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AN INVENTORY MODEL WITH IMPERFECT ITEMS, STOCK DEPENDENT DEMAND AND PERMISSIBLE DELAY IN PAYMENTS UNDER INFLATION

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Abstract. This paper develops an economic ordering quantity (EOQ) model with stock dependent demand and imperfect items under the effect of inflation and time value of money. In this model, the lifetime of a defective item follows a Weibull distribution and the imperfect items are reworked at a cost to become the perfect one. In this article, the model is considered with finite replenishment rate under progressive payment scheme within the cycle time and the retailer is allowed a trade-credit offer by the supplier to buy more items. During the credit period, the retailer can earn more by selling their products. The interest on purchasing cost is charged for the delay of payment by the retailer. The objective of this model is to minimize the total inventory cost of the retailer by finding the optimal cycle length and the optimal order quantity. Numerical examples are given to demonstrate the results. Sensitivity analysis of the model with respect to several system parameters has been carried out and the implications are discussed in detail.

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1. Introduction

Inflation plays an essential role for the optimal order policy and influences the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of saving and forces one for more current spending. Usually, these spending are on peripherals and luxury items that give rise to demand of these items. As a result, the effect of inflation and time value of the money cannot be ignored for determining the optimal inventory policy. As mentioned above, inflation has a major effect on the demand of the goods, especially for fashionable goods for middle and higher income groups. The concept of the inflation should be considered especially for long-term investment and forecasting. Buzacott [4] first developed the EOQ model taking inflation into account. Datta and Pal [15] considered the effects of inflation and time value of money on an inventory model with a linear time-dependent demand rate and shortages. Chung [10] derived an algorithm with finite replenishment and infinite planning horizon. Hariga [24] extended Datta and Pal's [15] model to consider flexible replenishment time and decreasing demand rate function. Bierman and Thomas [3] considered an inventory decision under inflationary conditions. Ray and Chaudhuri [44] provided an EOQ model with inflation and time discounting. Thangam and Uthayakumar [59] studied an inventory model for deteriorating

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items by considering inflation induced demand and exponential partial backorders. Valliathal and Uthayakumar [62] analyzed the production inventory model for both ameliorating and deteriorating items taking into account the non-linear shortage cost under inflation and time discounting. Sarkar *et al.* [54] framed an economic manufacturing quantity (EMQ) model for imperfect items allowing time varying demand under inflation and time value of money. Tolgari *et al.* [60] studied an inventory model by considering inspection errors for imperfect items under inflationary conditions. Guria *et al.* [22] gave an inventory policy with immediate part payment by considering inflation induced purchasing and selling prices. Gilding *et al.* [18] described a finite horizon inventory model and gave an optimal inventory replenishment schedule under the effect of inflation. Uthayakumar and Palanivel [61] derived an inventory model with trade credit and Inflation for defective items. Palanivel and Uthayakumar [41] proposed an economic production quantity (EPQ) model by taking into account variable production, probabilistic deterioration and partial backlogging under inflation. Palanivel and Uthayakumar [42] considered variable production cost, time dependent holding cost under inflation in an EPQ model.

In many real-life situations, for certain types of consumer goods (*e.g.*, fruits, vegetables, donuts, and others), the consumption rate is sometimes influenced by the stock-level. It is usually observed that a large pile of goods on shelf in a supermarket will lead the customer to buy more and then generate higher demand. The consumption rate may go up or down with the on-hand stock level. These phenomena attract many marketing researchers to investigate inventory models related to stock-level. The idea that the demand rate would decline along with stock-level throughout the cycle was reflected first in the model developed by Baker and Urban [2]. Datta and Pal [14] modified the concept of the Baker and Urban [2] by assuming that the demand rate would decline along with stock-level down to a certain level, and then it would become constant for the rest of the cycle. Pal *et al.* [40] extended the model to the case of deteriorating items. Later on, Mandal and Phaujdar [35] developed an inventory model with fixed production rate and linear deterioration rate in which demand is linearly dependent on the inventory level. Reddy and Sarma [45] presented a periodic review inventory problem with variable stock dependent demand. Jolai *et al.* [26] determined an economic production lot size model with deteriorating items, stock-dependent demand, inflation, and partial backlogging. Das *et al.* [13] gave a production policy for deteriorating items in a better way by considering permissible delay in payments and stock-dependent demand rate. Yang *et al.* [66] provided an inventory model in which the demand is stock-dependent and shortages are partially backlogged. Dye and Hsieh [17] derived the deterministic ordering policy with price and stock dependent demand when costs are fluctuating with time. Sarkar [50] presented a production in a EOQ environment taking into account delay in payments and stock dependent demand. Patra and Ratha [43] investigated an inventory replenishment policy for deteriorating items in a stock dependent consumption market under inflation. Duan *et al.* [16] studied the inventory models for perishable items with inventory level dependent demand rate. Lee and Dye [28] investigated an inventory model for deteriorating items under stock-dependent demand in which deterioration rate will be controllable. Chung and Cardenas-Barron [11] found out the simplified solution procedure for deteriorating items under stock-dependent demand using two-level trade credit in the supply chain management. Choudhury *et al.* [9] proposed an inventory model for deteriorating items by considering stock-dependent demand, time-varying holding cost in which shortages are allowed. Yang [67] framed an inventory model in which the both demand and holding cost are stock-dependent. Krommyda *et al.* [27] gave an optimal ordering policy with stock-dependent demand for substitutable products.

