RAIRO-Oper. Res. 49 (2015) 879–896
DOI: 10.1051/ro/2015034

RAIRO Operations Research
www.rairo-ro.org

RANKING UNITS AND DETERMINING DOMINANCE RELATIONS IN THE COST EFFICIENCY ANALYSIS

Roza Azizi[1], Reza Kazemi Matin[1]
and Reza Farzipoor Saen[2]

Abstract. The objective of this paper is to develop new models to compare cost efficiency of decision making units (DMUs) in all feasible input/output weights. Our focus is to determine cost ranking intervals and cost dominance relations in which the former show the best and the worst cost ranking of a special DMU in comparison with the other ones, and the latter specify the DMUs whose cost efficiencies are dominated by one special DMU. Finally, some new results on relevance of computed cost ranking intervals and cost dominance relations are presented.

Keywords. Data envelopment analysis (DEA), cost efficiency, ranking, dominance relation.

Mathematics Subject Classification. 90c11, 90c05, 90c90.

1. Introduction

Data envelopment analysis (DEA) is a nonparametric optimization approach which was developed in [4]. DEA assesses relative efficiency of decision making units (DMUs). DEA compares efficiency of DMUs by computing optimum weights of inputs and outputs of DMUs. These optimization models take into account all nonnegative weights without any restriction and do not deal with special cases. To resolve this issue, researchers have developed a couple of weight restriction models. Assurance region (AR) method [20], assumes a lower and upper bound for

Received March 27, 2015. Accepted September 16, 2015.

[1] Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran.
roza.azizi@kiau.ac.ir; rkmatin@kiau.ac.ir

[2] Department of Management, Karaj Branch, Islamic Azad University, Karaj, Iran.
farzipour@yahoo.com

Article published by EDP Sciences

the ratio of weights. Cone-ratio approach [5] generates AR method and assumed that the weights belong to defined cones. In [18] the idea of using virtual weights instead of absolute weights was presented and the reasons of using virtual weights were explained. In [23] DMUs were ranked by imposing a minimum weight constraint which is determined by the decision maker. The deviation of input and output weights were minimized in [11] from their minimum and maximum values which caused the DMUs to be efficient. In [17] the models which handle all feasible weights are introduced. Then the ranking intervals and efficiency dominance relationship of DMUs are achieved by using CCR (Charnes–Cooper–Rhodes) efficiency score for all feasible weights.

The initial DEA models assume no information on input costs and output prices. If costs and prices are available, cost, revenue, and profit models can be used to assess the DMUs. In [7] an approach for efficiency analysis in the presence of input prices is introduced. Then, cost efficiency model were introduced in [9, 10]. In [8] cost and profit functions were derived from the directional technology distance function. In recent years, there have been a few studies using the cost model in the absence or presence of price information (see [2, 15]).

Although all the mentioned models are useful for evaluating the performance of the systems, they cannot discriminate the DMUs precisely. Information about DMU's ranking and dominance relationship among DMUs enable researchers to discriminate DMUs and help managers to compare the performance of their DMUs with other ones. There are many studies to rank DMUs in the absence of prices. The most important approaches for ranking DMUs are super-efficiency and cross-efficiency methods (see [13, 16, 24] for more details). To rank efficient DMUs, in [1] the DMU under evaluation is omitted from technological matrix. This method is named super-efficiency one. However, this method has two drawbacks: (i) Since ranking is done based on optimal weights of each efficient or inefficient DMU, so there is no identical condition for comparing rank of DMUs. (ii) The model may be infeasible for some data set.

To rank DMUs with cross efficiency method [19], efficiency of each DMU is evaluated by taking into account optimal weight of all DMUs. Then, cross-efficiency matrix is built. However, cross-efficiency has some drawbacks. Firstly, it does not consider all feasible weights in evaluation. Secondly, there might be negative elements in cross-efficiency matrix. On the other hand, some researchers focused on dominance subject which can be useful in pairwise comparison of DMUs. In [3] a method is introduced which uses dominance techniques to form optimistic and pessimistic technologies for creating bounds for DEA models. To compute relative efficiency scores, [14] proposed a model which uses second-order stochastic dominance (SSD). In [12] a model to recognize the relationship among diversification, coherent measures of risk, and stochastic dominance is presented.

In this paper, we wish to bridge the gap in previous studies in the field of cost efficiency evaluation. The proposed ranking models show how the DMUs relate to each other when we consider all feasible weights not only the self appraisal DEA optimal weights. Note that the traditional DEA models just suggest the optimal weight to maximize the cost efficiency score of the under evaluation unit. In this paper, we analyze the rank position of every units based on their cost efficiency scores by taking all the feasible weights into account.

By considering all feasible weights, this paper presents the best and the worst cost ranking of each DMU based on cost efficiency score. To recognize the DMUs which have higher efficiency scores and also to determine the cost efficiency dominance relations, using pairwise comparison of all feasible weights, some DEA models are introduced. To this end, we follow the approach of [17] for cost model. Furthermore, in some special cases, the relationship of cost ranking and cost dominance is explored. The contributions of this paper are as follows:

- For the first time, we determine the best and the worst ranks of cost efficiency for all DMUs when the input price is the same or different across the units
- We represent the cost dominance relations of a DMU compared with other DMUs.
- We illustrate the relationship between cost ranking and cost dominance.

The rest of this paper is organized as follows: Section 2 gives a background. Section 3 presents cost ranking intervals, dominance relations of cost efficiency, and new results when the input prices are the same for all DMUs. Section 4 extends the proposed models for the case input prices are different across the units. Illustrative examples are presented in Section 5. Section 6 concludes.

2. Background

Assuming constant returns to scale (CRS) technology, consider n DMUs which consume m inputs to produce s outputs. Then, production possibility set (PPS)$T = \{(\boldsymbol{x}, \boldsymbol{y}) \in R_{+}^{m+s} |\ \boldsymbol{x}\ \textit{can produce}\ \boldsymbol{y}\}$ takes the form $T_{CRS} = \{(\boldsymbol{x}, \boldsymbol{y}) | \boldsymbol{x} \geq X\boldsymbol{\lambda}, \boldsymbol{y} \leq Y\boldsymbol{\lambda}, \boldsymbol{\lambda} \geq 0\}$. Here, X and Y are inputs and outputs matrix, respectively. $\mathbf{x}_j$ and $\mathbf{y}_j$ are inputs and outputs vectors for DMU_j, respectively. $\boldsymbol{\lambda}$ is a vector which enables us to construct unobserved but feasible DMUs by shrinking and expanding DMUs. Also, the corresponding input set for any output vector $\boldsymbol{y}$ is $L(\boldsymbol{y}) = \{\boldsymbol{x} | (\boldsymbol{x}, \boldsymbol{y}) \in T_{CRS}\}$.

