RAIRO-Oper. Res. 50 (2016) 611–625
DOI: 10.1051/ro/2015054

RAIRO Operations Research
www.rairo-ro.org

STRATEGIC INVESTMENTS IN R&D AND EFFICIENCY IN THE PRESENCE OF FREE RIDERS *

Mouna Ben Brahim[1], Georges Zaccour[2] and Fouad Ben Abdelaziz[3]

Abstract. We consider an industry composed of two types of firms, namely, innovators that invest in process research and development (R&D), and surfers that do not but benefit from knowledge spillover. We verify if the conclusions reached in the seminal paper by d'Aspremont and Jacquemin hold in this setting. We obtain that cooperation among innovators still lead to higher R&D and output levels than when they do not cooperate. Our main result is that the presence of surfers in an industry can be welfare improving under some conditions.

Mathematics Subject Classification. 91A10, 91A20.

Received April 10, 2015. Accepted November 12, 2015.

1. Introduction

In their seminal paper, d'Aspremont and Jacquemin [4] showed that cooperation in research and development (R&D) leads to higher investment levels in R&D than does noncooperation, when the (exogenous) knowledge spillover between firms is high enough (*i.e.*, larger or equal to one half). They also established that R&D cooperation is the most socially efficient arrangement. Our objective is to verify if these results still hold when some firms in the industry do not invest in R&D at all, but benefit to some extent from the R&D done by the other firms. It is an empirical fact that not all firms in an industry are active in R&D. As an illustrative example, we note that less than one in five companies invested in R&D in [15] 2007 in the Canadian manufacturing sector, which is the sector with the highest percentage of firms investing in R&D.[4]

Following d'Aspremont and Jacquemin [4], a series of subsequent studies investigated whether or not these results still hold under different assumptions related to the types of R&D and spillover (exogenous or endogenous, free or costly). Kamien *et al.* [12] considered various R&D cooperation scenarios, and obtained that a cartelized research joint venture (RJV), that is, a situation where all firms share a single laboratory, yields the best performance in terms of R&D investments, consumer surplus as well as producer surplus. The results in [4, 12]

Keywords. Investments in R&D, cooperation, noncooperation, Spillover, free riders, social Welfare.

* *The authors would like to thank an anonymous reviewer for helpful comments.*

1 École Supérieure de Tunis, Tunisia.
2 Chair in Game Theory and Management, GERAD, HEC Montréal, Canada.
3 NEOMA Business School, Mont Saint-Aignan, France. fouad.ben.abdelaziz@neoma-bs.fr

4 See http://www.impactg.com/pdf/corporaterdanalysis.pdf.

Article published by EDP Sciences

are based on a comparison of each scenario's symmetric equilibrium. In contrast, Amir and Wooders [13] obtained that noncooperation in R&D may result in higher profits than does cooperation in an asymmetric equilibrium. Suzumura [20] established that, for a certain general demand function, neither competitive nor cooperative R&D equilibria are socially efficient. Amir *et al.* [3] discussed different forms of cost functions for process R&D, and showed, among many other things, that d'Aspremont and Jacquemin's results still hold under a convex cost function that includes a fixed cost component.

Kaiser [10] introduced an initial stage where firms decide whether to cooperate or not. When spillovers are high, firms invest more under cooperation than under competition. Using an oligopoly model with differentiated products, Hinloopen [10] showed that (i) R&D cooperation is socially profitable if there is no market collusion, and (ii) that collusion increases investments and total surplus when research is complementary. Salant and Shaffer [17] considered asymmetric R&D investments and demonstrated that, even when there is no spillover, RJV increases social welfare. Silipo [18] looked at the factors affecting the incentives to cooperate in R&D, and the effects of cooperation on the incentives to innovate. One result is that R&D uncertainty and spillovers constitute the main factors leading to cooperation, and a second result is that RJV is the most preferred R&D agreement.

Other authors have considered endogenous spillovers. Katsouolacos and Ulph [13] is one of the early attemps and obtained the result that noncooperation can lead to maximal spillovers. Kamien and Zang [11] introduced absorptive capacity into d'Aspremont and Jacquemin's R&D production function. Absorptive capacity depends on own firm's R&D and on its R&D approach strategy. Firms adopt purely specific R&D strategies when they compete to offset spillovers, and broad strategies when they cooperate to maximize knowledge flows. Leahy and Neary [14] studied the effect of R&D on absorptive capacity and on a firm's profitability. They showed that R&D investments increase absorptive capacity but decrease the incentive to cooperate. Hammerschmidt [8] distinguished between two kinds of R&D investments: production-cost-reducing R&D and absorptive-capacity-improving R&D. She established that, when spillovers are high, firms invest more to improve their absorptive capacity. Martin [16] considered input and output spillovers, or appropriabilty, and dealt with an uncertain innovation process. The author established that social welfare is maximized when input spillovers are high and appropriability is low. Silipo and Weiss [19] studied R&D cooperation with spillovers and uncertainty and distinguished between incremental and offsetting spillovers. If spillovers are offsetting, then competition is preferred to cooperation, and the reverse if spillovers are incremental. Shravan (2005) retained a dynamic model where a laggard firm can spillover on the leader and characterized equilibrium strategies in this context. Ben Youssef *et al.* [6] considered the case where firms can invest in both innovative and absorptive R&D to reduce their unit production cost, and where they benefit from free spillovers between them. The authors obtained that investment in innovative R&D is always the highest and that the efficiency of investment in absorptive research has almost no impact on the equilibrium solution.

To the best of our knowledge, Ceccagnoli [7] and Ben Abdelaziz *et al.* [5] are the only studies to consider within this literature an heterogenous industry, with some firms investing in R&D and others not, but they did not look into the conditions under which the innovating firms formed a research joint venture. Ceccagnoli [7] analyzed the impact of the knowledge spillover to non-innovating firms, on the incentives of innovating firms to continue their cost-reducing R&D effort. Ben Abdelaziz *et al.* [5] obtained that the presence of non-innovating firms in an industry leads, at equilibrium, to lower individual investments in R&D, to a lower collective level of knowledge and to a higher product price.