The suppliers offer delay in payment to the retailers to buy more items and the retailers can sell the items before the closing of the delay time. As a result, the retailers sell the items and earn interests. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period. This provides opportunities to the retailers to accumulate revenue and earn interest by selling their items during the delay period. This permissible delay in payment provides benefit to the supplier by attracting new customers who consider it to be a type of price reduction and reduction in sells outstanding as some customers make payments on time in order to take advantage of permissible delay more frequently. In this direction, Goyal [19] extended the EOQ model under the conditions of permissible delay in payments. Liao *et al.* [29] developed an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible. Teng [58] developed another approach on the EOQ model under conditions of permissible delay in payments. Alfares [1] found out inventory model with stock dependent demand under variable holding cost. Musa and Sani [36] gave an inventory ordering policies of delayed deteriorating items under permissible delay in payments. Rezaei and Salimi [46] developed an EOQ model for imperfect quality items in which inspection shifts from buyer to supplier. Sarkar [51] presented an EOQ model with delay in payments and time varying deterioration rate. Maihami and Abadi [34] derived the pricing policies for non-instantaneously deteriorating items under permissible delay in payments. Lin *et al.* [31] developed an ordering policy for an integrated supplier–retailer inventory model with trade credit. Ouyang *et al.* [38] extended the optimal replenishment policies under two levels of trade credit depending on the order quantity. Liao *et al.* [30] proposed a two warehouse inventory model with trade credit in a supply chain system. Chen and Teng [7] analyzed an ordering policy for deteriorating items under supplier's trade credit financing. Chung *et al.* [12] framed an inventory model for an integrated three layer supply chain system under two levels of trade credit. Wu *et al.* [65] discussed a two-level trade credit financing with expiration dates of the items. Chen *et al.* [8] investigated the concept of conditionally permissible delay in payments link to order quantity. Sarkar *et al.* [56] analyzed a trade-credit policy for fixed lifetime products. Nita and Cardenas-Barron [37] described retailer's ordering policies for deteriorating items when a supplier offers order-linked credit period. Jaggi *et al.* [25] presented an EOQ model under trade credit in different scenarios. Ouyang *et al.* [39] developed an integrated inventory model with order-size dependent trade credit.

In most of the models, researchers considered that the produced products are perfect in nature. But, in real life situation, it is often seen that some of the items may be defective in nature which are reworked at a cost to make them perfect. Depending on this policy, some researchers like Salameh and Jaber [47], Cardenas-Barron [5] and Goyal and Cardenas-Barron [20] discussed an EPQ model for imperfect quality items. Salameh *et al.* [48] developed the model with the effect of deteriorating items on the instantaneous replenishment rate. Goyal *et al.* [21] discussed an EPQ model for imperfect quality items for a deterministic model. Sana *et al.* [49] investigated the EPQ model for deteriorating items using trended demand and allowed shortages. Wee *et al.* [63] proposed an inventory model for imperfect items with partial backordering. Lo *et al.* [32] framed an integrated production-inventory model under imperfect production processes by cosidering Weibull distribution deterioration. Maddah *et al.* [33] used Markov production process for a lot size model in which the imperfect items are scrapped. Sarkar and Moon [54] considered both stochastic and uniform demands over the time horizon in an EPQ system. Sarkar [52] presented an inventory model taking into account reliability in the imperfect production process. Widyadana and Wee [64] analyzed an EPQ model for deteriorating items with rework process. Haidar *et al.* [23] gave an instantaneous replenishment model under a sampling policy. Cardenas-Barron *et al.* [6] framed a economic order quantity model for Ford Whitman Harris. Taleizadeh *et al.* [57] developed a model by considering rework in manufacturing process. Sarkar *et al.* [55] gave an EPQ model with rework process for a single stage production system.