The CCR model [4] is introduced to evaluate the efficiency scores of DMUs in the absence of market prices. This model could be written as follows:

$$\begin{aligned} \max_{\boldsymbol{u},\boldsymbol{v}}\ & \boldsymbol{u}'\boldsymbol{y}_k \\ s.t.\ & \boldsymbol{v}'\boldsymbol{x}_k = 1 \\ & \boldsymbol{u}'\boldsymbol{y}_l - \boldsymbol{v}'\boldsymbol{x}_l \leq 0 \ l = 1, \ldots, n \\ & \boldsymbol{u} \geq 0, \boldsymbol{v} \geq 0 \end{aligned} \tag{2.1}$$

where $\boldsymbol{u}$ and $\boldsymbol{v}$ are weight vectors (shadow prices) for outputs and inputs, respectively. In optimality, the model (2.1) provides the set of most favorable weights for DMU_k and its objective values give a relative efficiency score called CCR-Technical

Efficiency. Note that the weights in this model are restricted to be non-negative and the optimal weights vary across DMUs.

Classical DEA models such as CCR assume that there is no price information. Given price information, if decision makers wish to know whether or not their resources are used efficiently, cost model can be employed. Also, cost model unlike other classical models is useful in competitive markets or tenders to analyze the methods for consuming resources to produce outputs with the lowest cost. Tenders are version of competition in which managers should evaluate and suggest the lowest cost for producing outputs or presenting projects to win the competition. In competitive markets, if one firm produces its products with lower cost than other firms, it will be able to present the products in lower price and pre-empt the market.

When input prices are available, measuring the cost efficiency lies in producing the output vector $\boldsymbol{y}$ at the minimum cost. Suppose that the input price vector for all DMUs is $\boldsymbol{c}$. Then, the actual cost for DMU_k is $\boldsymbol{c}'\boldsymbol{x}_k$ and the minimum cost of producing the target output $\boldsymbol{y}_k$ is $\min_{\boldsymbol{x}} \mathbf{c}'\boldsymbol{x}\ ; \boldsymbol{x} \in L(\boldsymbol{y}_k)$.

Like usual technical efficiency scores, computed by the model (2.1), the cost efficiency scores can be also used to rank the DMUs. However, the obtained ranking is changeable based on different weights and also these scores cannot discriminate all the DMUs.

In the next section, we answer the following questions:

(1) What is the best and worst rank for the DMUs when we use cost-efficiency score as a ranking criterion?
(2) Which DMUs dominate the given DMU in a pairwise cost-efficiency comparison?
(3) Is there any relationship between cost ranking and cost dominance and/or is there any special case for them?

As mentioned before, classical ratio models are not as much useful as cost model in competitive markets or in analyzing systems from economic aspect. Therefore, to answer to the above mentioned questions, we follow ratio-based multiplier method proposed in [17]. Then, we express relationships between cost ranking and cost dominance.

We start with the case where units have common input prices. The proposed models are then extended to the more general case, when prices are different for the units.

3. Cost efficiency with common input prices: A comparison criterion

Based on the linear structure of the CRS technology, the minimum cost could be obtained by solving the following linear programming (LP) when input vector price for all DMUs is shown by $\boldsymbol{c}$:

$$\begin{array}{lll} \mathbf{c}'\boldsymbol{x}^* = \min\limits_{\boldsymbol{\lambda},\boldsymbol{x}} \mathbf{c}'\boldsymbol{x} & & \\ s.t. & \sum_l \boldsymbol{x}_l\lambda_l \leq \boldsymbol{x} & \\ & \sum_l \boldsymbol{y}_l\lambda_l \geq \boldsymbol{y}_k & \\ & \lambda_l \geq 0 & l = 1,\ldots,n \end{array} \tag{3.1}$$

The cost efficiency score for DMU_k is then measured as $CE_k = \dfrac{\mathbf{c}'\boldsymbol{x}^*}{\mathbf{c}'\boldsymbol{x}_k}$. We have $0 \leq CE_k \leq 1$ and the DMU_k is cost efficient if and only if $CE_k = 1$ (see [9,10] for more details).

3.1. Ranking intervals

To get a ranking criterion based on cost efficiency scores, first we rewrite the model (3.1) in the following ratio form to compute the cost efficiency score of DMU_k directly:

$$\begin{array}{lll} CE_k = \min\limits_{\boldsymbol{\lambda},\boldsymbol{x}} \dfrac{\mathbf{c}'\boldsymbol{x}}{\mathbf{c}'\boldsymbol{x}_k} & & \\ s.t. & \sum_l \boldsymbol{x}_l\lambda_l \leq \boldsymbol{x} & \\ & \sum_l \boldsymbol{y}_l\lambda_l \geq \boldsymbol{y}_k & \\ & \lambda_l \geq 0 & l = 1,\ldots,n\cdot \end{array} \tag{3.2}$$

The dual formula of the model (3.2) is as follows:

$$\begin{array}{lll} CE_k(\boldsymbol{u}) = \max\limits_{\boldsymbol{u}} \boldsymbol{u}'\boldsymbol{y}_k & & \\ s.t. & \boldsymbol{u}'\boldsymbol{y}_l - \bar{\boldsymbol{c}}'\boldsymbol{x}_l \leq 0 & l = 1,\ldots,n \\ & \bar{\boldsymbol{c}} = \dfrac{\boldsymbol{c}}{\mathbf{c}'\boldsymbol{x}_k} & \\ & \boldsymbol{u} \geq 0. & \end{array} \tag{3.3}$$

Note that $\bar{\boldsymbol{c}}$ is the input cost vector which is normalized by the observed cost of the DMU under evaluation and from the second equation, we have $\bar{\boldsymbol{c}}'\boldsymbol{x}_k = 1$. Therefore, we can rewrite the model (3.3) in a ratio form as follows:

$$\begin{array}{lll} CE_k(\boldsymbol{u}) = \max\limits_{\boldsymbol{u}} \dfrac{\boldsymbol{u}'\boldsymbol{y}_k}{\bar{\boldsymbol{c}}'\boldsymbol{x}_k} & & \\ s.t. & \boldsymbol{u}'\boldsymbol{y}_l - \bar{\boldsymbol{c}}'\boldsymbol{x}_l \leq 0 & l = 1,\ldots,n \\ & \bar{\boldsymbol{c}} = \dfrac{\boldsymbol{c}}{\mathbf{c}'\boldsymbol{x}_k} & \\ & \boldsymbol{u} \geq 0. & \end{array} \tag{3.4}$$

The cost efficiency of each DMU depends only on the output weights (shadow prices) of the DMUs in the model (3.4). Note that the optimal output weights vary across the DMUs. To avoid zero weights and to impose decision maker's preferences on the outputs, we can impose more general constraints on output weights. For example, the output weights can be chosen from restricted sets $U \subseteq \Re^{s}_{++}$ derived in the cone-ratio approach [5] or they can be selected from assurance regions [20].

