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# ENVIRONMENTAL EFFICIENCY ANALYSIS BASED ON RELATIONAL TWO-STAGE DEA MODEL

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**Abstract.** Data envelopment analysis (DEA) efficiency measures integrated with the environmental DEA technology have gained popularity in environmental performance measurement of production system including undesirable outputs. Most studies treat the production system as a black box. This study measures the environmental performance considering the internal structure of production system as a two-stage process. The first stage is characterized as the production stage, and the second stage is the pollutant treatment stage. We extend the relational model from the constant returns to scale framework to the variable returns to scale version. The environmental efficiency of the entire two-stage production system for each DMU is a product of the environmental efficiencies of both stages, and a heuristic search is applied to the extended relational model. An example of industry system in some Chinese provinces shows applicability of the proposed approach. Since the model can effectively analyze a DMU's environmental efficiencies for the overall system as well as both sub-stages. It can imply more veracious decision-making information for environmental management.

**Mathematics Subject Classification.** 90B030.

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## 1. Introduction

In recent years, the growing awareness for environmental sustainability has made corporate environmental performance become the focus of public attention. The firms have recognized the fact that they can no longer concentrate solely on their economic growth while ignoring their impacts on environmental sustainability. At firm levels, it has been gradually recognized as business link to sustainable development because better environmental performance may bring managements and investors huge potential benefits (*e.g.* Anton *et al.* [1]; Labuschagne *et al.* [17]; Schmidheiny [27]). Not surprisingly, environmental performance evaluation has received significant attention in environmental science and management science (*e.g.* Brady *et al.* [5]; DeSimone and Popoff [9]; Reith and Guidry [25]). The term environmental performance has been widely advocated by decision makers and quoted by environmental policy analysts as it offers analysts and decision makers (DMs) information on environmental performance. Recently, the use of data envelopment analysis (DEA) has brought a new perspective to its study. DEA is a nonparametric approach to the efficiency evaluation of decision-making units. A main advantage of DEA is that it doesn't require any prior assumptions on the underlying functional relationships between inputs and outputs. It is a data-driven frontier analysis technique that floats a piecewise linear surface

*Keywords.* DEA, environmental efficiency, two stage, relational model.

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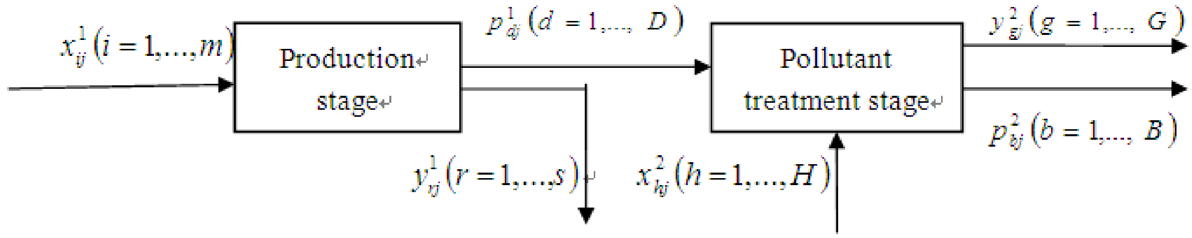


Figure 1. The production system.

to rest on top of the empirical observed quantities of the inputs and outputs (see [8]). So it can provide some advice for corporation's decision makers (DMs) to draw up a plan considering their inputs, desirable outputs and undesirable outputs based on the empirical piecewise linear function. DEA has gained popularity in environmental performance measurement.

In the existing DEA literatures, the common approaches for applying DEA to measure environmental performance are to first incorporate undesirable outputs in the traditional DEA framework, and then calculate the environmental efficiency. So far, many studies have been devoted to modeling undesirable factors in DEA, *e.g.* the data translation approach (*e.g.* Knox Lovell *et al.* [16]; Pastor [24]; Scheel [26]; Seiford and Zhu [29]), the utilization of undesirable outputs as inputs (*e.g.* Berg *et al.* [3]; Hailu and Veeman [21]; Liu and Sharp [14]; Milioni, Avellar, *et al.* [23]) and the utilization of environmental DEA technology (*e.g.* Fre *et al.* [11]; Fre *et al.* [12]; Yu [32]). Besides, some studies introduced game theory to model the undesirable outputs, for example, Gomes and Lins [13] applied the zero sum gains DEA (ZSG-DEA) models to access the performance in the presence of undesirable outputs and applied it to the evaluation of carbon dioxide emissions. Recently, Wu *et al.* [31] developed a fixed sum output DEA (FSODEA) model to evaluate the environmental efficiency of industry in China. All these methods treat each DMU as a black box and ignore the internal structure of the production system.

However, in reality, each DMU's production system is often comprised of two sub-stages. The specific form is shown in Figure 1. As shown in Figure 1, the entire production system is often comprised of the production stage and the pollutant treatment stage. In the production stage, the DMU utilizes resources (or inputs) to produce products (desirable outputs) as well as pollutants (undesirable outputs). The pollutants are then disposed in the pollutant treatment stage by using pollutant investments, the value of comprehensive utilization of the three wastes (VCU) is then treated as desirable output and the emitted pollutants are treated as undesirable outputs. To analyze the environmental performance of this kind of production system, we must consider the two-stage structure of the production system to fit the actual production well and evaluate the environmental efficiency of the production system more accurately. Recently, Bian [4] revised a non-cooperative two-stage DEA model to evaluate the environmental efficiency of regional industry systems in China, but neglected that the production stage and the pollutant treatment stage are of equally important. Ma *et al.* [30] attempted to propose a two-stage DEA approach to evaluate the environmental efficiency, but they treated the two-stage production system as two independent stages and ignored that the intermediate measures (generated pollution) must be consistent both as the outputs of stage 1 and inputs of stage 2.