It is clear that, most of the inventory models discussed above for defective item were not developed under situation in which the deterioration follows a Weibull distribution, and defective items are reworked at a cost under inflation and time value of money are investigated. This meticulous attempt on review of literature throw light in identifying that none of the authors considered the combination of trade credit, inflation, stock dependent demand, finite replenishment together with the factors. Hence, we have made an attempt to investigate these issues together and derive a comprehensive model to determine the optimal total cost, as this model has a new managerial approach that helps the retailer to reduce the total inventory cost.

The rest of the paper is organized as follows: in Section 2, the assumptions and notations, which are used throughout this article, are described. In Section 3, the mathematical model to minimize the total annual inventory cost is established. Section 4 presents solution procedure to find the optimal cycle length and optimal order quantity. Numerical examples are provided in Section 5 to illustrate the theory and the solution procedure. This is followed by sensitivity analysis and conclusion.

2. Assumptions and notations

To develop the mathematical model, the following assumptions are made.

2.1. Assumptions

1. A single item is considered over the fixed period.
2. The replenishment takes place at finite rate.
3. The permissible delay in payment is offered by the supplier to the retailer. The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the credit period given by the supplier.
4. There is a rework for imperfect items takes place in a given cycle. That is, the purchased defective units are spotted just after purchasing from the supplier, repaired against the cost and sold as perfect items to the customer.
5. There is no screening process for finding the defective product.
6. The lead time is zero and shortages are not permitted.
7. The inflation rate is constant and the demand is stock dependent.

2.2. Notations

In addition, the following notations are used throughout this paper:

t_1 Duration of the replenishment rate.
T The length of the production inventory cycle.
$q_1(t)$ The inventory level at time $t,\ 0 \leqslant t \leqslant t_1$.
$q_2(t)$ The inventory level at time $t,\ t_1 \leqslant t \leqslant T$.
D Stock dependent demand *i.e.* $D(t) = a + bq(t),\ a > 0,\ b > 0$, where $q(t)$ is the inventory level at time t.
K The constant supply rate of finished goods by the supplier to the retailer.
A The ordering cost (setup cost) per cycle.
C_1 The holding cost (excluding interest charges) per unit per unit time.
p The purchasing cost per unit.
s The selling price per unit, with $s > p$.
C_r The rework cost for the defective item.
M_1 The first offered trade credit period without any charge.
M_2 The second offered trade credit period with charge.
I_{C_1} The rate of interest charged in stock by the supplier for $[M_1,\ M_2]$.
I_{C_2} The rate of interest charged in stock by the supplier for $[M_2,\ T]$, $I_{C_2} > I_{C_1}$.
I_e The rate of interest earned due to financing inventory, $I_{C_2} > I_{C_1} > I_e$.
r The discount rate represents the time value of money.
f The inflation rate.
R The net discount rate of inflation *i.e.* $R = r - f$.
TC_i The total cost of the system for $i = \{1,\ 2,\ 3\}$.

3. Formulation of the model

The inventory system is developed as follows: the inventory cycle starts at $t = 0$ with zero inventory. In the time interval $[0,\ t_1]$, the inventory level increases at a rate K and decreases at a rate D simultaneously. The inventory level is decreasing only due to demand rate in the interval $[t_1, T]$, which finally reaches at the zero level at time T.

