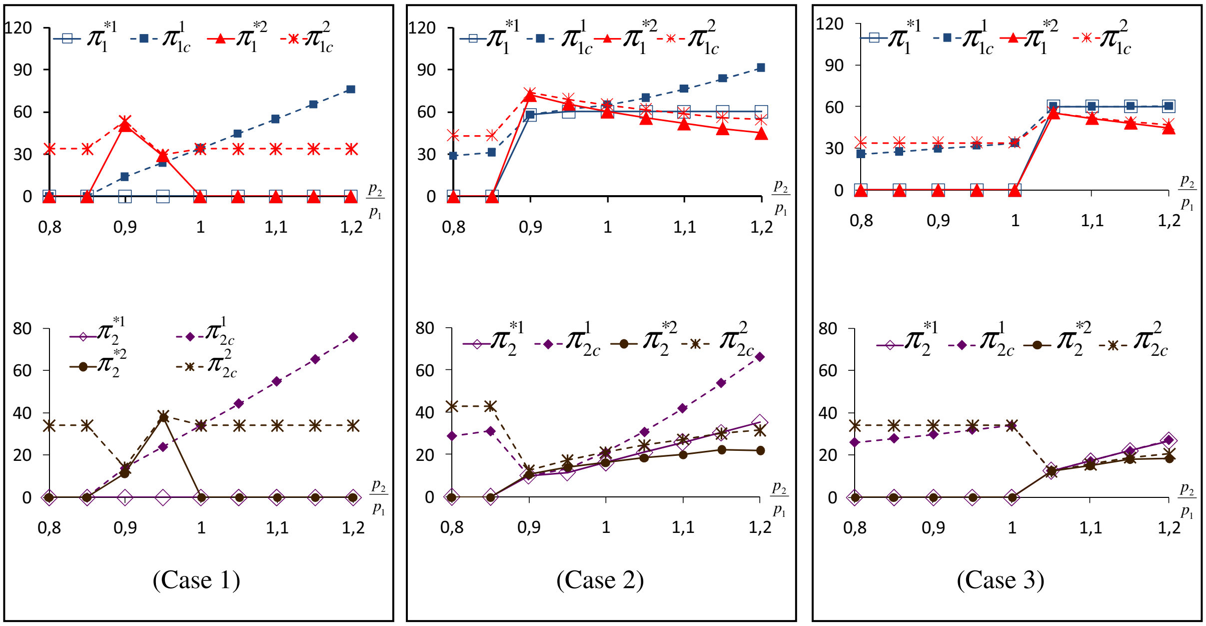

FIGURE 2. Suppliers profits (in K€) as functions of $\frac{p_2}{p_1}$: (Case 1) $\mu_1 = 15\,000$ items/year and $\mu_2 = 52\,000$ items/year, (Case 2) $\mu_1 = \mu_2 = 52\,000$ items/year, (Case 3) $\mu_1 = 52\,000$ items/year and $\mu_2 = 15\,000$ items/year.

of the adopted scheme. In this situation the best p_2 value is $p_2 = 1.2p_1$. Supplier-1 will then choose scheme-1. The profit of each suppler will be 75 K€.

In case 2, both suppliers have enough production capacities. Even without cooperation the deal will be accepted regardless the adopted scheme (except for the very small values of p_2 where the deal is unprofitable for supplier-2). When $p_2 < p_1$, supplier-1 would rather choose scheme-2. When $p_2 > p_1$, supplier-1 will prefer scheme-1. On the other hand, suppliers' profits are better after cooperation under both schemes. Note that in this case, if supplier-2 has to choose p_2 value, his best choice will be $p_2 = 1.2p_1$ in the competition and the cooperation cases. In this situation supplier-1 would rather choose scheme-1.

In case 3, it is not profitable for supplier-1 to get the entire offer. On the other hand, supplier-2 production capacity is small. Without cooperation, and for low values of p_2, supplier-2 refuses to be allocated any amount of the total demand volume. Thus, the new product offer is lost regardless the adopted scheme. However, for high values of p_2, the deal becomes profitable for supplier-2. Supplier-1 will then choose scheme-1. Note, that if supplier-2 has to choose p_2 value, he would rather choose $p_2 = 1.2\ p_1$. In this situation, supplier-1 is better off under scheme-1. In opposite, if suppliers decide to cooperate, supplier-2 would choose any value of p_2 such that $p_2 < p_1$. Consequently, supplier-1 would rather choose scheme-2.

7. Conclusion

In this paper, we investigated competitive and cooperative performances of a supply chain with two capacitated suppliers who are faced with the offer of a new product from a customer. Suppliers have the option to accept or reject the new product deal according to its profitability. In addition, they decide on their base stock levels. We studied the system from the perspectives of supplier-1 who acts as a Stackelberg leader. We considered two schemes: in the first scheme, supplier-1 gets the demand ratio he wishes to be allocated. The remaining quantity ratio is allocated to supplier-2. In the second scheme, supplier-1 decides to respond to the entire demand and to subcontract a part of it to supplier-2. Under the competitive setting, we derived analytical/numerical expressions of suppliers' decision variables. We gave conditions that allow supplier-1 to select the best scheme. We showed that supplier-1 would rather not get the whole demand amount although he has enough capacity.