This study aims to use a relational two-stage approach proposed by Kao and Hwang [15] to measure the environmental performance of Chinese regional industry systems, which considers the consistence of the intermediate measures both as outputs of stage 1 and inputs of stage 2, as well as the equal importance of these two stages. And, as the constant returns to scale (CRS) assumption is valid only when all DMUs are operating at an optimal scale. And the Chinese regional industry systems differ in size [33], the presumption that all regional industry systems under evaluation are already operating at an optimal scale may not be relevant. Thus, the application of variable returns to scale (VRS) setting will be more appropriate in environmental efficiency evaluation of regional industry systems. Therefore, this study also extends relational two-stage model from the constant returns to scale (CRS) to variable returns to scale (VRS). In this manner, when evaluating the environmental performance, we not only consider the two-stage structure of the DMUs but also consider the difference in the size of the DMUs. As the relational model is extended under the VRS framework, nonlinearity emerges. This nonlinearity problem is solved by a heuristic search as in Liang *et al.* [20] and Li *et al.* [18].

The rest of this study is as follows. In the next section, models for measuring environmental efficiency scores of the entire two-stage production system as well as the two individual stages are presented. Section 3 applies the new approach to industry systems in some Chinese provinces in 2009. Conclusions follow in Section 4.

## 2. Two-stage environmental efficiency methodology

Consider a two-stage production process shown in Figure 1. Suppose there are $n$ DMUs and denote each DMU as $DMU_j(j = 1, 2, \ldots, n)$. Each DMU uses inputs $x^1_{ij}(i = 1, \ldots, m)$ (such as employees, fixed assets, and electricity) to produce desirable outputs $y^1_{rj}(r = 1, \ldots, s)$ (such as GDP of the second industry) and undesirable outputs $p^1_{dj}(d = 1, \ldots, D)$ (such as COD generation, SO2 generation, and solid waste generation) in the production stage. The undesirable outputs of the production stage $p^1_{dj}(d = 1, \ldots, D)$ are referred to as intermediate measures, and are used as the inputs for the pollutant treatment stage. The pollutant treatment stage also has its exogenous inputs $x^2_{hj}(h = 1, \ldots, H)$ (such as pollutant-treatment investments). The desirable outputs of the pollutant treatment stage are denoted as $y^2_{gj}(g = 1, \ldots, G)$ (such as value of comprehensive utilization of the three wastes (VCU)) and the undesirable outputs are denoted as $p^2_{bj}(b = 1, \ldots, B)$ (such as COD emission, SO2 emission, and solid waste emission).

When treating the DMU as a black box, the inputs of $DMU_j(j = 1, 2, \ldots, n)$ are $x^1_{ij}(i = 1, \ldots, m)$ and $x^2_{hj}(h = 1, \ldots, H)$, the desirable outputs are $y^1_{rj}(r = 1, \ldots, s)$ and $y^2_{gj}(g = 1, \ldots, G)$, and the undesirable outputs are $p^2_{bj}(b = 1, \ldots, B)$. As the pollutants (undesirable outputs) are the by-products of the desirable outputs, we expect them as little as possible when evaluating the environmental efficiency. To do this, we follow a translation method to address the undesirable outputs proposed by Seiford and Zhu[28] as follows: each undesirable output is first multiplied by −1 and an appropriate translation vector $\upsilon^2$ is then added to the negative undesirable outputs to make them positive. That is, $\bar{p}^2_{bj} = -p^2_{bj} + \upsilon^2_b$, $b \in B$, which could be achieved by choosing $\upsilon^2_b = \max_j\{p^2_{bj}\} + 1$, $b \in B$. In this manner, the larger the $p^2_{bj}(b = 1, \ldots, B)$, the smaller the $\bar{p}^2_{bj}$. In the VRS setting, this transformation provides the identical efficient frontier. Thus, the black-box environmental efficiency of $DMU_k$ under evaluation can be obtained by applying BCC model (see Banker *et al.* [2]) as follows:

$$
\begin{aligned}
E_k^{BCC} &= \max \frac{\sum_{r=1}^{s}\mu_r y^1_{rk} + \sum_{g=1}^{G}\pi_g y^2_{gk} + \sum_{b=1}^{B}\xi_b \bar{p}^2_{bk} + u_k}{\sum_{i=1}^{m}\nu_i x^1_{ik} + \sum_{h=1}^{H} Q_h x^2_{hk}} \\
s.t.&\frac{\sum_{r=1}^{s}\mu_r y^1_{rj} + \sum_{g=1}^{G}\pi_g y^2_{gj} + \sum_{b=1}^{B}\xi_b \bar{p}^2_{bj} + u_k}{\sum_{i=1}^{m}\nu_i x^1_{ij} + \sum_{h=1}^{H} Q_h x^2_{hj}} \le 1 \\
&\mu_r, \upsilon_i, \pi_g, \xi_b, Q_h \ge 0, \forall r, i, g, b, h \\
&u_k free
\end{aligned}
\tag{2.1}
$$

where $\nu_i$ and $\mu_r$ are unknown non-negative weights attaching to inputs and desirable outputs of the production stage, respectively. $Q_h$, $\pi_g$ and $\xi_b$ are unknown non-negative weights attaching to inputs, desirable outputs and undesirable outputs of the pollutant treatment stage. Denote the optimal objective function value of model (2.1) as $e_k^{BCC*}$, which is the black-box environmental efficiency of $DMU_k$ while ignoring the internal two-stage network structure of the DMU. Therefore, it is difficult to identify the environmental efficiencies for both of the production stage and pollutant treatment stage.