Based on the above description, during the time interval $[0,\ t_1]$, the differential equation representing the inventory status is given by

$$\frac{dq_1(t)}{\mathrm{d}t} = K - D(q_1(t)), \quad 0 \leqslant t \leqslant t_1 \tag{3.1}$$

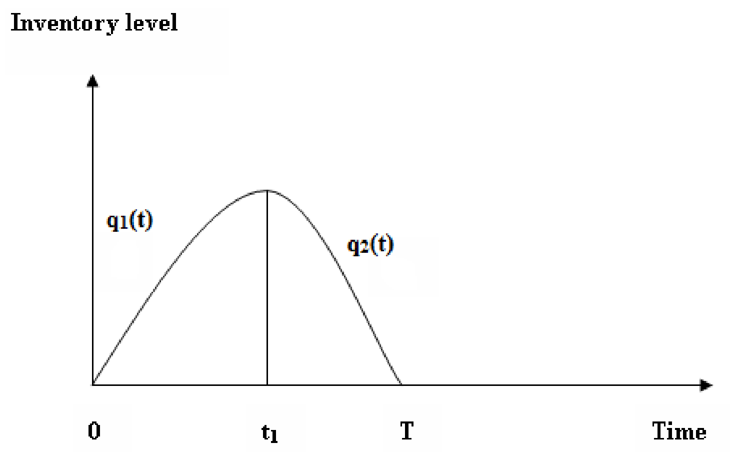

FIGURE 1. Graphical representation of the inventory system.

With the condition $q_1(0) = 0$, the solution of equation (3.1) is

$$q_1(t) = \frac{K-a}{b}\left(1-\mathrm{e}^{-b\,t}\right). \quad 0 \leqslant t \leqslant t_1 \tag{3.2}$$

In the second interval $[t_1,\ T]$, the inventory level decreases due to demand. Thus, the differential equation below represents the inventory status:

$$\frac{\mathrm{d}q_2(t)}{\mathrm{d}t} = -D(q_2(t)), \quad t_1 \leqslant t \leqslant T \tag{3.3}$$

with the condition $q_2(T) = 0$, we get the solution of equation (3.3) is

$$q_2(t) = \frac{a}{b}\left(\mathrm{e}^{b(T-t)} - 1\right) \quad t_1 \leqslant t \leqslant T \tag{3.4}$$

Put $t = t_1$ in equations (3.2) and (3.4) we find the value of t_1 as

$$t_1 = \frac{1}{b}\ln\left(1 + \frac{a\left(\mathrm{e}^{bT}-1\right)}{K}\right)\cdot \tag{3.5}$$

Now we want to find the different inventory costs with the effect of inflation as:

$$\text{Ordering cost (OC)} = \frac{A}{T}\cdot \tag{3.6}$$

$$\text{Inventory holding cost}(HC) = \frac{C_1}{T}\int\limits_0^{t_1} q_1(t)\,\mathrm{e}^{-R\,t}\mathrm{d}t + \int\limits_{t_1}^{T} q_2(t)\,\mathrm{e}^{-R\,t}\mathrm{d}t$$

$$HC = \frac{C_1}{T\,b}\left\{\frac{\mathrm{e}^{-(R+b)\ t_1}}{R+b}\left[K + a\left(\mathrm{e}^{bT-1}\right)\right] + \frac{1}{R+b}\left[a\left(1-\mathrm{e}^{-RT}\right) - K\right] + \frac{1}{R}\left[K\left(1-\mathrm{e}^{-R\,t_1}\right) + a\left(\mathrm{e}^{-RT}-1\right)\right]\right\} \tag{3.7}$$

Along with the delay-in-payments, the paper considers the imperfect items which follow the Weibull distribution. The inventory cycle with imperfect products starts with a constant supply rate $K$and continues up to time t_1. During $[0,\ t_1]$, the inventory piles up after adjusting the stock dependent demand D and after reaching time t_1 it decreases gradually until the zero level at time T. The defective rate which follows a Weibull distribution as $\theta(t) = \alpha\,t^{\beta},\ \beta > -1$, where α and β are two parameters and t is the time at which the item becomes failure [53].

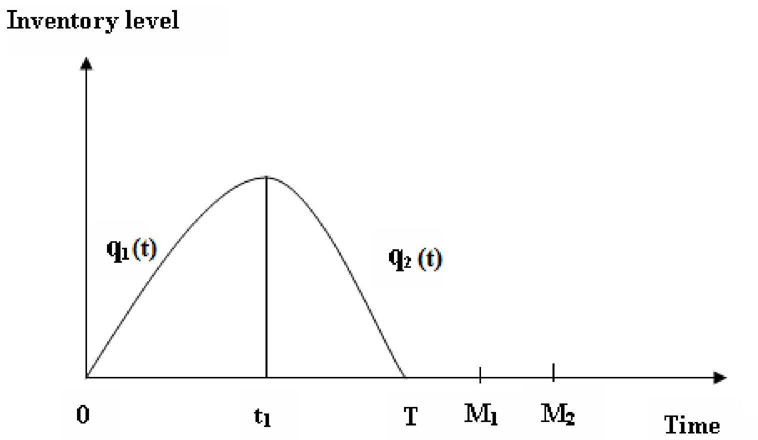

FIGURE 2. $T \leqslant M_1$ (**Case 1**).