We consider the case of cooperation among firms active in R&D, and compare it to the noncooperative case. When surfers are present, we obtain that cooperation among innovators still leads to higher R&D investment (by innovator) and market outputs. Our main result is that surfers presence in an industry could enhance R&D investment level and welfare in some region of the parameter space.

The rest of the paper is organized as follows: in Section 2, we introduce the model and determine equilibrium strategies in the two scenarios, namely, when firms cooperate and when they do not in setting their R&D investments. Section 3 discusses social welfare implications of having surfers in an industry, and Section 4 briefly concludes.

2. The Model

The model is an extension of [4, 5]. The difference with d'Aspremont and Jacquemin [4] is the assumption that some firms do not engage in R&D, and the difference with Ben Abdelaziz *et al.* [5] is in the form of the R&D cost function, that is, quadratic instead of linear.

Denote by $\mathcal{N} = \{1, \ldots, N\}$ the set of firms producing a homogenous product. We distinguish between two types of firms: a subset $\mathcal{I}$ of firms that invest in R&D, or the *innovators*, and a subset $\mathcal{S}$ of firms that do not carry out any research activity, called the *surfers*. Denote by x_i the investment in R&D of firm $i \in \mathcal{I}$, and by q_i,the output of firm $i \in \mathcal{N}$. Following the tradition initiated by d'Aspremont and Jacquemin [4], we model the decision process as a two-stage game. In stage 1, the innovators choose their investment levels, and in stage 2, all firms select their output levels. The innovators may cooperate or act noncooperatively when making their R&D decisions in the first stage. The mode of play is noncooperative in the second stage.

We make the classical assumption that knowledge cannot be fully appropriated. Denote by β the exogenous spillover rate between innovators, and by θ the rate of knowledge that spills over to the surfers, with $0 \leq \theta < \beta \leq 1$. This ranking of parameters can be justified by the fact that, in the absence of any research effort, a surfer cannot absorb as much knowledge as an innovator. Note that the special case where $\beta = 1$ corresponds to research joint venture (RJV), which will be also considered. Denote by X_k the total level of knowledge available to firm $k \in \mathcal{N}$, that is,

$$X_k = \begin{cases} x_k + \beta \sum_{j \in \mathcal{I}, j \neq k} x_j, & k \in \mathcal{I} \\ \theta \sum_{j \in \mathcal{I}} x_j, & k \in \mathcal{S}. \end{cases} \tag{2.1}$$

We suppose that the unit production cost decreases with the level of knowledge. To keep things simple while still being able to make our point, we suppose as in [4] that the unit production cost can be well approximated by the following linear function:

$$F_k(X_k) = A - X_k,$$

where $A > 0$ is the initial cost. The above cost function can be written equivalently as

$$F_k(\bar{x}) = \begin{cases} A - \left(x_k + \beta \sum_{j \in \mathcal{I}, j \neq k} x_j \right), & k \in \mathcal{I} \\ A - \theta \sum_{j \in \mathcal{I}} x_j, & k \in \mathcal{S}, \end{cases} \tag{2.2}$$

where $\bar{x} = (x_i)_{i \in \mathcal{I}}$. We assume that $F_k(\bar{x})$ is positive in all computed solutions.[5]

We consider a linear inverse demand function $p = a - Q$, where Q is the total market-output level given by $Q = \sum_{i \in \mathcal{N}} q_i$, and $a > A$. The R&D cost is quadratic and given by $\frac{\gamma}{2} x_k^2$, with $\gamma > 0$. The profit function of firm k reads as follows:

$$\pi_k = \begin{cases} \left(a - \sum_{j \in \mathcal{N}} q_j - F_k(\bar{x}) \right) q_k - \frac{\gamma}{2} x_k^2, & k \in \mathcal{I} \\ \left(a - \sum_{j \in \mathcal{N}} q_j - F_k(\bar{x}) \right) q_k, & k \in \mathcal{S}. \end{cases} \tag{2.3}$$

2.1. Equilibria

We analyze the two scenarios considered in the literature, namely, that firms (innovators) compete or cooperate in the R&D stage. To derive a subgame-perfect Nash equilibrium, we solve the two-stage game backward.

[5] An alternative to this assumption is to suppose that the cost is given by an hyperbolic function, which is intrinsically positive for all values of total knowledge. However, taking this route prevents us from making direct comparisons with the literature, which is a main objective of this paper.

2.2. Noncooperation in R&D

Suppose that the innovators decide on their R&D investment levels noncooperatively and maximize their individual profits. The following proposition characterizes the resulting equilibrium, where the superscript n stands for *n*oncooperation in R&D.

Proposition 2.1. *Assuming an interior solution, the unique symmetric subgame-perfect equilibrium is given by*

$$\begin{aligned} x^n &= \frac{2(a-A)Z}{\gamma(N+1)^2 - 2ZY}, \\ q_i^n &= (a-A)\,\frac{\gamma(N+1)}{\gamma(N+1)^2 - 2ZY}, \qquad i \in \mathcal{I}, \\ q_s^n &= (a-A)\left(\frac{\gamma(N+1) - 2ZW}{\gamma(N+1)^2 - 2ZY}\right), \quad s \in \mathcal{S}, \end{aligned}$$

where

$$\begin{aligned} Y &= (S+1)(1-\beta) + I((\beta-\theta)S + \beta) > 0, \\ Z &= N - \beta I - \theta S + \beta > 0, \\ W &= I(\beta-\theta) + 1 - \beta > 0, \end{aligned}$$

with I being the number of innovators and S, the number of surfers.