To solve this kind of environmental efficiency evaluation problem, this study measures the environmental performance of a DMU in a two-stage process. In previous two-stage DEA methods, to obtain the environmental efficiency of the entire two-stage process, there are two common manners to combine the efficiencies of individual stages: weight additive manner and multiplicative manner. But in the additive model, the efficiency score of an individual stage would have less impact on the efficiency score of the entire two-stage production system as the efficiency score of the entire two-stage production system is not only determined by the efficiencies of both stages but also the weights to each stage. Besides, the additive model biases the efficiency assessments as it is in favor of the second stage [10]. Therefore, we adopt the multiplicative manner in this study and define the environmental efficiency of the entire two-stage production system as a product of the two stages environmental efficiencies, namely $e_k = e_k^1 * e_k^2$ as the relational model proposed by Kao *et al.* [15]. The model from the constant returns to scale (CRS) framework to the variable returns to scale (VRS) version is as follows:

$$
\begin{aligned}
e_k &= \max e_k^1 * e_k^2 \\
&= \max \frac{\sum_{r=1}^{s} \mu_r y_{rk}^1 + \sum_{d=1}^{D} w_d \bar{p}_{dk}^1 + u_k^1}{\sum_{i=1}^{m} \nu_i x_{ik}^1} * \frac{\sum_{g=1}^{G} \pi_g y_{gk}^2 + \sum_{b=1}^{B} \xi_b \bar{p}_{bk}^2 + u_k^2}{\sum_{d=1}^{D} w_d \bar{p}_{dk}^1 + \sum_{h=1}^{H} Q_h x_{hk}^2} \\
s.t.&\frac{\sum_{r=1}^{s} \mu_r y_{rj}^1 + \sum_{d=1}^{D} w_d \bar{p}_{dj}^1 + u_k^1}{\sum_{i=1}^{m} \nu_i x_{ij}^1} \leq 1, \forall j \\
&\frac{\sum_{g=1}^{G} \pi_g y_{gj}^2 + \sum_{b=1}^{B} \xi_b \bar{p}_{bj}^2 + u_k^2}{\sum_{d=1}^{D} w_d \bar{p}_{dj}^1 + \sum_{h=1}^{H} Q_h x_{hj}^2} \leq 1, \forall j \\
&\mu_r, \upsilon_i, \pi_g, \xi_b, Q_h, w_d \geq 0, \forall r, i, g, b, h \\
&u_k^1, u_k^1 free.
\end{aligned}
\tag{2.2}
$$

The first two constraints ensure the environmental efficiency scores of individual stages do not excess one. The objective function expresses the environmental efficiency of the entire two-stage production system as a product of the individual stages' environmental efficiencies. And the weight attaching to the intermediate measure is the same regardless of it is viewed as the output of the production stage or the input of the pollutant treatment stage according to Kao *et al.* [2] and Liang *et al.* [19]. This assumption is important because it links the two stages and represents the serial relationship between the two stages (Chen *et al.* [7]. Moreover, the intermediate measures $p_{dj}^1$ $(d = 1, \ldots, D)$ (such as COD generation, SO2 generation and Solid waste generation) are pollutants, so we expect them as little as possible both as outputs of the first stage and inputs of the second stage. Thus, these intermediate measures serve as undesirable outputs of the production stage as well as the undesirable inputs of the pollutant treatment stage according to Liu *et al.* [22]. Therefore, we expect them as little as possible whether they serve as undesirable outputs or undesirable inputs. We transform them as follows: $\bar{p}_{dj}^1 = -p_{dj}^1 + v_d^1$, $d = 1, \ldots, D$, which could be achieved by choosing $v_d^1 = \max_j\{p_{dj}^1\} + 1$, $d = 1, \ldots, D$. Similarly, the undesirable outputs of the pollutant treatment stage $p_{bj}^2 (b = 1, \ldots, B)$ could also be transformed as $\bar{p}_{bj}^2 = -p_{bj}^2 + v_b^2$, $b = 1, \ldots, B$ following the method above. That is, $\bar{p}_{bj}^2 = -p_{bj}^2 + v_b^2$, $b \in B$, which could be achieved by choosing $v_b^2 = \max_j\{p_{bj}^2\} + 1$, $b \in B$.

Because model (2.2) is a fractional programming and the product of free variables and the exogenous inputs of pollutant treatment stage exist in the model. It is hard to transform it into a linear model. To facilitate the linearization of this model, the heuristic search procedure of Liang *et al.* [20] and Li *et al.* [18] is applied. Consequently, the following model is considered:

$$
\begin{aligned}
e^1_{k\max} = \max & \frac{\sum_{r=1}^{s}\mu_r y^1_{rk} + \sum_{d=1}^{D} w_d \bar{p}^1_{dk} + u^1_k}{\sum_{i=1}^{m}\nu_i x^1_{ik}} \\
s.t. & \frac{\sum_{r=1}^{s}\mu_r y^1_{rj} + \sum_{d=1}^{D} w_d \bar{p}^1_{dj} + u^1_k}{\sum_{i=1}^{m}\nu_i x^1_{ij}} \leq 1, \forall j \\
& \frac{\sum_{g=1}^{G}\pi_g y^2_{gj} + \sum_{b=1}^{B}\xi_b \bar{p}^2_{bj} + u^2_k}{\sum_{d=1}^{D} w_d \bar{p}^1_{dj} + \sum_{h=1}^{H} Q_h x^2_{hj}} \leq 1, \forall j \\
& \mu_r, \upsilon_i, \pi_g, \xi_b, Q_h, w_d \geq 0, \forall r, i, g, b, h \\
& u^1_k, u^1_k free.
\end{aligned} \tag{2.3}
$$