Now, as a criterion, DMUs can be ranked based on their best and worst cost efficiency scores computed over the set of all feasible output weights in the model (3.4).

For the sake of simplicity, we consider following indices to distinguish DMUs which have strictly higher cost efficiency score than the DMU_k or at least as high cost efficiency score as DMU_k under a common set of output weights:

$$CR_k^{<} = \{l \in \{1,\dots,n\} | CE_k(\boldsymbol{u}) < CE_l(\boldsymbol{u})\}$$
$$CR_k^{\leq} = \{l \in \{1,\dots,n\} - \{k\} | CE_k(\boldsymbol{u}) \leq CE_l(\boldsymbol{u})\}.$$

With these notations, we can define the corresponding cost efficiency ranking as:

$$cr_k^{<} = 1 + |CR_k^{<}|,$$
$$cr_k^{\leq} = 1 + |CR_k^{\leq}|,$$

where $|CR|$ shows the cardinality of the set CR. Based on the above relations, if there exist some feasible weights for DMU_k to makes it cost-efficient, *i.e.* there is no DMUs with strictly higher cost efficiency score than DMU_k, then $cr_k^{<} = 1$ and in this case $cr_k^{\leq}$ is equal to the number of all cost efficient DMUs under the selected output weights. On the other hand, if with some feasible output weights, DMU_k becomes cost inefficient, then the indices $cr_k^{<} (cr_k^{\leq})$ will be increased in terms of number of DMUs, which have higher cost efficiency score than DMU_k (at least have the same cost efficiency score).

To determine the best and the worst rank for DMU_k and to get its ranking interval, it is sufficient to find minimum $cr_k^{<}$ and maximum of $cr_k^{\leq}$ as the lower and upper bounds of ranking interval, respectively.

Following the proposed technique in [17], we present two mixed integer linear programs (MILP) to determine the ranking interval of cost efficiency.

$$\begin{array}{lll} \min\limits_{\boldsymbol{u},\boldsymbol{z}} & 1 + \sum_{l \neq k} z_l & \\ s.t. & \boldsymbol{u}'\boldsymbol{y}_l - \bar{\boldsymbol{c}}'\boldsymbol{x}_l \leq M z_l & l \neq k \\ & \boldsymbol{u}'\boldsymbol{y}_k = 1 & \\ & z_l \in \{0,1\} & l \neq k \\ & \boldsymbol{u} \in U & \end{array} \tag{3.5}$$

and

$$\begin{array}{lll} \max\limits_{\boldsymbol{u},\boldsymbol{z}} & 1 + \sum_{l \neq k} z_l & \\ s.t. & -\boldsymbol{u}'\boldsymbol{y}_l + \bar{\boldsymbol{c}}'\boldsymbol{x}_l \leq M(1 - z_l) & l \neq k \\ & \boldsymbol{u}'\boldsymbol{y}_k = 1 & \\ & z_l \in \{0,1\} & l \neq k \\ & \boldsymbol{u} \in U. & \end{array} \tag{3.6}$$

Here, M is a sufficiently large positive constant. The output weight set is assumed to be closed and bounded by the constraint $\boldsymbol{u}'\boldsymbol{y}_k = 1$. Theorems 3.1 and 3.2

TABLE 1. Data set.

DMU	Input	Output 1	Output 2
A	1	1	1
B	1	4	3
C	1	2	2
D	1	2	4
E	1	4	5

TABLE 2. The worst and the best rankings.

DMU	min $cr_k^{<}$	max $cr_k^{\leq}$
A	5	5
B	1	3
C	3	4
D	2	4
E	1	2

help to determine ranking interval in the cost efficiency evaluation. The proofs are presented in Appendix.

Theorem 3.1. *The optimal objective value of the model* (3.5) *is the best cost-ranking of DMU_k.*

Theorem 3.2. *The optimal objective value of the model* (3.6) *is the worst cost-ranking of DMU_k.*

To illustrate the Theorems 3.1 and 3.2 consider Example 3.3:

Example 3.3. Assume that there are 5 DMUs to consume one input to produce two outputs. The input price is 2. Table 1 depicts the data set.

Using the models (3.5) and (3.6), Table 2 shows the best and the worst ranking of these DMUs.

For instance, consider the DMUs B and E. As is seen, the best ranks of these DMUs are 1. It means there are output weights which make them as the best performers. Since the worst rank of the DMU E is less than the DMU B, it has better performance. The result also shows that DMU A has the worst performance since its best rank is 5.

3.2. Cost efficiency dominance relation

Based on the ranking intervals, we can study a pairwise comparison of cost efficiency scores across feasible weights and determine the dominance correlation between two DMUs. To this end, for each pair of DMUs we need to specify whether or not a DMU dominates the other DMU. Here, we modify the dominance relationship proposed in [17]. Consider the following definition.

Definition 3.4. DMU_k dominates DMU_l ($DMU_l \prec DMU_k$)

$$\text{if and only if } \begin{cases} CE_k(\boldsymbol{u}) \geq CE_l(\boldsymbol{u}) & \text{for all} \boldsymbol{u} \in U \\ CE_k(\boldsymbol{u}) > CE_l(\boldsymbol{u}) & \text{for some} \boldsymbol{u} \in U \end{cases}$$

Dominance relationship between DMUs k and l can be shown using their cost efficiency ratio as $CD_{k,l}(\boldsymbol{u}) = \dfrac{CE_k(\boldsymbol{u})}{CE_l(\boldsymbol{u})}$. To analyze the dominance relationship of cost efficiency between DMU_k and DMU_l the extreme values of this ratio should be determined. Let $\overline{CD}_{k,l}(\boldsymbol{u})$ and $\underline{CD}_{k,l}(\boldsymbol{u})$ denote the maximum and minimum amount of $CD_{k,l}(\boldsymbol{u})$, respectively. Based on these values, if $\underline{CD}_{k,l}(\boldsymbol{u}) > 1$ then in all feasible output weights we have $CE^*_k(\boldsymbol{u}) > CE^*_l(\boldsymbol{u})$ and DMU_k dominates DMU_l. In the case of $\underline{CD}_{k,l}(\boldsymbol{u}) = 1$, the value of $\overline{CD}_{k,l}(\boldsymbol{u})$ should be computed and if $\overline{CD}_{k,l}(\boldsymbol{u}) > 1$, the DMU_k dominates DMU_l. Otherwise, $CE^*_k(\boldsymbol{u}) = CE^*_l(\boldsymbol{u})$ is hold for all feasible weights and there is no dominance relationship for these DMUs. Finally, condition $\underline{CD}_{k,l}(\boldsymbol{u}) < 1$ imply no dominance relationship.