Proof. See Appendix. □

The following corollary states the result for the case where the firms compete, while seeking maximum spillover in knowledge ($\beta = 1$). This corresponds to a competitive RJV in the terminology of [12].

Corollary 2.2. *Under RJV competition, the unique symmetric subgame-perfect equilibrium is given by*

$$\begin{aligned} \tilde{x}^n &= \frac{2(a-A)\tilde{Z}}{\gamma(N+1)^2 - 2\tilde{Z}\tilde{Y}}, \\ \tilde{q}_i^n &= \frac{(a-A)}{N+1}\,\frac{\gamma(N+1)^2}{\gamma(N+1)^2 - 2\tilde{Z}\tilde{Y}}, \qquad i \in \mathcal{I}, \\ \tilde{q}_s^n &= (a-A)\left[\frac{\gamma(N+1) - 2\tilde{Z}\tilde{W}}{\gamma(N+1)^2 - 2\tilde{Z}\tilde{Y}}\right], \quad s \in \mathcal{S}, \end{aligned}$$

where

$$\begin{aligned} \tilde{Y} &= I((1-\theta)S + 1) > 0, \\ \tilde{Z} &= S\,(1-\theta) + 1 > 0, \\ \tilde{W} &= I(1-\theta) > 0. \end{aligned}$$

Proof. It suffices to set $\beta = 1$ in the proof of Proposition 2.1 to get the results. □

2.3. Cooperation in R&D

Assume now that the innovators cooperate in the first stage, *i.e.*, they select the R&D levels that maximize their joint profit. The following proposition characterizes the resulting equilibrium, where the superscript c stands for *c*ooperation in R&D.

Proposition 2.3. *Assuming an interior solution, the unique symmetric subgame-perfect equilibrium when the innovators cooperate in R&D is given by*

$$x^c = \frac{2(a-A)Y}{\gamma(N+1)^2 - 2Y^2} \quad i \in \mathcal{I},$$
$$q_i^c = (a-A)\left(\frac{\gamma(N+1)}{\gamma(N+1)^2 - 2Y^2}\right), \quad i \in \mathcal{I}$$
$$q_s^c = (a-A)\left[\frac{\gamma(N+1) - 2WY}{\gamma(N+1)^2 - 2Y^2}\right], \quad s \in \mathcal{S}.$$

The profits are given by

$$\pi_i^c = \frac{(Y-2Z)(N+1)^2 + 4Z^2(a-A)}{4YZ^2}, \quad i \in \mathcal{I}, \tag{2.4}$$
$$\pi_s^c = \frac{\left(Y(N+1) - W\left((N+1)^2 - 2Z(a-A)\right)\right)^2}{4Y^2Z^2}, \quad s \in \mathcal{S}, \tag{2.5}$$

Proof. See Appendix. □

The following corollary states the results in the scenario of a cooperative RJV.

Corollary 2.4. *When innovators form a research joint venture,* i.e., $\beta = 1$, *the unique symmetric subgame-perfect equilibrium when the innovators cooperate in the first stage is given by*

$$\tilde{x}^c = \frac{2(\tilde{Z}+\tilde{T})(a-A)}{\gamma(N+1)^2 - 2(\tilde{Z}+\tilde{T})\tilde{Y}} \quad i \in \mathcal{I},$$
$$\tilde{q}_i^c = (a-A)\left(\frac{\gamma(N+1)}{\gamma(N+1)^2 - 2(\tilde{Z}+\tilde{T})\tilde{Y}}\right), \quad i \in \mathcal{I}$$
$$\tilde{q}_s^c = (a-A)\left[\frac{\gamma(N+1) - 2\left(\tilde{Z}+\tilde{T}\right)W}{\gamma(N+1)^2 - 2\left(\tilde{Z}+\tilde{T}\right)\tilde{Y}}\right], \quad s \in \mathcal{S},$$

where

$$\tilde{T} = (I-1)(S(1-\theta)+1) > 0.$$

The profits are given by

$$\pi_i^c = \frac{(Y-2(Z+T))(N+1)^2 + 4(Z+T)^2(a-A)}{4Y(Z+T)^2}, \quad s \in \mathcal{I}, \tag{2.6}$$
$$\pi_s^c = \frac{\left(Y(N+1) - W\left((N+1)^2 - 2(Z+T)(a-A)\right)\right)^2}{4Y^2(Z+T)^2}, \quad s \in \mathcal{S}. \tag{2.7}$$

Proof. It suffices to set $\beta = 1$ in Proposition 2.3 to get the results. □

The above two propositions are stated under the assumption that the solution is interior. Now, we derive the conditions on parameter values needed for that. In the noncooperative scenario, the following two conditions are required:

$$x^n \text{ and } q_i^n > 0 \Leftrightarrow \gamma(N+1)^2 - 2ZY > 0,$$
$$q_s^n > 0 \Leftrightarrow \gamma(N+1) - 2ZW > 0,$$

which can be combined into

$$Z < \frac{\gamma(N+1)}{2} \cdot \min\left(\frac{N+1}{Y}, \frac{1}{W}\right) \cdot \tag{2.8}$$

In the cooperative scenario, we first recall that

$$Y = (S+1)(1-\beta) + \beta I + (\beta - \theta) IS > 0,$$

and consequently, we have

$$\begin{aligned} x^c \text{ and } q_i^c > 0 &\Leftrightarrow \gamma(N+1)^2 - 2Y^2 > 0, \\ q_s^c > 0 &\Leftrightarrow \gamma(N+1) - 2WY > 0. \end{aligned}$$

The above two conditions can then be compacted as follows:

$$Y < \frac{\gamma(N+1)}{2} \min\left(\frac{(N+1)}{Y}, \frac{1}{W}\right) \cdot \tag{2.9}$$

As

$$\min\left(\frac{(N+1)}{Y}, \frac{1}{W}\right) = \frac{1}{W},$$

then (2.8)–(2.9) become

$$\max(Z, Y) < \frac{\gamma(N+1)}{2W} \cdot \tag{2.10}$$

Note that the sign of the difference

$$Y - Z \triangleq T = (I-1)(S(\beta - \theta) + 2\beta - 1),$$

depends on the parameter values. In the absence of surfers, the condition for having $T \geq 0$ reduces to $\beta \geq 1/2$.