Denote the optimal objective function value of model (2.3) as $e^1_{k\max}$, then the possible maximum environmental efficiency score of the production stage is $e^1_{k\max}$. Thus, the environmental efficiency score of the production stage $e^1_k$ would be determined in the interval of $[0, e^1_{k\max}]$. Model (2.3) is a fractional programming, but it can be converted into a linear model via the Charnes-Cooper (C-C) transformation. Then model (2.3) is equivalent to the following linear programming model.

$$
\begin{aligned}
e^1_{k\max} = \max & \sum_{r=1}^{s}\mu_r y^1_{rk} + \sum_{d=1}^{D} w_d \bar{p}^1_{dk} + u^1_k \\
s.t. & \sum_{i=1}^{m}\nu_i x^1_{ik} = 1 \\
& \sum_{r=1}^{s}\mu_r y^1_{rj} + \sum_{d=1}^{D} w_d \bar{p}^1_{dj} + u^1_k - \sum_{i=1}^{m}\nu_i x^1_{ij} \leq 0, \forall j \\
& \sum_{g=1}^{G}\pi_g y^2_{gj} + \sum_{b=1}^{B}\xi_b \bar{p}^2_{bj} + u^2_k - \sum_{d=1}^{D} w_d \bar{p}^1_{dj} - \sum_{h=1}^{H} Q_h x^2_{hj} \leq 0, \forall j \\
& \mu_r, \upsilon_i, \pi_g, \xi_b, Q_h, w_d \geq 0, \forall r, i, g, b, h \\
& u^1_k, u^1_k free.
\end{aligned} \tag{2.4}
$$

Since $e_k^1 \in [0, e_{k\max}^1]$, the expression of $e_k^1$ can be considered as a variable in measuring the environmental efficiency score of the entire two-stage production system. Therefore, model (2.4) can be rewritten as:

$$\begin{aligned}
e_k &= \max e_k^1 * e_k^2 \\
&= \max e_k^1 * \frac{\sum\limits_{g=1}^{G} \pi_g y_{gk}^2 + \sum\limits_{b=1}^{B} \xi_b \bar{p}_{bk}^2 + u_k^2}{\sum\limits_{d=1}^{D} w_d \bar{p}_{dk}^1 + \sum\limits_{h=1}^{H} Q_h x_{hk}^2} \\
s.t. & \frac{\sum\limits_{r=1}^{s} \mu_r y_{rj}^1 + \sum\limits_{d=1}^{D} w_d \bar{p}_{dj}^1 + u_k^1}{\sum\limits_{i=1}^{m} \nu_i x_{ij}^1} \leq 1, \forall j \\
& \frac{\sum\limits_{g=1}^{G} \pi_g y_{gj}^2 + \sum\limits_{b=1}^{B} \xi_b \bar{p}_{bj}^2 + u_k^2}{\sum\limits_{d=1}^{D} w_d \bar{p}_{dj}^1 + \sum\limits_{h=1}^{H} Q_h x_{hj}^2} \leq 1, \forall j \\
& e_k^1 \in [0, e_{k\max}^1] \\
& \mu_r, \upsilon_i, \pi_g, \xi_b, Q_h, w_d \geq 0, \forall r, i, g, b, h, d \\
& u_k^1, u_k^1 free.
\end{aligned} \tag{2.5}$$

Model (2.5) can be converted into model (2.6) through the C-C transformation:

$$\begin{aligned}
e_k &= \max e_k^1 * e_k^2 \\
&= \max e_k^1 * (\sum_{g=1}^{G} \pi_g y_{gj}^2 + \sum_{b=1}^{B} \xi_b \bar{p}_{bj}^2 + u_k^2) \\
s.t. & \sum_{d=1}^{D} w_d \bar{p}_{dj}^1 + \sum_{h=1}^{H} Q_h x_{hj}^2 = 1 \\
& \sum_{r=1}^{s} \mu_r y_{rj}^1 + \sum_{d=1}^{D} w_d \bar{p}_{dj}^1 + u_k^1 - \sum_{i=1}^{m} \nu_i x_{ij}^1 \leq 0, \forall j \\
& \sum_{g=1}^{G} \pi_g y_{gj}^2 + \sum_{b=1}^{B} \xi_b \bar{p}_{bj}^2 + u_k^2 - \sum_{d=1}^{D} w_d \bar{p}_{dj}^1 - \sum_{h=1}^{H} Q_h x_{hj}^2 \leq 0, \forall j \\
& \sum_{r=1}^{s} \mu_r y_{rk}^1 + \sum_{d=1}^{D} w_d \bar{p}_{dk}^1 + u_k^1 - e_k^1 \sum_{i=1}^{m} \nu_i x_{ik}^1 = 0 \\
& \mu_r, \upsilon_i, \pi_g, \xi_b, Q_h, w_d, \forall r, i, g, b, h \\
& e_k^1 \in [0, e_{k\max}^1] \\
& \mu_r, \upsilon_i, \pi_g, \xi_b, Q_h, w_d, \forall r, i, g, b, h, d \\
& u_k^1, u_k^1 free.
\end{aligned} \tag{2.6}$$

In order to calculate the optimal solution of model (2.6), we set $e_k^1 = e_{k\max}^1 - t\varepsilon$. Here $\varepsilon$ is a step size for the heuristic search procedure[2], and $t = 0, 1, 2, \ldots, [t_{\max}] + 1$, where $[t_{\max}]$ is the maximum integer of $e_{k\max}^1 / \varepsilon$.

[2]The smaller the $\varepsilon$ value we select, the more precise results we obtain.