Theorem 3.5. *The maximum (minimum) of $CD_{k,l}(\boldsymbol{u})$ will be obtained by solving the following linear model.*

$$\begin{array}{lll} \max\limits_{\boldsymbol{u}}(\min\limits_{\boldsymbol{u}}) & \boldsymbol{u}'\boldsymbol{y}_k & \\ s.t. & \boldsymbol{u}'\boldsymbol{y}_l = \bar{\boldsymbol{c}}'\boldsymbol{x}_l & \\ & \boldsymbol{u} \in U. & \end{array} \tag{3.7}$$

Note that DMU_A dominates DMU_B, if its cost efficiency score is more than or equal to DMU_B for all feasible weights and at least for one set of weights the inequality holds strictly. Therefore, cost efficiency dominance does not necessarily happen for all pairs of DMUs. For more illustrations, consider the DMUs in Table 1. Without using information on input cost, and with a simple vector analysis we have the following dominance relationship:

- A is dominated by other DMUs
- C is dominated by B, D, and E
- B and D are dominated by E
- Pairs B and D have no dominance relationship

Given input price, using the model (3.7) with non-negative weights yields the same relations in cost efficiency dominance. To show that the results depend on the weights, the results of this model with extra output restriction $u_1/u_2 \geq 0.5$ are reported in Table 3.

Table 3 shows that DMU_D is only dominated by E should we assume non-negative output weights. However, if an extra condition like $u_1/u_2 \geq 0.5$ is added on the weights, DMUs B and E will dominate DMU_D.

3.3. Further results

By computing the best and the worst cost rankings of DMUs and the cost dominance relationships, some new results are obtained.

TABLE 3. Dominance relations of cost efficiency among 5 DMUs.

	Dominated by	Dominated by
DMU	$u_1, u_2 \geq 0$	$u_1/u_2 \geq 0.5$
A	B-C-D-E	B-C-D-E
B	E	E
C	B-D-E	B-D-E
D	E	B-E
E	–	–

If the best and the worst cost rank of DMU_k is d and s and for optimal weights of models (3.5) and (3.6) there are b and h DMUs respectively which their cost efficiency score is similar to DMU_k, then

- DMU_k at most will dominate $n - s + h$ DMUs.
- DMU_k at most will be dominated by $d + b - 1$ DMUs.

In case of one output and multiple inputs, since the output weights are defined and fixed by $\boldsymbol{u}'\boldsymbol{y}_k = 1$, the maximizing or minimizing models play no roles in weight assessment. Theorem 3.6 shows the feature of this case.

Theorem 3.6. *Assume that there are no DMUs which have equal cost efficiency except for efficient DMUs. In the case of single output and multiple inputs, the best rank of cost-inefficient DMUs are similar to their worst rank. On the other hand, for cost efficient DMUs, the best rank is one and their worst rank is equal to the number of efficient DMUs.*

4. Cost efficiency with different input prices: A comparison criterion

Now, suppose that the cost per input is different across the units and each DMU has its own prices. Based on the linear structure of the CRS technology, the minimum cost of DMU_k could be obtained by solving the following LP when the input prices are available:

$$\begin{array}{lll} CE_k = \min\limits_{\boldsymbol{\lambda},\overline{\boldsymbol{x}}} & \dfrac{\overline{\boldsymbol{x}}}{\overline{\boldsymbol{x}}_k}, & \\ s.t. & \sum_l \overline{\boldsymbol{x}}_l \lambda_l \leq \bar{\boldsymbol{x}}, & \\ & \sum_l \boldsymbol{y}_l \lambda_l \geq \boldsymbol{y}_k, & \\ & \lambda_l \geq 0 & l = 1, \ldots, n, \end{array} \tag{4.1}$$

where $\overline{\boldsymbol{x}}_l = (c_{1l}x_{1l}, c_{2l}x_{2l}, \ldots, c_{ml}x_{ml})', l = 1, \ldots, n$ (See [21]).

The dual model of model (4.1) is as follows:

$$\begin{array}{lll} CE_k(\boldsymbol{u}) = \max\limits_{\boldsymbol{u}} & \boldsymbol{u}'\boldsymbol{y}_k, & \\ s.t. & \boldsymbol{u}'\boldsymbol{y}_l - \tilde{\boldsymbol{c}}'\bar{\boldsymbol{x}}_l \leq 0, & l = 1, \ldots, n, \\ & \tilde{\boldsymbol{c}} = \dfrac{1}{\mathbf{c}_k'\boldsymbol{x}_k}, & \\ & \boldsymbol{u} \geq 0. & \end{array} \tag{4.2}$$

Note from the second constraint, we have $\tilde{\boldsymbol{c}}'\bar{\boldsymbol{x}}_k = \frac{1}{\mathbf{c}_k'\boldsymbol{x}_k}\mathbf{c}_k'\boldsymbol{x}_k = 1$. Therefore, we can rewrite the model (4.2) in a ratio form as follows:

$$\begin{array}{lll} CE_k(\boldsymbol{u}) = \max\limits_{\boldsymbol{u}} & \dfrac{\boldsymbol{u}'\boldsymbol{y}_k}{\tilde{\boldsymbol{c}}'\bar{\boldsymbol{x}}_k}, & \\ s.t. & \boldsymbol{u}'\boldsymbol{y}_l - \tilde{\boldsymbol{c}}'\bar{\boldsymbol{x}}_l \leq 0, & l = 1, \ldots, n, \\ & \tilde{\boldsymbol{c}} = \dfrac{1}{\mathbf{c}_k'\boldsymbol{x}_k}, & \\ & \boldsymbol{u} \geq 0. & \end{array} \tag{4.3}$$

To present the best and the worst ranking of DMUs based on their cost efficiency scores over the set of all feasible output weights, the same approach as proposed in Section 3.1 can be used. So, the best and the worst cost rank of a production unit with its own prices can be determined by the optimal solution of models (4.4) and (4.5), respectively.