2.4. Comparison

In D&J [4], it was shown that if the spillover parameter β is large enough, then firms invest more in R&D when they cooperate than when they set noncooperatively their R&D expenditures. More specifically, they obtain that if $\beta \geq 1/2$, then $x^c \geq x^n$. The following proposition states a similar result in our scenario, where surfers are present in the industry.

Proposition 2.5. *If the intra innovators knowledge spillover is sufficiently high,* i.e., $\beta \geq \beta^S = \frac{1+\theta S}{S+2}$, *then each innovator invests more in R&D when firms cooperate than when they do not. Moreover, if the knowledge spillover to surfers is sufficiently low,* i.e., $\theta \leq 1/2$, *then the interval under which* $x^c > x^n$ *is larger when surfers are present than when they are absent, that is,*

$$\theta \leq 1/2 \Rightarrow [1/2, 1] \subseteq \left[\beta^S, 1\right] \cdot$$

Proof. The difference in R&D investments is given by:

$$x^c - x^n = \frac{2(a-A)T\gamma(N+1)^2}{\left[\gamma(N+1)^2 - 2(Z+T)Y\right]\left[\gamma(N+1)^2 - 2ZY\right]}.$$

As the denominator is positive by the assumption of interior solution and $a > A$, we conclude that

$$\operatorname{sign}(x^c - x^n) = \operatorname{sign} T,$$

which is equivalent to the statement

$$x^c - x^n \geq 0 \Leftrightarrow T \geq 0 \Leftrightarrow \beta \geq \frac{1+\theta S}{S+2} = \beta^S.$$

Finally, we have

$$\beta^S - \frac{1}{2} = \frac{(2\theta - 1)\, S}{2\,(S+2)},$$

which is less or equal to zero for $\theta \leq 1/2$. □

Note that for $S = 0$, we recover D&J [4] result. The main message from the above proposition is that the presence of surfers in the market does not change qualitatively the result in D&J [4], that is, the ordering of investment levels is the same, but induces a quantitative change in the threshold.

In order to assess the impact of surfers on investments in R&D under cooperation and noncooperation, respectively, we compare our results to those in D&J [4].

Proposition 2.6. *Independently on the mode of play (noncooperative or cooperative) for setting R&D investments, a sufficient condition for having a higher investment by an innovator when surfers are present is* $\beta < 1/2$.

Proof. See Appendix. □

The result shows that for low spillover, innovators tend to invest more when there are surfers than when there are not. One explanation of this result is as follows: first, recall that the unit production cost decreases with the total knowledge produced in the industry. In the D&J model, as well in all others that followed, there are N firms participating in this effort; here there are I innovators, with $I < N$. Consequently, for any given desired level of knowledge, each innovator has to do more in our setting to reach it. Second, this increase will make sense only if the firm can still appropriate a large share of the benefit of R&D, which occurs when the spillover is low.

3. Social Welfare analysis

In this section we do two things. First, we determine the socially efficient R&D investments and outputs when surfers are present, and second, we compare welfare under the two cases, that is, with and without surfers.

Proposition 3.1. *The socially efficient R&D investment* x^* *and total output* Q^* *in the presence of surfers are given by*

$$x^* = \frac{N(a-A)\,(S\theta + 1 + \beta(I-1))}{N^2\gamma - I\,(S\theta + 1 + \beta(I-1))^2}, \tag{3.1}$$

$$Q^* = a - A + (S\theta + 1 + \beta\,(I-1))\,\frac{Ix^*}{N}\cdot \tag{3.2}$$

Proof. See the Appendix. □

Note that for $N = I = 2$ and $S = 0$, we recover the same expression for efficient output in D&J [4].

Proposition 3.2. *Optimal socially efficient R&D investment and output are always lower when there are surfers in the market than when there are not.*

Proof. See the Appendix. □

Proposition 3.3. *If*

$$\gamma \in \left[\frac{I\left(S\theta+1+\beta\left(I-1\right)\right)^{2}}{N^{2}},\ \frac{N\left(1+\beta\left(N-1\right)\right)^{2}}{N^{2}}\right], \tag{3.3}$$

then the presence of surfers in the industry is welfare improving.

Proof. See the Appendix. □

We make two observations on the above interval. First, the higher the number of innovators in the industry, the narrower the interval in (3.3). Indeed, the derivative of the lower bound with respect to I is clearly positive. Second, when innovators form an RJV ($\beta = 1$), the expression of the interval given above simplifies to

$$\gamma \in \left[\frac{I\left(S\theta+I\right)^{2}}{N^{2}},\ N\right].$$

Clearly, the larger the spillover of knowledge to surfers, the narrower the interval where their presence is welfare improving.

From the proof of Proposition 3.1, we have that the consumer surplus is given by $CS = \frac{1}{2}Q^2$. As the total output is always higher in the absence of surfers, any loss in total welfare is then due to a decrease in producer's surplus. The innovators profits at the social optimum are given by

$$\Pi^{*} = p^{*}Q^{*} - c_{k}Q^{*} - \gamma I\frac{x^{*2}}{2}, \tag{3.4}$$

where c_k is the unit industry production cost is given by

$$c_{k} = A - \left(\frac{(S\theta+1+\beta(I-1))I}{N}\right)x^{*}.$$

Denote by $K = \frac{(S\theta+1+\beta(I-1))I}{N}$, then $c_k = A - Kx^*$. The socially optimal output and price are given by

$$Q^{*} = a - A + Kx^{*} \Leftrightarrow Kx^{*} = Q^{*} - (a-A),$$
$$p^{*} = a - Q^{*}.$$

Substituting in (3.4), we get

$$\Pi = \left((a-Q^{*}) - (A-Kx^{*})\right)Q^{*} - \gamma I\frac{x^{*2}}{2} = -\gamma I\frac{x^{*2}}{2},$$

which clearly shows that the social profit is negative.