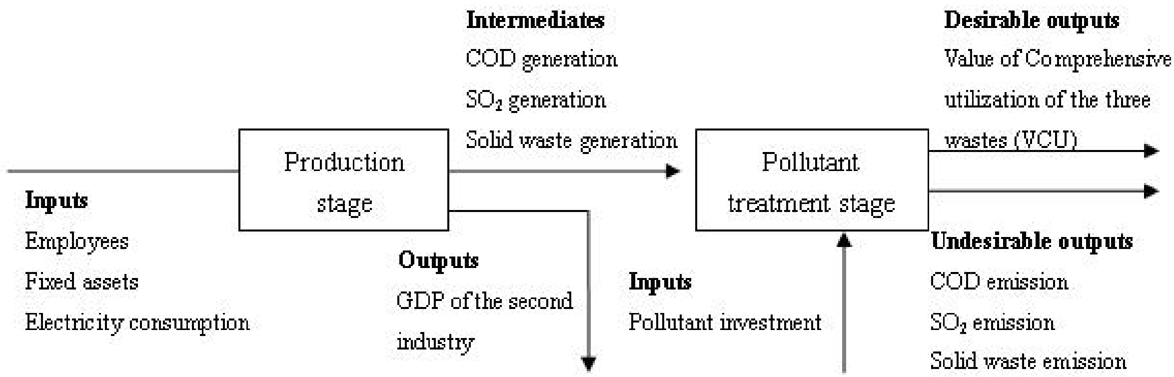


FIGURE 2. Internal structure of regional industry systems in China.

In solving model (2.6), we increase t from the initial value 0 to $[t_{\max}]+1$ with the step size $\varepsilon$. Thus for each t, a given $e_k^1(t)$ is obtained and then model (2.6) can be solved as a linear programming. Denote the optimal objective function value of model (2.6) corresponding to each t as $e_k^{v1}(t)$. Then, the global optimal solution of the entire two-stage production system can be obtained as $e_k^{v1*} = \mathrm{Max}_t e_k^{v1}(t)$. Therefore, the maximum environmental efficiency score of the entire two-stage production system is $e_k^{v1*}$ when the environmental efficiency of the production stage is considered as a variable.

When the two-stage production system obtains its maximum environmental efficiency as $e_k^{v1*}$, the maximum environmental efficiency score of the production stage is $\bar{e}_k^1 = e_k^1(t^*) = e_{k\max}^1 - t^*\varepsilon$, where $t^* = \mathrm{Min}\{t|e_k^{v1*} = e_k^{v1}(t)\}$. As a result, the corresponding minimum environmental efficiency score of the pollutant treatment stage is $\underline{e}_k^2 = e_k^{v1*}/\bar{e}_k^1$ and we have $e_k^{v1*} = \underline{e}_k^1 * \bar{e}_k^2$.

Similarly, we can also treat the environmental efficiency of pollutant treatment stage as a variable. The optimal environmental efficiency of the pollutant treatment stage $e_{k\max}^2$ can be calculated using a model similar to model (2.2). Then, according to the above-mentioned algorithm, we can get the global environmental efficiency $e_k^{v2*}$ and its corresponding maximal environmental efficiency of the pollutant treatment stage $\bar{e}_k^2$, respectively. And then the minimal environmental efficiency of the production stage is $\underline{e}_k^1 = e_k^{v2*}/\bar{e}_k^2$.

**Theorem 2.1.** *For each DMU, it would be $e_k^{v1*} = e_k^{v2*}$, where $e_k^{v1*}$ and $e_k^{v2*}$ are optimal environmental efficiency scores of the entire two-stage production system when the environmental efficiency of the production stage and the environmental efficiency of the pollutant treatment stage are considered as a variable, respectively.*

*Proof.* If the environmental efficiency of the production stage or the environmental efficiency of the pollutant treatment stage is considered as a variable, the optimal environmental efficiency score of the same two-stage production system is unique (Li *et al.* [18]). Thus, we have $e_k^{v1*} = e_k^{v2*}$. □

## 3. ILLUSTRATIONS

In this section, we will analyze the environmental efficiency of industry systems in some Chinese provinces in 2009 by using our relational two-stage DEA approach. Each industry system consists of two processes arranged in series: production stage and pollutant treatment stage as shown in Figure 2.

We try to follow existing work, such as Bian [4] and Ma *et al.* [30], for variables selection of the two-stage production system of Chinese regional industry system. In the production stage, we select the total amount of employees, fixed assets, and electricity used in the secondary industry as the inputs. Gross domestic product (GDP) can reflect the production stage more intuitively, so we select GDP as the desirable output of the

TABLE 1. Descriptive statistics of raw data.

| Variable | Unit | Min | Max | Average | Std.Dev |
|---|---|---|---|---|---|
| Employees | 10 000 persons | 2030.9 | 19.38 | 672.32 | 108.72 |
| Fixed assets | 1 billion Yuan | 10 191.19 | 111.75 | 3014.43 | 445.40 |
| Electricity consumption | 100 million kWh | 3609.64 | 20.41 | 1180.58 | 162.26 |
| COD generation | 10 000 tons | 247.75 | 0.12 | 56.81 | 10.25 |
| SO2 generation | 10 000 tons | 409.94 | 0.1 | 153.41 | 18.37 |
| Solid waste generation | 10 000 tons | 21 975.81 | 11 | 6578.82 | 937.25 |
| GDP of the second industry | 1 billion Yuan | 18 091.56 | 39.73 | 5080.78 | 855.58 |
| Pollutant investment | 1 billion Yuan | 515 831.6 | 3562.5 | 147 540.2 | 19 884.71 |
| VCU | 10 000 Yuan | 2 513 210 | 239 | 518 788.4 | 104 116.7 |
| COD emission | 10 000 tons | 51.88 | 0.11 | 14.18 | 2.00 |
| SO2 emission | 10 000 tons | 136.62 | 0.1 | 60.19 | 6.71 |
| Solid waste emission | 10 000 tons | 7261.1 | 1.47 | 1532.22 | 344.33 |