$$\begin{array}{lll} \min\limits_{\boldsymbol{u},\boldsymbol{z}} & 1 + \sum_{l \neq k} z_l, & \\ s.t. & \boldsymbol{u}'\boldsymbol{y}_l - \tilde{\boldsymbol{c}}'\bar{\boldsymbol{x}}_l \leq M z_l, & l \neq k \\ & \boldsymbol{u}'\boldsymbol{y}_k = 1, & \\ & z_l \in \{0, 1\}, & l \neq k \\ & \boldsymbol{u} \in U, & \end{array} \tag{4.4}$$

and

$$\begin{array}{lll} \max\limits_{\boldsymbol{u},\boldsymbol{z}}, & 1 + \sum_{l \neq k} z_l & \\ s.t. & -\boldsymbol{u}'\boldsymbol{y}_l + \tilde{\boldsymbol{c}}'\bar{\boldsymbol{x}}_l \leq M(1 - z_l), & l \neq k \\ & \boldsymbol{u}'\boldsymbol{y}_k = 1, & \\ & z_l \in \{0, 1\}, & l \neq k \\ & \boldsymbol{u} \in U. & \end{array} \tag{4.5}$$

The optimal solution of model (4.6) determines whether a DMU is dominated by other DMUs based on its cost efficiency score.

$$\begin{array}{ll} \overline{CD}_{k,l}(\boldsymbol{u})(\underline{CD}_{k,l}(\boldsymbol{u})) = \max\limits_{\boldsymbol{u}}(\min\limits_{\boldsymbol{u}}) & \boldsymbol{u}'\boldsymbol{y}_k \\ s.t. & \boldsymbol{u}'\boldsymbol{y}_l = \tilde{\boldsymbol{c}}'\bar{\boldsymbol{x}}_l \\ & \boldsymbol{u} \in U \end{array} \tag{4.6}$$

The same results as Section 3.3 are available for the case of different input prices.

5. Illustrative applications

In this section, we illustrate the ranking approach in cost efficiency and cost dominance by three numerical examples.

Example 5.1. In Table 4, the data set related to 12 hospitals which have been taken from [6] is depicted. There are two inputs (number of doctors and nurses) and two outputs (number of outpatients and inpatients). For all DMUs, price of doctors and nurses are assumed to be 436 and 86, respectively.

Computed cost efficiency scores are reported in Table 5. The best and the worst cost efficiency rankings of the hospitals are represented in the third and fourth

TABLE 4. Data from 12 hospitals.

Hospitals	Number of doctors	Number of nurses	Number of outpatients	Number of inpatients
A	20	151	100	90
B	19	131	150	50
C	25	160	160	55
D	27	168	180	72
E	22	158	94	66
F	55	255	230	90
G	33	235	220	88
H	31	206	152	80
I	30	244	190	100
J	50	268	250	100
K	53	306	260	147
L	38	284	250	120

TABLE 5. Cost efficiency, the best and the worst cost ranking, and the dominance relationship of cost efficiency of 12 hospitals.

Hospital	Cost efficiency	min $cr_k^{\leq}$	max $cr_k^{\leq}$	Dominated by
A	**1**	**1**	**11**	–
	0.8556	5	9	B-D-G-L
B	**1**	**1**	**8**	–
	1	1	1	–
C	**0.8560**	**3**	**10**	**B-D**
	0.8522	4	6	B-D-G
D	**0.9648**	**2**	**6**	–
	0.9395	2	2	B
E	**0.7574**	**5**	**12**	**A-I-K-L**
	0.6747	12	12	A-B-C-D-F-G-H-I-J-K-L
F	**0.6974**	**9**	**12**	**B-C-D-G-I-J-K-L**
	0.6813	11	11	A-B-C-D-G-H-I-J-K-L
G	**08936**	**3**	**9**	**B-D**
	0.8702	3	4	B-D
H	**0.7781**	**7**	**11**	**D-I-K-L**
	0.7261	10	10	A-B-C-D-G-I-J-K-L
I	**0.8917**	**3**	**7**	–
	0.8322	6	7	B-C-D-G-L
J	**0.7834**	**7**	**11**	**B-C-D-G-I-L**
	0.7629	8	9	B-C-D-G-I-K-L
K	**0.8725**	**2**	**8**	–
	0.8050	7	8	B-C-D-G-I-L
L	**0.9318**	**2**	**5**	–
	0.8823	3	5	B-D

columns of the Table 5. The dominance relationships among hospitals are reported in the last column of the Table 5. We run the introduced models under two different weight restriction scenarios. The first scenario does not consider any preference relationships for outputs and in the second scenario, we assume the existence of non-negative weights with $2 \geq u_1/u_2 \geq 1$. Due to the fact that common prices are considered in this example, the models of Section 3 are used to present the results.

In Table 5, there are two rows for every hospital which are associated with the two scenarios. The first row of each hospital with the bold components shows the results of hospitals evaluation in which $u_1, u_2 \geq 0$ are incorporated. The second row of each hospital with the simple components is associated with the case in which $2 \geq u_1/u_2 \geq 1$ is incorporated.

We illustrate the approach by considering the first weight restriction scenario. The interpretation of our cost ranking and dominance models is the same as any weight restriction scenario. Cost-efficient hospitals A and B achieve the best performance for some feasible weights. The cost ranking intervals of A and B are [1,11] and [1,8], respectively. It means that their ranking is flexible in these intervals based on the different feasible weights. These DMUs are unable to have the worst rank among all feasible output weights. Hospital A, for some feasible weights, in the worst case has better performance than only one DMU. Given their cost ranking intervals, we conclude that hospital B has better performance than the hospital A. Consequently, B has the best performance among all hospitals. Hospitals A and B are not cost-dominated. However, the last column of Table 5 shows that hospital A dominates one and hospital B dominates four hospitals.

Now, we consider hospitals E and F. Using non-negative output weights, hospitals E and F are the only hospitals that get the worst possible rank 12. As is seen in Table 5, the maximum number of the DMUs which dominate E and F are 4 and 8, respectively. The worst rank of F is 12 and the best rank of F is 9 which are not good. Therefore, it can be concluded that the hospital F has the worst performance among inefficient hospitals.

Example 5.2. Table 6 reports the results of analyzing data set of Indian Life Insurance Corporation (LIC) operations in 19 annual periods (DMUs) by the models of Section 3. The data set is taken from [22]. We have four inputs and one output. The inputs are business services, labor, debt capital, and equity capital, respectively. The output is losses as the claims settled during the year including claims written back. The common price is computed with the simple average of all input prices among DMUs. So, the input prices are considered 0.0000211, 0.0000948, 0.112 and 0.175, respectively.