The policy implication is that a benevolent regulator would need to compensate innovators for their losses if he wants to implement the socially optimal solution; otherwise, innovating firms would be better off not investing in R&D.

4. Conclusion

We generalize D&J model introducing free riders in the industry. Computing cooperative and noncooperative equilibria, we confirm earlier results that for sufficiently high spillovers, cooperation dominates competition. Comparing R&D investment levels with and without surfers, we establish that under competition in R&D, both for low spillover β and for high spillovers β (intra innovators) and θ (to surfers), innovators invest more when there are surfes than when there are not. Important spillovers encourage innovators to invest more even in the presence of surfers. Our result contrasts with Ben Abdelaziz *et al.* who established that innovators invest less when surfers exist. This shows the positive role of surfers in enhancing investment activity in an industry. Also, we obtained that in a region of the parameter values, the presence of surfers is welfare improving.

This study can be extended in at least two directions. First, the assumption that surfers can freely absorb partially the knowledge created by others could be soften by imposing a certain cost on surfers. Second, the assumption that the type of each firm is exogenously given should be relaxed. One can then add a stage to the game in which the firm chooses to be active or not in R&D.

Appendix

A.1. Proof of Proposition 2.1

From Kamien *et al.* [12], the profit functions can be written as

$$\pi_k = \begin{cases} q_k^2\left(\bar{x}\right) - \frac{\gamma}{2} x_k^2, & k \in \mathcal{I}, \\ q_k^2\left(\bar{x}\right), & k \in \mathcal{S}. \end{cases}$$

Assuming an interior solution, differentiating the profit function of an innovator with respect to quantity and equating to zero yields

$$q_i\left(\bar{x}\right) = \frac{a - NF_i\left(\bar{x}\right) + \sum_{j \in \mathcal{N}, j \neq i} F_j\left(\bar{x}\right)}{N+1}, k \in N.$$
$$F_k\left(\bar{x}\right) = \begin{cases} A - \left(x_k + \beta \sum_{j \in \mathcal{I}, j \neq k} x_j\right), & k \in \mathcal{I} \\ A - \theta \sum_{j \in \mathcal{I}} x_j, & k \in \mathcal{S}. \end{cases}$$

Assuming symmetry, that is, $x_i = x$, $\forall i \in \mathcal{I}$,the above expression simplifies to

$$q_i\left(\bar{x}\right) = q\left(\bar{x}\right) = \frac{a - A + Yx}{N+1},$$

where

$$Y = (S+1)(1-\beta) + I((\beta-\theta)S + \beta) > 0.$$

Inserting in the profit function, we get

$$\pi_i = \left(q_i\left(\bar{x}\right)\right)^2 - \frac{\gamma}{2} x_i^2.$$

Assuming an interior solution, the first-order condition is given by:

$$2q_i \frac{\partial q_i}{\partial x_i} - \gamma x_i = 0, \tag{A.1}$$

with

$$\frac{\partial q_i}{\partial x_i} = Z,$$
$$Z = N - \beta I - \theta S + \beta > 0.$$

Assuming symmetry, (A.1) becomes

$$2q_i \frac{\partial q_i}{\partial x_i} - \gamma x_i = \frac{2}{(N+1)^2}[a - A + Yx]Z - \gamma x = 0,$$

and consequently

$$x^n = \frac{2(a-A)Z}{\gamma(N+1)^2 - 2ZY}.$$

Substituting for x^n in the expression of q_i, we get

$$q_i^n = \frac{(a-A)}{N+1}\frac{\gamma(N+1)^2}{\gamma(N+1)^2 - 2ZY}.$$

It can be easily established that a surfer's equilibrium output is given by

$$q_s^n = (a-A)\left[\frac{\gamma(N+1) - 2ZW}{\gamma(N+1)^2 - 2ZY}\right],\ s \in \mathcal{S},$$

where

$$W = I(\beta - \theta) + 1 - \beta > 0.$$

A.2. Proof of Proposition 2.5

Assuming an interior solution, the second-stage output levels and profits are given, as in the noncooperative R&D case, by:

$$q_k\left(\bar{x}\right) = \frac{a - NF_k\left(\bar{x}\right) + \sum_{j\in\mathcal{N}, j\neq k} F_j\left(\bar{x}\right)}{N+1}, k \in N, \tag{A.2}$$

$$\pi_k\left(\bar{x}\right) = \begin{cases} q_k^2\left(\bar{x}\right) - x_k, & k \in \mathcal{I} \\ q_k^2\left(\bar{x}\right), & k \in \mathcal{S}. \end{cases} \tag{A.3}$$

In the first stage, the innovators maximize their joint profit, that is,

$$\max_x \mathit{\Pi} = \max_x \sum_{i\in\mathcal{I}} \pi_i\left(\bar{x}\right) = \max_x \sum_{i\in I}\left(q_i^2\left(\bar{x}\right) - \frac{\gamma}{2}x_i^2\right).$$

Assuming an interior solution, the first-order conditions are given by

$$\frac{\partial \mathit{\Pi}}{\partial x_i} = 2q_i\left(\bar{x}\right)\frac{\partial q_i\left(\bar{x}\right)}{\partial x_i} + \sum_{j\in\mathcal{I}, j\neq i} 2q_j\left(\bar{x}\right)\frac{\partial q_j\left(\bar{x}\right)}{\partial x_i} - \gamma x_i = 0, \quad i \in \mathcal{I}. \tag{A.4}$$