Hence, the total number of defective items is:

$$= K \int_0^{t_1} \theta(t)\, \mathrm{e}^{\left[-\int_0^t \theta(u)\mathrm{d}u\right]} \mathrm{d}t = K \left(1 - \mathrm{e}^{\left[-\frac{\alpha\, t_1^{\beta+1}}{\beta+1}\right]}\right). \tag{3.8}$$

Therefore the reworking cost RC with the effect of inflation is

$$RC = \frac{KC_r}{T}\left(\frac{1-\mathrm{e}^{-RT}}{R}\right)\left(1 - \mathrm{e}^{\left[-\frac{\alpha\, t_1^{\beta+1}}{\beta+1}\right]}\right). \tag{3.9}$$

There may arise some cases in delay periods.

Case (3.1). If the retailer pays the purchasing cost within the time M_1 (*i.e.*, $T \leqslant M_1$), then there is no interest charged.

Case (3.2). If the retailer pays the purchasing cost after M_1 and before M_2 (*i.e.*, $M_1 \leqslant T \leqslant M_2$), then the supplier can charge a rate of interest I_{C_1} to the retailer.

Case (3.3). If the retailer pays the purchasing cost after M_2 and before T (*i.e.*, $M_2 \leqslant T$) for the purchased units from the supplier, the supplier can charge a rate of interest I_{C_2} to the retailer.

Therefore the total cost of the system per unit time is given by

$$TC = \begin{cases} TC_1, & T \leqslant M_1 \\ TC_2, & M_1 \leqslant T \leqslant M_2 \\ TC_3, & M_2 \leqslant T \end{cases} \tag{3.10}$$

3.1. Case 1: $T \leqslant M_1$.

In this case, the first period of delay in payment (M_1) is more than the cycle length. The retailer has received all the payment of sales goods from the customers at time T, and makes payment for purchased goods to the supplier at the end of the credit period M_1, but he/she does not need to pay the supplier until the end of the inventory cycle, T. Therefore the retailer uses the sales revenue to earn interest at a rate of I_e up to M_1. And in this case no interest is payable.

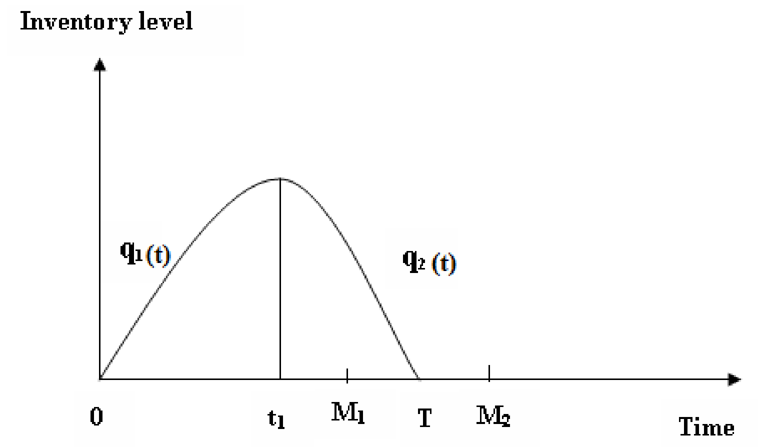

FIGURE 3. $M_1 \leqslant T \leqslant M_2$ (**Case 2**).