Note that in the case of one output and multiple inputs we cannot define any special constraints on weights because it may lead to infeasibility. As is shown in Table 6, the DMU_{13} is the only DMU which is cost-efficient. Therefore, given the Theorem 3.6, its best and worst rank is one. Also, for all DMUs the best and the worst ranks are equal. Based on the discussions in Section 3, DMU_{11} is dominated by 5 DMUs and it dominates 13 DMUs. The DMU_1 with the worst performance

Table 6. Cost efficiency, the best and the worst cost ranking, and the dominance relationship of cost efficiency of 19 DMUs.

DMU	Cost efficiency	min $cr_k^{<}$	max $cr_k^{\leq}$	Dominated by
1	0.7164	19	19	2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19
2	0.7414	18	18	3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19
3	0.8206	17	17	4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19
4	0.8262	15	15	5-6-7-8-9-10-11-12-13-14-15-16-17-18-19
5	0.8236	16	16	4-6-7-8-9-10-11-12-13-14-15-16-17-18-19
6	0.8538	14	14	7-8-9-10-11-12-13-14-15-16-17-18-19
7	0.8879	10	10	11-12-13-14-15-16-17-18-19
8	0.8738	13	13	7-9-10-11-12-13-14-15-16-17-18-19
9	0.8826	12	12	7-10-11-12-13-14-15-16-17-18-19
10	0.8876	11	11	7-11-12-13-14-15-16-17-18-19
11	0.9568	6	6	12-13-15-16-19
12	0.9760	4	4	13-15-19
13	1	1	1	–
14	0.9233	8	8	11-12-13-15-16-18-19
15	0.9774	3	3	13-19
16	0.9587	5	5	12-13-15-19
17	0.9213	9	9	11-12-13-14-15-16-18-19
18	0.9365	7	7	11-12-13-15-16-19
19	0.9921	2	2	13

cannot dominate any DMU and it is dominated by all DMUs. Finally, DMU_{13} which has the best performance can dominate all the other 18 DMUs.

In this example, it was shown that the minimum number of DMUs which have strictly higher cost efficiency scores than the DMU under evaluation is equal to the maximum number of DMUs which have at least as high efficiency score as the DMU under evaluation. Thus, the different sets of feasible weights do not have any effect on the cost rank of DMUs, and all the feasible weights make the same cost ranking for all DMUs. The results show that there is no interval ranking. Therefore, decision makers should be aware that there are not any alternative positions which make their DMUs has better or worse ranking in comparison with the position reported in the Table 6. For example, the worst cost rank of DMU_{14} is 8. It means there is no set of feasible weights to make better cost rank because its best cost ranking is 8. Consequently, considering all feasible weights, it can be said that there is just one position for cost ranking of each DMU which is ideal in performance analysis of DMUs because all position makes the same results.

Example 5.3. To illustrate the results of the new proposed models in Section 4, we consider the same units of Example 5.1 with different input prices. The new data set is presented in Table 7.

Second column of Table 8 reports cost efficiency score of 12 hospitals computed by model (4.2). The next two columns show the best and worst cost ranking of

TABLE 7. Input prices of 12 hospitals.

Hospitals	Cost of doctors	Cost of nurses
A	500	100
B	350	80
C	450	90
D	600	120
E	300	70
F	450	80
G	500	100
H	450	85
I	380	76
J	410	75
K	440	80
L	400	70

TABLE 8. Cost efficiency, the best and the worst ranks, and dominance relationship of cost efficiency of 12 hospitals.

Hospital	Cost efficiency	min $cr_k^{<}$	max $cr_k^{\leq}$	Dominated by
A	0.9594	2	12	E
B	1	1	6	–
C	0.7239	4	10	B-I-L
D	0.6241	10	12	B-C-E-F-G-I-J-K-L
E	1	1	8	–
F	0.6343	9	11	B-C-E-G-I-J-K-L
G	0.6934	6	10	B-I-J-L
H	0.7259	7	11	B-E-I-L
I	0.9533	3	4	L
J	0.7763	4	8	B-I-L
K	0.8627	5	7	I-L
L	1	1	3	–

DMUs achieved by models (4.4) and (4.5), respectively. The last column reports the dominance relationship among hospitals which is determined by model (4.6).

The weights are constrained to be nonnegative and no preference for outputs is assumed. Table 8 shows that hospitals B, E and L are cost efficient, so, their best ranking is 1. Now we consider hospital L. Hospital L performs better than other ones because its worst ranking is better than hospitals B and E. The ranking interval for hospital L is [1,3], this means that there are some sets of feasible weights for which the rank of hospital L is 1, 2 and 3. In other words, there is no set of feasible weights for which the ranking of hospital L be worse than 3. As is shown in the last column of Table 8, hospital L is not dominated by other DMUs but it dominates 8 other ones.

6. Conclusion

In this paper, basic DEA models were reviewed to assess production units with complete information on inputs and outputs without prior need on prices. When the input prices are available the cost model is a better choice to evaluate the performance of the DMUs. In this paper, we analyzed the relative performance of DMUs based on their cost efficiency scores by taking into account all feasible input/output weights. First, we determined the best and the worst cost ranking of a given DMU with respect to the existence of DMUs alternate cost efficiency scores in all feasible weights. Furthermore, a pairwise comparison was designed to show whether a DMU can dominate any specific DMU based on their cost efficiency score while all feasible output weights are considered, not just the optimal ones. Then, some relations between cost ranking intervals and cost dominance were presented. The models and results are presented when the cost per input is the same or different across the units. Finally, some numerical examples were given to show the advantages of our developed models.

Similar models can be developed for variable returns to scale technology. To address more practical problems, we suggest further research to be conducted for determining cost dominance relationship in variable returns to scale technology. It is also worth to determine efficiency bounds which show how much more efficient a given DMU can be relative to some other DMU, for different efficiency measures.

Appendix

Proof of Theorem 3.1. Let the best ranking of DMU_k is achieved at $\boldsymbol{u} \in U$. Therefore, there exists $L = CR_k^<$ so that

$$\begin{cases} CE_k^*(\boldsymbol{u}) < CE_l^*(\boldsymbol{u}) \text{ for all } l \in L, \\ CE_k^*(\boldsymbol{u}) \geq CE_l^*(\boldsymbol{u}) \text{ for all } l \notin L. \end{cases}$$

Let $\hat{\boldsymbol{u}} = \dfrac{\boldsymbol{u}}{\boldsymbol{u}'\boldsymbol{y}_k}$. Then $\hat{\boldsymbol{u}} \in U$ *and* $\hat{\boldsymbol{u}}'\boldsymbol{y}_k = 1$.