The optimal output level can be written as:

$$q_i\left(\bar{x}\right) = \frac{a - NF_i\left(\bar{x}\right) + \sum_{j\in\mathcal{I}, j\neq i} F_j\left(\bar{x}\right) + \sum_{s\in\mathcal{S}} F_s\left(\bar{x}\right)}{(N+1)}.$$

Recalling that the unit production costs are given by:

$$F_k\left(\bar{x}\right) = \begin{cases} A - \left(x_k + \beta\sum_{j\in\mathcal{I}, j\neq k} x_j\right), & k \in \mathcal{I} \\ A - \left(\theta\sum_{j\in\mathcal{I}} x_j\right), & k \in \mathcal{S}, \end{cases} \tag{A.5}$$

the condition in (A.4) becomes

$$\frac{2q_i(\bar{x})}{N+1}\left[-N\frac{\partial F_i\left(\bar{x}\right)}{\partial x_i} + \sum_{j\in\mathcal{I}, j\neq i,}\frac{\partial F_j\left(\bar{x}\right)}{\partial x_i} + \sum_{s\in\mathcal{S}}\frac{\partial F_s\left(\bar{x}\right)}{\partial x_i}\right]$$
$$+ \sum_{j\in\mathcal{I}, j\neq i}\frac{2q_j\left(\bar{x}\right)}{N+1}\left[-N\frac{\partial F_j\left(\bar{x}\right)}{\partial x_i} + \sum_{j\in\mathcal{I}, j\neq i,}\frac{\partial F_j\left(\bar{x}\right)}{\partial x_i} + \sum_{s\in\mathcal{S}}\frac{\partial F_s\left(\bar{x}\right)}{\partial x_i}\right] - \gamma x_i = 0,\ i \in \mathcal{I}. \tag{A.6}$$

In symmetric case, the above equation reduces to:

$$\frac{2q_i\left(\bar{x}\right)}{N+1}Y-\gamma x=0, \tag{A.7}$$

which is equivalent

$$\frac{2Y}{(N+1)^2}(a-A+xY)-\gamma x=0,$$

and consequently

$$x^c=\frac{2Y(a-A)}{\gamma(N+1)^2-2Y^2}.$$

Inserting x^c in (A.7), we obtain the following equilibrium output for an innovator:

$$q_i^c=(a-A)\left(\frac{\gamma(N+1)}{\gamma(N+1)^2-2Y^2}\right), \quad i\in\mathcal{I}.$$

For a surfer, we get

$$q_s^c=(a-A)\left[\frac{\gamma(N+1)-2YW}{\gamma(N+1)^2-2Y^2}\right],\ s\in\mathcal{S}.$$

It suffices to substitute for the quantities and R&D investments in the profit functions to obtain the expressions in the proposition.

A.3. Proof of Proposition 2.6

We first consider the case where the firms set noncooperatively their R&D investments. The equilibrium R&D expenditures in D&J and in our case are given by

$$x^n_{D\&J}=\frac{2(a-A)Z_D}{\gamma(N+1)^2-2Z_DY_D},$$
$$x^n=\frac{2(a-A)Z}{\gamma(N+1)^2-2ZY},$$

where

$$Z_D=N(1-\beta)+\beta,$$
$$Y_D=(1-\beta)+N\beta.$$

Compute the difference

$$x^n-x^n_{D\&J}=\frac{(a-A)\left(\gamma\left(N+1\right)^2\left(S\left(\beta-\theta\right)\right)-2ZZ^D\left(Y^D-Y\right)\right)}{\left(\gamma(N+1)^2-2ZY\right)\left(\gamma(N+1)^2-2Y^DZ^D\right)}$$
$$=\frac{(a-A)}{\left(\gamma(N+1)^2-2ZY\right)\left(\gamma(N+1)^2-2Y^DZ^D\right)}$$
$$\times\left(\gamma\left(N+1\right)^2\left(S\left(\beta-\theta\right)\right)+2\left(N-\beta I-\theta S+\beta\right)\left(N(1-\beta)+\beta\right)\left(S\left(1-2\beta+I\left(\beta-\theta\right)\right)\right)\right).$$

For interior solutions, we have

$$\gamma(N+1)^2-2ZY>0\ \text{ and }\ \gamma(N+1)^2-2Y^DZ^D>0,$$

and therefore the denominator of $x^n-x^n_{D\&J}$ is positive. Consequently, we have

$$\operatorname{sign}\left(x^n-x^n_{D\&J}\right)=\operatorname{sign}\Gamma,$$

where

$$\Gamma = \gamma\left(N+1\right)^2\left(S\left(\beta-\theta\right)\right) + 2\left(N-\beta I-\theta S+\beta\right)\left(N(1-\beta)+\beta\right)\left(S\left(I\left(\beta-\theta\right)+1-2\beta\right)\right).$$

Clearly, a sufficient (not necessary) condition to have $\Gamma > 0$ is to have $S\left(I\left(\beta-\theta\right)+1-2\beta\right) > 0$, which holds true for $\beta < 1/2$.

(ii) Under cooperative R&D, we have

$$x^c = \frac{2\left(a-A\right)Y}{\gamma(N+1)^2-2Y^2},$$
$$x^c_{D\&J} = \frac{2\left(a-A\right)\left(N\beta+1-\beta\right)}{\gamma(N+1)^2-2\left(N\beta+1-\beta\right)^2}.$$

The difference is given by

$$x^c - x^c_{D\&J} = \frac{2\left(a-A\right)\left(\gamma(N+1)^2+2Y\left(N\beta+1-\beta\right)\right)\left(Y-\left(N\beta+1-\beta\right)\right)}{\left(\gamma(N+1)^2-2Y^2\right)\left(\gamma(N+1)^2-2\left(N\beta+1-\beta\right)^2\right)}.$$

It is then easy to check that

$$\begin{aligned}\operatorname{sign}\left(x^c - x^c_{D\&J}\right) &= \operatorname{sign}\left(Y-\left(N\beta+1-\beta\right)\right)\\ &= \operatorname{sign}\left(S\left(I\left(\beta-\theta\right)+1-2\beta\right)\right),\end{aligned}$$

which leads to the same result as in (i).