We investigated the corresponding cooperative system. We showed that, the new product offer can be lost, while, it is accepted if the supply chain is managed in a cooperative manner. We presented the equal allocation rule to share the benefits of collaboration among the two suppliers. We showed that under this allocation rule, each supplier gets a better profit and that the corresponding benefit can be important.

As perspectives of our work, it would be interesting to include the customer in the game and to examine the interaction between the three actors of the defined supply chain. The case of multiple suppliers could be also another interesting extension of our model.

References

[1] S. Anily and M. Haviv, Cooperation in service systems. *Oper. Res.* **58** (2010) 660–673.

[2] Y. Arda and J.-C. Hennet, Inventory control in a decentralized two stage make to stock queueing system. *Int. J. Syst. Sci.* **39** (2008) 741–750.

[3] S. Benjaafar, E. Elahi and K.L. Donohue, Outsourcing via service competition. *Manage. Sci.* **53** (2007) 241–259.

[4] S. Benjaafar, M. ElHafsi and F. de Véricourt, Demand Allocation in Multiple-Product, Multiple-Facility. Make-to-Stock Systems. *Manage. Sci.* **50** (2004) 1431–1448.

[5] S. Benjaafar, Y. Li, D. Xu and S. Elhedhli, Demand allocation in systems with multiple inventory locations and multiple demand sources. *Manufact. Service Operat. Manage.* **10** (2008) 43–60.

[6] F. Bernstein and F. de Véricourt, Competition for procurement contracts with service guarantees. *Oper. Res.* **56** (2008) 562–575.

[7] J. Buzacott and J. Shanthikumar, *Stochastic models of manufacturing systems*, 1st edition. Printice Hall, New Jersey (1993).

[8] G. Cachon, M. Lariviere, Supply Chain Coordination with Revenue Sharing Contracts: Strengths and Limitations. *Manage. Sci.* **51** (2005) 30–44.

[9] G. Cachon, F. Zhang, Procuring Fast Delivery: Sole Sourcing with Information Asymmetry. *Manage. Sci.* **52** (2006) 881–896.

[10] G. Cachon and F. Zhang, Obtaining Fast Service in a Queueing System via Performance-Based Allocation of Demand. *Manage. Sci.* **53** (2007) 408–420.

[11] G. Cachon and P. Zipkin, Competitive and cooperative inventory schemes in a two stages supply chain. *Manage. Sci.* **45** (1999) 936–953.

[12] R. Caldentey, L. Wein, Analysis of a decentralized production − inventory system. *Manufact. Service Oper. Manage.* **5** (2003) 1–17.

[13] S. Carr and I. Duenyas, Optimal admission control and sequencing in a make-To-Stock/make-To-Order production system. *Oper. Res.* **48** (2000) 709–720.

[14] W. K. Ching, S. M.Choi and X. Huang, Inducing high service capacities in outsourcing via penalty and competition. *Int. J. Prod. Res.* **49** (2011) 5169–5182.

[15] E. Elahi, Outsourcing through competition: What is the best competition parameter? *Int. J. Prod. Econ.* **144** (2013) 370–382.

[16] A. Elomri, Z. Jemai, A. Ghaffari and Y. Dallery, Stability of Hedonic Coalition Structures: Application to a Supply Chain Game. in Applications of Multi-Criteria and Game Theory Approaches, edited by L. Benyoucef *et al. Springer Series in Advanced Manufacturing*. Springer-Verlag, London (2014) 337–363.

[17] A. Y. Ha, L. Li and S.-M. Ng, Price and Delivery Logistics Competition in a Supply Chain. *Manage. Sci.* **49** (2003) 1139–1153.

[18] Z. Jemai and F. Karaesmen, Decentralized Inventory Control in a Two-Stage Capacitated Supply Chain. *IIE Transactions* **39** (2007) 501–512.

[19] P. Kaminsky, O. Kaya, Combined Make-to-Order/Make-to-Stock Supply Chains. *IIE Transactions* **41** (2009) 103–119.

[20] M. Ruth and K. Stanley, Factors Affecting the Selection of Optimal Suppliers in Procurement Management. *Int. J. Humanit. Soc. Sci.* **14** (2012) 189–193.

[21] G. Sošiæ, Transshipment of Inventories among Retailers: Myopic vs. Farsighted Stability. *Manage. Sci.* **52** (2006) 1493–1508.

[22] Y. Yimin, S. Benjaafar and Y. Gerchak, *Capacity Sharing and Cost Allocation among Independent Firms in the Presence of Congestion*. Working paper, Department of Mechanical Engineering, University of Minnesota (2009).