Let $z_l = 1, (l \neq k)$ for $l \in L$ and $z_l = 0, l \neq k$ for $l \notin L$. $z_l(l \neq k)$ is the lth component of $\boldsymbol{z}$. Therefore, for any $l \notin L$ we have

$$CE_k^*(\boldsymbol{u}) \geq CE_l^*(\boldsymbol{u}) \Longrightarrow 1 \geq \frac{CE_l^*(\boldsymbol{u})}{CE_k^*(\boldsymbol{u})} = \frac{CE_l^*(\hat{\boldsymbol{u}})}{CE_k^*(\hat{\boldsymbol{u}})} = \frac{\dfrac{\hat{\boldsymbol{u}}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l}}{\dfrac{\hat{\boldsymbol{u}}'\boldsymbol{y}_k}{\bar{\boldsymbol{c}}'\boldsymbol{x}_k}} = \frac{\hat{\boldsymbol{u}}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l}$$

$$\Longrightarrow \hat{\boldsymbol{u}}'\boldsymbol{y}_l \leq \bar{\boldsymbol{c}}'\boldsymbol{x}_l \Longrightarrow \hat{\boldsymbol{u}}'\boldsymbol{y}_l - \bar{\boldsymbol{c}}'\boldsymbol{x}_l \leq 0$$

for any $l \in L$ we have

$$CE_k^*(\boldsymbol{u}) < CE_l^*(\boldsymbol{u}) \Longrightarrow 1 < \frac{CE_l^*(\boldsymbol{u})}{CE_k^*(\boldsymbol{u})} = \frac{CE_l^*(\hat{\boldsymbol{u}})}{CE_k^*(\hat{\boldsymbol{u}})} = \frac{\frac{\hat{\boldsymbol{u}}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l}}{\frac{\hat{\boldsymbol{u}}'\boldsymbol{y}_k}{\bar{\boldsymbol{c}}'\boldsymbol{x}_k}} = \frac{\hat{\boldsymbol{u}}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l}$$

$$\Longrightarrow \hat{\boldsymbol{u}}'\boldsymbol{y}_l > \bar{\boldsymbol{c}}'\boldsymbol{x}_l \Longrightarrow \hat{\boldsymbol{u}}'\boldsymbol{y}_l - \bar{\boldsymbol{c}}'\boldsymbol{x}_l > 0.$$

Given $z_l = 0, (l \neq k)$ for $l \notin L$ and $z_l = 1, (l \neq k)$ for $l \in L$ and multiplying z_l by M the first constraint is established. Moreover, the solution of the model (3.5) is not larger than the best ranking of DMU_k, because $1 + \sum_{l \neq k} z_l = 1 + |L| = 1 + |CR_k^<| = min\ cr_k^<$. Conversely, let the optimal solution of the model (3.5) is attained at $(\boldsymbol{u},\boldsymbol{z})$. Let $L = \{l | z_l = 1, (l \neq k)\}$ and for all $l \notin L$ then $z_l = 0, l \neq k$.

So, for all $l \notin L$ it follows that the first constraint of the model (3.5) reaches to $\boldsymbol{u}'\boldsymbol{y}_l \leq \bar{\boldsymbol{c}}'\boldsymbol{x}_l$. Therefore, $\frac{CE_l^*(\boldsymbol{u})}{CE_k^*(\boldsymbol{u})} = \frac{\boldsymbol{u}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l} \leq 1$ is held because of the second constraint of the model (3.5) and the expressions of $\bar{\boldsymbol{c}}$. For $l \in L$, the $\boldsymbol{u}'\boldsymbol{y}_l \leq \bar{\boldsymbol{c}}'\boldsymbol{x}_l \Longleftrightarrow CE_l^*(\boldsymbol{u}) \leq CE_k^*(\boldsymbol{u})$ is not hold, otherwise optimality of $\boldsymbol{z}$ makes $z_l = 1, (l \neq k)$ could be changed to $z_l = 0, l \neq k$ (it causes the first constraint remains satisfied, but the objective function is decreased). Thus, $CR_k^< = L$ and $min\ cr_k^< = 1 + |CR_k^<| = 1 + |L| = 1 + \sum_{l \neq k} z_l$. □

Proof of Theorem 3.2. The procedure of proof of this theorem is similar to the Theorem 3.1. □

Proof of Theorem 3.5. Choose $\boldsymbol{u}^* \in U$ so that $CD_{k,l}(\boldsymbol{u}^*) \geq CD_{k,l}(\boldsymbol{u})$ for all $\boldsymbol{u} \in U$. Define $\hat{\boldsymbol{u}} = \frac{\boldsymbol{u}^* \times (\bar{\boldsymbol{c}}'\boldsymbol{x}_l)}{\boldsymbol{u}^{*\prime}\boldsymbol{y}_l}$. Thus, $\hat{\boldsymbol{u}}$ satisfies in the constraints of the model (3.7) and $CD_{k,l}(\boldsymbol{u}^*) = CD_{k,l}(\hat{\boldsymbol{u}}) = \hat{\boldsymbol{u}}'\boldsymbol{y}_k$, because the model is unit invariant. Therefore, the optimal amount of the model (3.7) is not smaller than $CD_{k,l}(\boldsymbol{u}^*)$. Conversely, let the optimal solution of the model (3.7) is attained at $\hat{\boldsymbol{u}}$. Therefore, because of the first constraint of the model (3.7) and the expressions of $\bar{\boldsymbol{c}}$, we have

$$\hat{\boldsymbol{u}} \in U \text{ and } CD_{k,l}(\hat{\boldsymbol{u}}) = \frac{CE_k(\hat{\boldsymbol{u}})}{CE_l(\hat{\boldsymbol{u}})} = \frac{\frac{\hat{\boldsymbol{u}}'\boldsymbol{y}_k}{\bar{\boldsymbol{c}}'\boldsymbol{x}_k}}{\frac{\hat{\boldsymbol{u}}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l}} = \hat{\boldsymbol{u}}'\boldsymbol{y}_k. \tag{A.1}$$

Thus, the maximum of $CD_{k,l}(\boldsymbol{u})$ over U is at least as high as the solution of the model (3.7). The same procedure can be applied to prove the minimum case. □

Proof of Theorem 3.6. It is clear that in both models (3.5) and (3.6), maximizing and minimizing do not have any effect in the amount of $\boldsymbol{u}$ and $\bar{\boldsymbol{c}}$, because $\bar{\boldsymbol{c}}$ is obtained by data and is not a variable. $\boldsymbol{u}$ which is obtained by models is unique in both of them because we have one output.