A.4. Proof of Proposition 3.1

The consumer surplus CS is given by

$$CS = U(Q) - pQ,$$

that is, the difference between consumer's utility $U(Q)$ and expenses. In our (linear-demand) case, CS is given by

$$CS = \int_p^a (a-z)\mathrm{d}z = \frac{1}{2}\left(a-p\right)^2 = \frac{1}{2}Q^2.$$

Producer's surplus is given by the industry profit

$$\pi\left(Q\right) = pQ - CT(Q),$$

where $CT(Q)$ is the total cost given by

$$CT(Q) = FC + VC\left(Q\right),$$

where FC represents the fixed costs, which are nil in this case, and VC designates the variable costs. The average unit production cost of the industry is:

$$c_k = \frac{S\cdot c_S + I\cdot C_i}{N}, \quad i\in\mathcal{I}, \quad s\in\mathcal{S},$$

with

$$c_i = A - [1+\beta(I-1)]x,$$
$$c_s = A - \theta I x.$$

Total costs are given by the sum of total variable production cost and the R&D costs for the I innovators:

$$CT(Q) = c_k Q + I\gamma \frac{x^2}{2}\cdot$$

Consequently, the total surplus can be written

$$\begin{aligned} W(Q) &= U(Q) - c_k Q - I\gamma \frac{x^2}{2} \\ &= \frac{1}{2}Q^2 + pQ - CT(Q). \end{aligned}$$

Using $p = a - Q$, we get

$$W(Q) = (a - A)\, Q - \frac{1}{2}Q^2 + \frac{IQ}{N}\left(S\theta + (1 + \beta(I-1))\right) x - I\gamma \frac{x^2}{2}\cdot$$

Differentiating with respect to Q and equating to zero yields the optimal efficient output level

$$Q^* = a - A + (S\theta + 1 + \beta\,(I - 1))\, \frac{Ix^*}{N}.$$

A.5. Proof of Proposition 3.2

To get the efficient R&D investment and total output in an industry having N firms all investing in R&D, it suffices to set $I = N$ and $S = 0$ in (3.1)–(3.2), that is,

$$x^*_{D\&J} = \frac{(a-A)\,(1 + \beta(N-1))}{N\gamma - (1 + \beta(N-1))^2}, \tag{A.8}$$

$$Q^*_{D\&J} = a - A + (1 + \beta\,(N-1))\, x^*_{D\&J}. \tag{A.9}$$

Compute the difference

$$x^* - x^*_{D\&J} = \frac{(a-A)}{\left(N^2\gamma - I\,(S\theta + 1 + \beta(I-1))^2\right)\left(N\gamma - (1 + \beta(N-1))^2\right)} \times \Lambda,$$

where

$$\begin{aligned} \Lambda = {} & (S\theta + 1 + \beta(I-1))\left(N^2\gamma - N\,(1 + \beta(N-1))^2\right) \\ & - (1 + \beta(N-1))\left(N^2\gamma - I\,(S\theta + 1 + \beta(I-1))^2\right). \end{aligned}$$

As the denominator is positive, the sign of $x^* - x^*_{D\&J}$ is the same as the sign of Λ. Let

$$\begin{aligned} A &= (S\theta + 1 + \beta(I-1)) \\ B &= \left(N^2\gamma - N\,(1 + \beta(N-1))^2\right) \\ C &= (1 + \beta(N-1)) \\ D &= \left(N^2\gamma - I\,(S\theta + 1 + \beta(I-1))^2\right). \end{aligned}$$

Compute the differences

$$\begin{aligned} A - C &= -S\,(\beta - \theta) < 0, \\ B - D &= S^2 I\left(\theta^2 - \beta^2\right) + 2SI\left((\theta - \beta)\,\beta I - \theta\,(1-\beta) - S\beta^2\right) \\ & \quad - S^3\beta^2 - S\beta^2 I^2 + S\,(1-\beta)\,(-2S\beta - 4\beta I + \beta - 1) < 0. \end{aligned}$$

Therefore

$$\Lambda = AB - CD,$$

is negative because

$$A, B, C, D > 0$$
$$A < C \text{ and } B < D.$$

Hence the result.

For the quantities, we have

$$\begin{aligned} Q^* - Q^*_{D\&J} &= \left(a - A + (S\theta + 1 + \beta (I-1)) \frac{Ix^*}{N}\right) - (a - A + (1 + \beta (N-1)) x^*_{D\&J}) \\ &= (S\theta + 1 + \beta (I-1)) \frac{Ix^*}{N} - (1 + \beta (N-1)) x^*_{D\&J}. \end{aligned}$$

As

$$(S\theta + 1 + \beta(I-1)) - (1 + \beta(N-1)) = -S(\beta - \theta) < 0,$$
$$\frac{Ix^*}{N} < x^*_{D\&J},$$

we get $Q^* < Q^*_{D\&J}$.