Hence the retailer's interest earned per unit time is

$$IE_1 = \frac{sI_e}{T}\left[\int_0^{t_1}(t_1-t)D(q_1(t))\,\mathrm{e}^{-R\,t}\mathrm{d}t + \int_{t_1}^{T}(T-t)D(q_2(t))\,\mathrm{e}^{-R\,t}\mathrm{d}t + \int_{T}^{M_1}K\,t_1\,\mathrm{e}^{-R\,t}\mathrm{d}t\right]$$

$$IE_1 = \frac{sI_e}{T}\left\{\frac{K}{R^2}\left[\mathrm{e}^{-R\,t_1} + Rt_1\left(1+\mathrm{e}^{-RT}-\mathrm{e}^{-RM_1}\right)-1\right] + \frac{at_1}{R+b}\left[1-\frac{K}{a}+\left(\frac{T}{t_1}-1\right)\mathrm{e}^{bT-(R+b)t_1}\right]\right.$$
$$\left. + \frac{1}{(R+b)^2}\left[(K-a)\quad\left(1-\mathrm{e}^{-(R+b)t_1}\right)\ +a(\mathrm{e}^{-RT}-\mathrm{e}^{bT-(R+b)t_1})\right]\right\}. \tag{3.11}$$

Total cost per cycle = ordering cost + inventory holding cost + rework cost − interest earned.

So, the total cost per unit time is

$$TC_1 = OC+HC+RC-IE_1 = \frac{A}{T}+\frac{C_1}{T\,b}\left\{\frac{\mathrm{e}^{-(R+b)\ t_1}}{R+b}\ \left[K+a\left(\mathrm{e}^{bT-1}\right)\right]\right.$$
$$\left. + \frac{1}{R+b}\left[a\left(1-\mathrm{e}^{-RT}\right)-K\right] + \frac{1}{R}\left[K\left(1-\mathrm{e}^{-R\,t_1}\right)+a\left(\mathrm{e}^{-RT}-1\right)\right]\right\}$$
$$+ \frac{KC_r}{T}\left(\frac{1-\mathrm{e}^{-RT}}{R}\right)\left(1-\mathrm{e}^{\left[-\frac{\alpha\,t_1^{\beta+1}}{\beta+1}\right]}\right) - \frac{sI_e}{T}\left\{\frac{K}{R^2}\left[\mathrm{e}^{-R\,t_1}+Rt_1\left(1+\mathrm{e}^{-RT}-\mathrm{e}^{-RM_1}\right)-1\right]\right.$$
$$+ \frac{at_1}{R+b}\left[1-\frac{K}{a}+\left(\frac{T}{t_1}-1\right)\mathrm{e}^{bT-(R+b)t_1}\right]$$
$$\left. + \frac{1}{(R+b)^2}\left[(K-a)\left(1-\mathrm{e}^{-(R+b)t_1}\right)\ +a(\mathrm{e}^{-RT}-\mathrm{e}^{bT-(R+b)t_1})\right]\right\}. \tag{3.12}$$

3.2. Case 2: $M_1 \leqslant T \leqslant M_2$.

In this case the interest charged by the supplier to the retailer is

$$IC_2 = \frac{pI_{C_1}}{T}\int_{M_1}^{T} q_2(t)\,\mathrm{e}^{-R\,t}\mathrm{d}t$$

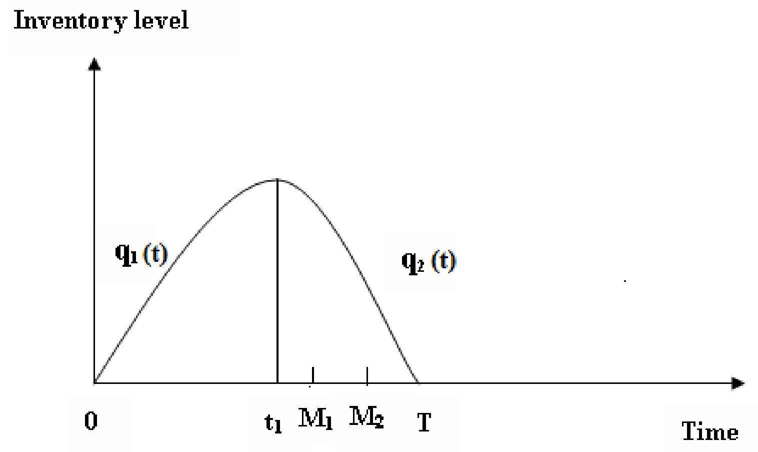

FIGURE 4. $M_2 \leqslant T$ (**Case 3**).

$$IC_2 = \frac{pI_{C_1}a}{bT}\left[\frac{1}{R+b}\left(\mathrm{e}^{bT-(R+b)M_1} - \mathrm{e}^{-RT}\right) + \frac{1}{R}\left(\mathrm{e}^{-RT} - \mathrm{e}^{-RM_1}\right)\right]. \tag{3.13}$$