TABLE 2. Efficiency tendency of Guangxi province (DMU 21) based on model (2.6).

| $t$ | $e_k^1(t) = e_{k\max}^1 - t * 0.001$ | $e_k^{v1}(t)$ |
|---|---|---|
| 0 | 0.6322 | 0.3525 (global optimal efficiency) |
| 1–10 | 0.6222–0.5322 | 0.3525–0.2967 |
| 11–20 | 0.5222–0.4322 | 0.2912–0.2410 |
| 21–30 | 0.4222–0.3322 | 0.2354–0.1852 |
| 31–40 | 0.3222–0.2322 | 0.1796–0.1295 |
| 41–50 | 0.2222–0.1322 | 0.1239–0.0737 |
| 51–60 | 0.1222–0.0322 | 0.0681–0.0179 |
| 61–70 | 0.0222–0.0000 | 0.0124–0.0012 |

industry. And COD, SO2, and solid waste are harmful to humans health, so we select generated COD, SO2, and solid waste as undesirable outputs. Also, the generated COD, SO2, and solid waste are disposed in the pollutant treatment stage, so we select them as re-input indicators. Besides, in the pollutant treatment stage, pollutant investment is chosen as exogenous inputs, the value of comprehensive utilization of the three wastes is chosen as desirable output and the emitted COD, SO2, and solid waste are chosen as undesirable outputs. And the data set is obtained from China Statistical Yearbook 2010 published by China National Bureau of statistics in 2010.

The description of the inputs and outputs of industry system in each province are shown in Table 1. It shows that the data are heterogeneous. For example, the amount of electricity consumption ranges from 20.41 to 3609.64, and the standard deviation is 162.2626. So the effect of scale on the environmental efficiency score should be considered. The VRS assumption should be imposed on the model.

To illustrate the proposed computation procedure in estimating the global optimal environmental efficiency of each province's industry system, a heuristic search is conducted. Consider Guangxi Province (DMU 20). Its maximum environmental efficiency score of the production stage is $e_{k\max}^1 = 0.6830$ according to model (2.2). Let $e_k^1 = e_{k\max}^1 - t\varepsilon$ , $t = 0, 1, 2, \ldots, [t_{\max}] + 1$, and set the step size to $\varepsilon = 0.01$. The value of t changes from the initial 0 to the maximum of 64 because $[t_{\max}] + 1 = [e_{k\max}^1/\varepsilon] + 1$. Table 2 shows the optimal objective function value of model (2.6) for Guangxi Province (DMU 20) corresponding to each $t$. The global optimal environmental efficiency of Guangxi Province (DMU 21) is $e_k^{v1*} = 0.3525$ at $t = 0$.

Figure 3 shows the change of the optimal environmental efficiency score of Guangxi province (DMU 20) according to model (2.6) that corresponds to each $t$. It could be seen that the optimal environmental efficiency decreases monotonously as $t$ increases from 0 to 64. The entire two-stage production system obtains its global

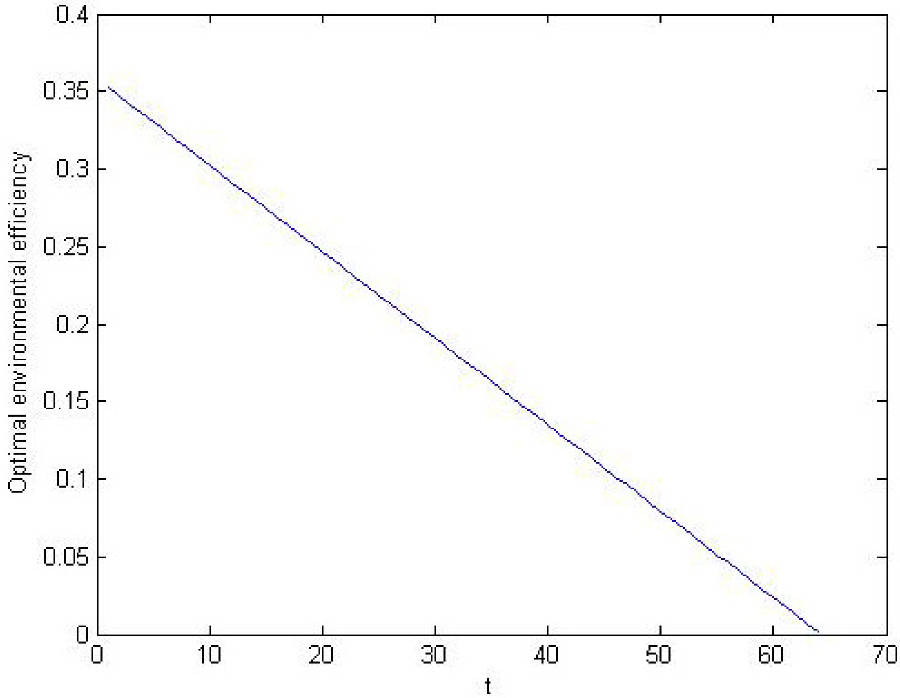


FIGURE 3. Efficiency changes of Guangxi province (DMU 21) based on model (6).

optimal environmental efficiency score when $t = 0$. Therefore, the maximum environmental efficiency score of Guangxi province (DMU 21) is $e_k^{v1*} = 0.3525$ when $t = 0$.