A.6. Proof of Proposition 3.3

The difference in welfare is given by

$$\begin{aligned} W(Q^*_{D\&J}) - W(Q^*) = (a-A)(Q^*_{D\&J} - Q^*) - \frac{1}{2}\left((Q^*_{D\&J})^2 - (Q^*)^2\right) \\ + Q^*_{D\&J}(Q^*_{D\&J} - a + A) - Q^*(Q^* - a + A) - \frac{\gamma}{2}\left(N(x^*_{D\&J})^2 - I(x^*)^2\right), \end{aligned}$$

which can be rewritten as

$$W(Q^*_{D\&J}) - W(Q^*) = \frac{1}{2}\left((Q^*_{D\&J})^2 - (Q^*)^2 - \gamma\left(N(x^*_{D\&J})^2 - I(x^*)^2\right)\right). \tag{A.10}$$

Let

$$\Phi = (S\theta + 1 + \beta(I-1)); \quad \Delta = (1 + \beta(N-1)).$$

The socially efficient quantities and investments in the two scenarios can then be written as

$$x^* = \frac{N(a-A)\Phi}{N^2\gamma - I\Phi^2}, \; x^*_{D\&J} = \frac{(a-A)\Delta}{N\gamma - \Delta^2}, \tag{A.11}$$

$$Q^* = (a-A)N\gamma\left(\frac{N}{N^2\gamma - I\Phi^2}\right) \tag{A.12}$$

$$Q^*_{D\&J} = (a-A)N\gamma\left(\frac{1}{N\gamma - \Delta^2}\right). \tag{A.13}$$

Substituting in (A.10), we get

$$W(Q^*_{D\&J}) - W(Q^*) = \frac{(a-A)^2\gamma N}{2}\left(\frac{N\Delta^2 - I\Phi^2}{(N\gamma - \Delta^2)(N^2\gamma - I\Phi^2)}\right).$$

Since $\Delta > \Phi$, we have $N\Delta^2 - I\Phi^2 > 0$. Therefore, the sign of $W(Q^*_{D\&J}) - W(Q^*)$ is the same as the denominator. Writing the latter in long, we have

$$\left(N\gamma - \Delta^2\right)\left(N^2\gamma - I\Phi^2\right) = N^3\gamma^2 - \left(N^2\Delta^2 + IN\Phi^2\right)\gamma + I\Phi^2\Delta^2,$$

which is polynomial of degree 2 in γ. Its roots are given by

$$\gamma = \frac{\left(N^2\Delta^2 + IN\Phi^2\right) \pm \sqrt{\left(N^2\Delta^2 + IN\Phi^2\right)^2 - 4N^3 I\Phi^2\Delta^2}}{2N^3},$$

that is

$$\gamma^- = \frac{I\Phi^2}{N^2}; \qquad \gamma^+ = \frac{N\Delta^2}{N^2}.$$

We conclude that

$$W(Q^*_{D\&J}) - W(Q^*) \leq 0 \text{ for } \gamma \in \left[\frac{I\Phi^2}{N^2},\ \frac{N\Delta^2}{N^2}\right],$$

that is,

$$\Leftrightarrow \gamma \in \left[\frac{I\left(S\theta + 1 + \beta\left(I-1\right)\right)^2}{N^2},\ \frac{N\left(1 + \beta\left(N-1\right)\right)^2}{N^2}\right].$$

References

[1] R. Amir, Modeling imperfectly appropriable R&D via spillovers. *Int. J. Ind. Organ.* **18** (2000) 1013–1032.

[2] R. Amir, I. Evstigneev and J. Wooders, Noncooperative versus cooperative R&D with endogenous rates. *Games Econ. Behav.* **42** (2003) 183–207.

[3] R. Amir, J.Y. Jin and M. Troege, On additive spillovers and returns to scale in R&D. *Int. J. Ind. Organ.* **26** (2008) 695–703.

[4] C. D'Aspremont and A. Jacquemin, Cooperative and Non cooperative R&D in Duopoly with Spillovers. *Am. Econ. Rev.* (1988) 1133–1137.

[5] F. Ben Abdelaziz, M. Ben Brahim and G. Zaccour, R&D strategic equilibrium with surfers. *J. Optim. Theory Appl.* **136** (2008) 1–13.

[6] S. Ben Youssef, M. Breton and G. Zaccour, Cooperating Firms in Inventive and Absorptive Research. *J. Optim. Theory Appl.* **156** (2013) 229–251.

[7] M. Ceccagnoli, Firm Heterogeneity, Imitation and The Incentives for Cost Reducing R&D Effort. *J. Ind. Econ.* **53** (2005) 83–100.

[8] A. Hammerschmidt, No Pain, No Gain: An R&D Model with Endogenous Absorptive Capacity. *J. Inst. Theory Econ.* **165** (2009) 418–437.

[9] J. Hinloopen, Strategic R&D cooperatives. *Res. Econ.* **54** (2000) 153–185.

[10] U. Kaiser, R&D with spillovers and endogenous absorptive capacity. *J. Inst. Theory Econ.* **158** (2002) 286–303.

[11] M.I. Kamien and I. Zang, Meet me halfway: research joint ventures and absorptive capacity, *Int. J. Ind. Organ.* **18** (2000) 995–1012.

[12] M.I. Kamien, E. Muller and I. Zang, Research Joint Ventures and R&D Cartels. *Am. Econ. Rev.* (1992) 1293–1306.

[13] Y. Katsoulacos and D. Ulph, Endogenous Spillovers and the Performance of Research Joint Ventures. *J. Ind. Econ.* XLVI **3** (1998) 333–357.

[14] D. Leahy and J. Neary, Absorptive capacity, R&D spillovers and public policy. *Int. J. Ind. Organ.* **25** 1089–1108.

[15] S. Luckraz, Dynamic Noncooperative R&D in Duopoly with Spillovers and Technology Gap. *Game Theor. Appl.* **11** (2007) 139–161.

[16] S. Martin, Spillovers, appropriability and R&D. *J. Econ.* **75** (2002) 1–32.

[17] S.W. Salant and G. Shaffer, Optimal asymmetric strategies in research joint ventures. *Int. J. Ind. Organ.* **16** (1998) 195–208.

[18] D. Silipo, Incentives and forms of cooperation in research and development. *Res. Econ.* **62** (2008) 101–119.

[19] D. Silipo and A. Weiss, Cooperation and competition in an R&D market with spillovers. *Res. Econ.* **59** (2005) 41–57.

[20] K. Suzumura, Cooperative and Noncooperative R&D in an Oligopoly with spillovers. *Am. Econ. Rev.* (1992) 1307–1320.