Interest earned by the retailer is

$$IE_2 = \frac{sI_e}{T}\left[\int_0^{t_1}(t_1-t)D(q_1(t))\,\mathrm{e}^{-R\,t}\mathrm{d}t + \int_{t_1}^{T}(T-t)D(q_2(t))\,\mathrm{e}^{-R\,t}\mathrm{d}t\right]$$

$$\begin{aligned}IE_2 =& \frac{sI_e}{T}\Bigg\{\frac{K}{R^2}\left[\mathrm{e}^{-R\,t_1} + Rt_1 - 1\right] + \frac{at_1}{R+b}\left[1 - \frac{K}{a} + \left(\frac{T}{t_1} - 1\right)\mathrm{e}^{bT-(R+b)t_1}\right]\\ &+ \frac{1}{(R+b)^2}\left[(K-a)(1-\mathrm{e}^{-(R+b)t_1}) \quad + a(\mathrm{e}^{-RT} - \mathrm{e}^{bT-(R+b)t_1})\right]\Bigg\}.\end{aligned} \tag{3.14}$$

Total cost per cycle = ordering cost + inventory holding cost + rework cost + interest payable – interest earned.

So, the total cost per unit time is

$$\begin{aligned}&TC_2 = OC + HC + RC + IC_2 - IE_2\\ &= \frac{A}{T} + \frac{C_1}{T\,b}\left\{\frac{\mathrm{e}^{-(R+b)\;t_1}}{R+b}\left[K + a\left(\mathrm{e}^{bT-1}\right)\right] + \frac{1}{R+b}\left[a\left(1-\mathrm{e}^{-RT}\right) - K\right] + \frac{1}{R}\left[K\left(1-\mathrm{e}^{-R\,t_1}\right) + a\left(\mathrm{e}^{-RT} - 1\right)\right]\right\}\\ &+ \frac{KC_r}{T}\left(\frac{1-\mathrm{e}^{-RT}}{R}\right)\left(1 - \mathrm{e}^{\left[-\frac{\alpha\,t_1^{\beta+1}}{\beta+1}\right]}\right) + \frac{pI_{C_1}a}{bT}\left[\frac{1}{R+b}\left(\mathrm{e}^{bT-(R+b)M_1} - \mathrm{e}^{-RT}\right) + \frac{1}{R}\left(\mathrm{e}^{-RT} - \mathrm{e}^{-RM_1}\right)\right]\\ &- \frac{sI_e}{T}\Bigg\{\frac{K}{R^2}\left[\mathrm{e}^{-R\,t_1} + Rt_1 - 1\right] + \frac{at_1}{R+b}\left[1 - \frac{K}{a} + \left(\frac{T}{t_1} - 1\right)\mathrm{e}^{bT-(R+b)t_1}\right] + \frac{1}{(R+b)^2}\Big[(K-a)(1-\mathrm{e}^{-(R+b)t_1})\\ &+ a(\mathrm{e}^{-RT} - \mathrm{e}^{bT-(R+b)t_1})\Big]\Bigg\}.\end{aligned} \tag{3.15}$$

3.3. Case 3: $M_2 \leqslant T$.

In this case, the retailer pays to the supplier at the end of the 2nd offered credit period, M_2, which is before the inventory is depleted completely. Hence, the retailer still has some stock on hand during the time interval $[M_2, T]$. Therefore, the interest charged by the supplier to the retailer per unit time is

$$IC_3 = \frac{pI_{C_2}}{T} \int_{M_2}^{T} q_2(t)\, \mathrm{e}^{-R\,t} \mathrm{d}t$$

$$IC_3 = \frac{pI_{C_2} a}{bT} \left[\frac{1}{R+b} \left(\mathrm{e}^{bT-(R+b)M_2} - \mathrm{e}^{-RT} \right) + \frac{1}{R} \left(\mathrm{e}^{-RT} - \mathrm{e}^{-RM_2} \right) \right]. \tag{3.16}$$