In Table 3, we document the black-box environmental efficiency scores and the environmental efficiency scores based on our proposed relational two-stage DEA models. The black-box environmental efficiency $e_k^{BCC}$ in the column 9 of Table 3 is calculated via model (2.1), which treats the DMU as a black box. Based on extended relational models and a step size of $\varepsilon = 0.00001$, the global optimal environmental efficiency score of the entire two-stage production system and the related environmental efficiency scores of the individual stages are shown in columns 3 to 8. With the environmental efficiency of the production stage considered as a variable, the corresponding maximum environmental efficiency score of this stage $\bar{e}_k^1$, the minimum environmental efficiency score of the pollutant treatment stage $\underline{e}_k^2$, and the global optimal environmental efficiency of the entire two-stage production system $e_k^{v1*}$ are presented in columns 3 to 5. Similarly, considering the environmental efficiency of the pollutant treatment stage as a variable, we can obtain the minimum environmental efficiency score of the production stage $\underline{e}_k^1$, the maximum environmental efficiency score of the pollutant treatment stage $\bar{e}_k^2$, and the global optimal environmental efficiency score $e_k^{v2*}$ are shown in columns 6 to 8 of Table 3. The environmental efficiency scores of the entire two-stage production systems for all provinces' industry systems accord with Theorem 2.1 as the environmental efficiency score of the entire two-stage production system for each province's industry system is unique when either the environmental efficiency of the production stage or that of the pollutant treatment stage is considered as a variable.

Moreover, the discriminating power of the relational two-stage model is stronger than that of the black-box model. As shown in Table 3, 14 DMUs are efficient when the province's industry system is treated as a black box while only one province's industry system is efficient using the proposed relational two-stage DEA model. It is due to the fact that the two-stage DEA model can identify more sources of inefficiency than the conventional BCC model.

TABLE 3. Environmental efficiency results of all regions.

| DMU | Region | Stage 1 as a variable | | | Stage 2 as a variable | | | |
|---|---|---|---|---|---|---|---|---|
| | | $e_k^{v1*}$ | $\bar{e}_k^1$ | $\underline{e}_k^2$ | $e_k^{v2*}$ | $\underline{e}_k^1$ | $\bar{e}_k^2$ | $e_k^{BCC}$ |
| 1 | Beijing | 0.2154 | 1.0000 | 0.2154 | 0.2154 | 1.0000 | 0.2154 | 1.0000 |
| 2 | Tianjin | 0.2282 | 1.0000 | 0.2282 | 0.2282 | 1.0000 | 0.2282 | 1.0000 |
| 3 | Hebei | 0.3261 | 0.5893 | 0.5533 | 0.3261 | 0.5893 | 0.5533 | 0.8502 |
| 4 | Liaoning | 0.2057 | 0.5648 | 0.3642 | 0.2057 | 0.5648 | 0.3642 | 0.7391 |
| 5 | Shanghai | 0.1992 | 1.0000 | 0.1992 | 0.1992 | 1.0000 | 0.1992 | 1.0000 |
| 6 | Jiangsu | 0.1875 | 0.7139 | 0.2627 | 0.1875 | 0.7139 | 0.2627 | 0.8578 |
| 7 | Zhejiang | 0.4131 | 0.9297 | 0.4443 | 0.4131 | 0.9297 | 0.4443 | 1.0000 |
| 8 | Fujian | 0.2717 | 0.8838 | 0.3075 | 0.2717 | 0.8838 | 0.3075 | 0.9684 |
| 9 | Guangdong | 0.2051 | 1.0000 | 0.2051 | 0.2051 | 1.0000 | 0.2051 | 1.0000 |
| 10 | Hainan | 0.4670 | 0.8125 | 0.5748 | 0.4670 | 0.8125 | 0.5748 | 1.0000 |
| 11 | Shandong | 0.7900 | 0.7900 | 1.0000 | 0.7900 | 0.7900 | 1.0000 | 1.0000 |
| 12 | Shanxi | 0.3355 | 0.6496 | 0.5165 | 0.3355 | 0.6496 | 0.5165 | 0.8075 |
| 13 | Inner Mongolia | 0.3366 | 0.8331 | 0.4040 | 0.3366 | 0.8331 | 0.4040 | 0.9039 |
| 14 | Jilin | 0.7755 | 0.8155 | 0.9510 | 0.7755 | 0.8155 | 0.9510 | 1.0000 |
| 15 | Heilongjiang | 0.3682 | 0.9432 | 0.3904 | 0.3682 | 0.9432 | 0.3904 | 1.0000 |
| 16 | Anhui | 0.2685 | 0.7581 | 0.3542 | 0.2685 | 0.7581 | 0.3542 | 0.9163 |
| 17 | Jiangxi | 0.3651 | 0.7760 | 0.4704 | 0.3651 | 0.7760 | 0.4704 | 0.8688 |
| 18 | Henan | 0.4559 | 0.7864 | 0.5797 | 0.4559 | 0.7864 | 0.5797 | 0.9342 |
| 19 | Hubei | 0.1786 | 1.0000 | 0.1786 | 0.1786 | 1.0000 | 0.1786 | 1.0000 |
| 20 | Hunan | 0.3525 | 0.6322 | 0.5576 | 0.3525 | 0.6322 | 0.5576 | 0.7171 |
| 21 | Guangxi | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 22 | Sichuan | 0.3862 | 0.9078 | 0.4254 | 0.3862 | 0.9078 | 0.4254 | 1.0000 |
| 23 | Guizhou | 0.3512 | 0.6969 | 0.5039 | 0.3512 | 0.6969 | 0.5039 | 0.8564 |
| 24 | Yunnan | 0.2525 | 0.4260 | 0.5928 | 0.2525 | 0.4260 | 0.5928 | 0.5820 |
| 25 | Shaanxi | 0.5016 | 0.5016 | 1.0000 | 0.5016 | 0.5016 | 1.0000 | 1.0000 |
| 26 | Gansu | 0.2078 | 0.7997 | 0.2598 | 0.2078 | 0.7997 | 0.2598 | 0.8163 |
| 27 | Qinghai | 0.4098 | 0.4803 | 0.8533 | 0.4098 | 0.4803 | 0.8533 | 0.8624 |
| 28 | Ningxia | 0.5788 | 0.8860 | 0.6533 | 0.5788 | 0.8860 | 0.6533 | 0.8941 |
| 29 | Xinjiang | 0.5554 | 0.6709 | 0.8278 | 0.5554 | 0.6709 | 0.8278 | 0.8426 |
| 30 | Chongqing | 0.3887 | 0.8526 | 0.4559 | 0.3887 | 0.8526 | 0.4559 | 1.0000 |