We know model (3.5) sets $z_l = 1$ for $l \in L = \{l \in \{1,\dots,n\} | CE_k^*(\boldsymbol{u}) < CE_l^*(\boldsymbol{u})\}$, and model (3.6) sets $z_l = 1$ for $l \in L = \{l \in \{1,\dots,n\} - \{k\} | CE_k^*(\boldsymbol{u}) \leq CE_l^*(\boldsymbol{u})\}$, therefore,

The model (3.5) sets $z_l = 1$ if $CE_k^*(\boldsymbol{u}) < CE_l^*(\boldsymbol{u}) \Longrightarrow 1 < \dfrac{CE_l^*(\boldsymbol{u})}{CE_k^*(\boldsymbol{u})} = \dfrac{\boldsymbol{u}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l}$ $\quad (A)$

The model (3.6) sets $z_l = 1$ if $CE_k^*(\boldsymbol{u}) \leq CE_l^*(\boldsymbol{u}) \Longrightarrow 1 \leq \dfrac{CE_l^*(\boldsymbol{u})}{CE_k^*(\boldsymbol{u})} = \dfrac{\boldsymbol{u}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l}$ $\quad (B)$

First we prove for inefficient DMUs. According to the assumptions of the theorem we have

The model (3.6) sets $z_l = 1$ if $CE_k^*(\boldsymbol{u}) < CE_l^*(\boldsymbol{u}) \Longrightarrow 1 < \dfrac{CE_l^*(\boldsymbol{u})}{CE_k^*(\boldsymbol{u})} = \dfrac{\boldsymbol{u}'\boldsymbol{y}_l}{\bar{\boldsymbol{c}}'\boldsymbol{x}_l}$ $\quad (C)$

Since $\boldsymbol{u}$ and $\bar{\boldsymbol{c}}$ are equal in both models, the constraints (A) and (C) are equivalent. Thus, the models (3.5) and (3.6) set $z_l = 1$ for identical DMUs. Therefore, the best and the worst cost rankings are equal. For cost efficient DMUs, it is clear that the model (3.6) set $z_l = 1$ for all DMU_l which has the same cost efficiency score as DMU_k (see (B)) and because of the equality of $\boldsymbol{u}$ and $\bar{\boldsymbol{c}}$ in both models, the only difference of the best and the worst ranking of DMUs is the number of the cost efficient DMUs minus 1. □

References

[1] P. Andersen and N.C. Petersen, A procedure for ranking efficient units in data envelopment analysis. *Manage. Sci.* **39** (1993) 1261–1264.

[2] A.S. Camahno and R.G. Dyson, Cost efficiency measurement with price uncertainty: a DEA application to bank branch assessments. *Eur. J. Oper Res.* **161** (2005) 432–446.

[3] R.G. Chambers, R. Fre, E. Jaenicke and E. Lichtenberg, Using dominance in forming bounds on DEA models: The case of experimental agricultural data. *J. Econ.* **85** (1998) 189–203.

[4] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of the decision making units. *Eur. J. Oper Res.* **2** (1978) 429–444.

[5] A. Charnes, W.W. Cooper, Q.L. Wei and Z.M. Huang, Cone Ratio Data Envelopment Analysis and Multi-objective Programming. *Int. J. Syst. Sci.* **20** (1989) 1099–1118.

[6] W.W. Cooper, L. M. Seiford and K. Tone, Data Envelopment Analysis, A Comprehensive Text with Models, Applications, References and DEA-Solver Software, 2th edition. Kluwer Academic Publishers, Boston (2007).

[7] M.J. Farrell, The measurement of productive efficiency. *J. R. Stat. Soc. Series A* **120** (1957) 253–281.

[8] R. Färe and S. Grosskopf, Profit efficiency, Farrell decompositions and the Mahler inequality. *Econ. Lett.* **57** (1997) 283–287.

[9] R. Färe, S. Grosskopf and C.A.K. Lovell, Measurement of Efficiency of Production. Kluwer-Nijhoff Publishing Co., Inc, Boston (1985).

[10] R. Färe, S. Grosskopf and C.A.K. Lovell, Production Frontiers. Cambridge University Press, Cambridge (1994).

[11] K.F. Lam and F. Bai, Minimizing deviations of input and output weights from their means in data envelopment analysis. *Comput. Ind. Eng.* **60** (2011) 527–533.

[12] J.D. Lamb and K.H. Tee, Data envelopment analysis models of investment funds. *Eur. J. Oper. Res.* **216** (2012) 687–696.

[13] H.Sh. Lee, Ch.W. Chu and J. Zhu, Super-efficiency DEA in the presence of infeasibility. *Eur. J. Oper. Res.* **212** (2011) 141-147.

[14] S. Lozano and E. Gutirrez, Data envelopment analysis of mutual funds based on second-order stochastic dominance. *Eur. J. Oper. Res.* **189** (2007) 230–244.

[15] C.S. Ray and H.G. Kim, Cost efficiency in the US steel industry: A nonparametric analysis using data envelopment analysis. *Eur. J. Oper. Res.* **80** (1995) 654–671.

[16] W. Rodder and E. Reucher, A consensual peer-based DEA-model with optimized cross-efficiencies Input allocation instead of radial reduction. *Eur. J. Oper. Res.* **212** (2011) 148–154.

[17] A. Salo and A. Punkka, Ranking intervals and dominance relations for ratio-based efficiency analysis. *Manage. Sci.* **57** (2011) 200–214.

[18] C.S. Sarrico and R.G. Dyson, Restricting virtual weights in data envelopment analysis. *Eur. J. Oper. Res.* **159** (2004) 17–34.

[19] T.R. Sexton, R.H. Silkman and A.J. Hogan, Data Envelopment Analysis: Critique and Extensions. In Measuring Efficiency: An Assessment of Data Envelopment Analysis, edited by R.H. Silkman. Jossey-Bass, San Francisco, CA (1986).

[20] R.G. Thompson, Jr.F.D Singleton, R.M. Thrall and B.A. Smith, Comparative site evaluations for locating a high-energy physics lab in Texas. *Interfaces* **16** (1986) 35–49.

[21] K. Tone, A strange case of the cost and allocative efficiencies in DEA. *J. Oper. Res. Soc.* **53** (2002) 1225–1231.

[22] K. Tone, B.K. Sahoo, Evaluating cost efficiency and returns to scale in the Life Insurance Corporation of India using data envelopment analysis. *Soc. Econ. Plan. Sci.* **39** (2005), 261-285.

[23] Y.M. Wang, Y. Luo and L. Liang, Ranking decision making units by imposing a minimum weight restriction in the data envelopment analysis. *J. Comput. Appl. Math.* **223** (2009) 469–484.

[24] B. Xu and J. Ouenniche, A data envelopment analysis-based framework for the relative performance evaluation of competing crude oil prices volatility forecasting models. *Energ. Econ.* **34** (2012) 576–583.