Directions for improving the environmental performance of each province's industry system can also be identified. As shown in Figure 4, the horizontal and vertical axes of the efficiency matrix represent the environmental efficiencies of the production stage and the pollutant treatment stage, respectively. Each province is located in the matrix. Using both average environmental efficiencies of the production stage and the pollutant treatment stage, we can divide the efficiency matrix into four sub-matrices. Based on the location of the province in the four sub-matrices, policy implications for improving environmental performance can be proposed. The provinces in the first quadrant have high environmental efficiencies in both stages, while the provinces in the third quadrant have low environmental efficiencies in both stages. But the provinces in the second quadrant have high environmental efficiencies in the pollutant treatment stage but low environmental efficiencies in the production stage. It is converse for the provinces in the fourth quadrant which have low environmental efficiencies in the pollutant treatment stage but high environmental efficiencies in the production stage. For example, Tianjin (DMU 2) is in the fourth quadrant, which means that its industry system has a high environmental efficiency

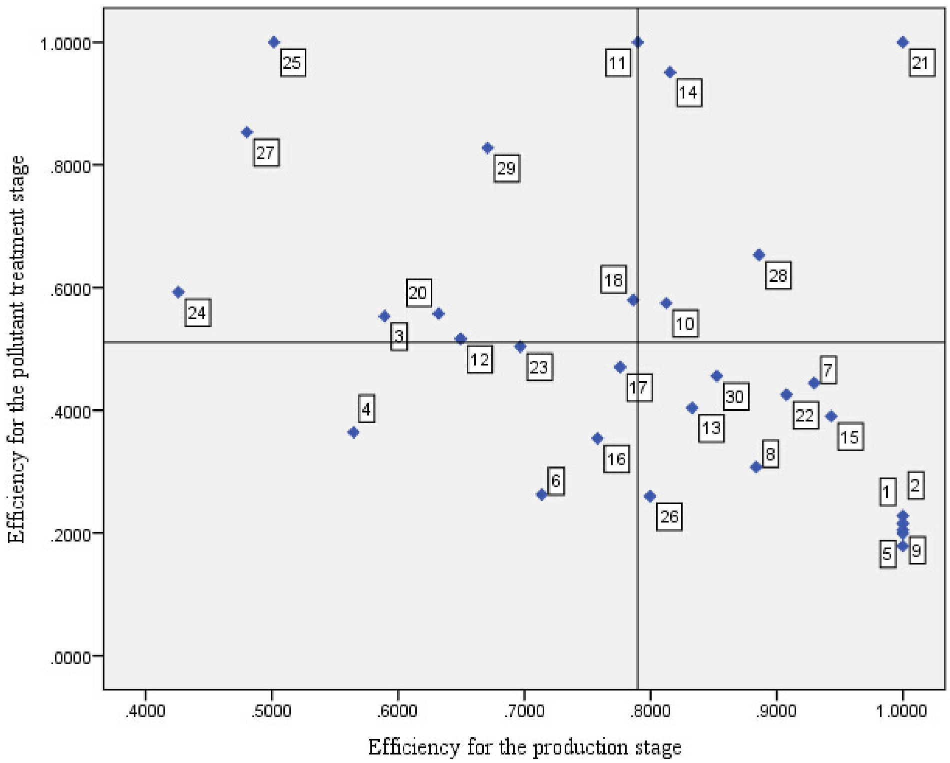


FIGURE 4. Plot of environmental efficiencies for both stages.

of 1 in the production stage but a low environmental efficiency of 0.2282 in the pollutant treatment stage. So it is advisable for Tianjin (DMU 2) to improve its environmental performance in the entire industry system by exerting considerable effort in pollutant treatment stage.

## 4. CONCLUSION

The environmental performance evaluation of the industry systems of Chinese provinces is a classical two-stage network process. Hoverer, in previous studies on the environmental efficiency analysis, the production system is treated as a black box ignoring the internal structure. This study measures the environmental performance of environmental efficiency considering the internal structure of production system as a two-stage network process. The first stage is characterized as production stage, and the second stage is the pollutant treatment stage. Variable return to scale is assumed. A heuristic search is used to estimate the environmental efficiency score of the entire two-stage production system because of nonlinearity in the extended relational model. The case of the industry system in some Chinese provinces is given using this newly developed approach. Its results are compared with those of the traditional BCC model with undesirable outputs. It is found that the environmental efficiency values of our approach are lower than those of the traditional BCC model with undesirable outputs, because the former opens the black box of the production system and pays more attention to the influence of the environment. It provides more accurate decision-making information for environmental management.

In current paper, a radial relational two-stage DEA model is applied. Recently, the non-radial DEA model SBM model has been applied to the environmental performance measurement (*e.g.* Zhou *et al.* [34]; Zhou *et al.* [35]; Chang *et al.* [6]. Thus, developing a non-radial network DEA model (*e.g.* network SBM model) may be a possible direction of future studies.

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