Furthermore, for the items that have already sold but have not yet been paid during the time interval $[0,\ M_2]$. That is, the retailer sells a certain amount of units and deposits the revenue into the interest earning account at a rate I_{e}. Thus, the retailer earn an interest per unit time is

$$IE_3 = \frac{sI_e}{T} \left[\int_0^{t_1} (t_1 - t) D(q_1(t))\, \mathrm{e}^{-R\,t} \mathrm{d}t + \int_{t_1}^{M_2} (T - t) D(q_2(t))\, \mathrm{e}^{-R\,t} \mathrm{d}t \right]$$

$$\begin{aligned} IE_3 = \frac{sI_e}{T} \Bigg\{ & \frac{K}{R^2} \left[\mathrm{e}^{-R\,t_1} + Rt_1 - 1 \right] + \frac{1}{R+b} \left[(a-K)t_1 + a(M_2 - T)\, \mathrm{e}^{bT-(R+b)M_2} + a(T - t_1)\mathrm{e}^{bT-(R+b)t_1} \right] \\ & + \frac{1}{(R+b)^2} \left[(K-a)(1 - \mathrm{e}^{-(R+b)t_1}) + a\mathrm{e}^{bT} (\mathrm{e}^{-(R+b)M_2} - \mathrm{e}^{-(R+b)t_1}) \right] \Bigg\}. \end{aligned} \tag{3.17}$$

Total cost per cycle = ordering cost + inventory holding cost + rework cost + interest payable – interest earned.

So, the total cost per unit time is

$$\begin{aligned} TC_3 = & OC + HC + RC + IC_3 - IE_3 \\ = & \frac{A}{T} + \frac{C_1}{T\,b} \Bigg\{ \frac{\mathrm{e}^{-(R+b)\ t_1}}{R+b} \left[K + a \left(\mathrm{e}^{bT-1} \right) \right] \\ & + \frac{1}{R+b} \left[a \left(1 - \mathrm{e}^{-RT} \right) - K \right] + \frac{1}{R} \left[K \left(1 - \mathrm{e}^{-R\,t_1} \right) + a \left(\mathrm{e}^{-RT} - 1 \right) \right] \Bigg\} \\ & + \frac{KC_r}{T} \left(\frac{1 - \mathrm{e}^{-RT}}{R} \right) \left(1 - \mathrm{e}^{\left[-\frac{\alpha\, t_1^{\beta+1}}{\beta+1} \right]} \right) + \frac{pI_{C_2} a}{bT} \left[\frac{1}{R+b} \left(\mathrm{e}^{bT-(R+b)M_2} - \mathrm{e}^{-RT} \right) + \frac{1}{R} \left(\mathrm{e}^{-RT} - \mathrm{e}^{-RM_2} \right) \right] \\ & - \frac{sI_e}{T} \Bigg\{ \frac{K}{R^2} \left[\mathrm{e}^{-R\,t_1} + Rt_1 - 1 \right] + \frac{1}{R+b} \left[(a-K)t_1 + a(M_2 - T)\, \mathrm{e}^{bT-(R+b)M_2} + a(T - t_1)\mathrm{e}^{bT-(R+b)t_1} \right] \\ & + \frac{1}{(R+b)^2} \left[(K-a)(1 - \mathrm{e}^{-(R+b)t_1}) + a\mathrm{e}^{bT} (\mathrm{e}^{-(R+b)M_2} - \mathrm{e}^{-(R+b)t_1}) \right] \Bigg\} \end{aligned} \tag{3.18}$$

4. Solution procedure

In order to find the optimal solution T^* and to minimize the annual total relevant cost, we take the first and second order derivatives of $TC_i(T)$ with respect to T, where $i = \{1,\ 2, 3\}$. In other words, the necessary and sufficient conditions for minimization of $TC_i(T)$ are respectively $\frac{\mathrm{d}TC_i(T)}{\mathrm{d}T} = 0$ and $\frac{\mathrm{d}^2 TC_i(T)}{\mathrm{d}T^2} > 0$, where $i = \{1,\ 2, 3\}$.

4.1. Case 1: $T \leqslant M_1$.

The necessary and sufficient conditions to minimize $TC_1(T)$ are respectively $\frac{\mathrm{d}TC_1(T)}{\mathrm{d}T}=0$ and $\frac{\mathrm{d}^2TC_1(T)}{\mathrm{d}T^2}>0$. Now $\frac{\mathrm{d}TC_1(T)}{\mathrm{d}T}=0$ gives the following equation in T.

$$
\begin{aligned}
&\frac{-1}{T}\left\{A+\frac{C_1}{b}\left\{\frac{\mathrm{e}^{-(R+b)t_1}}{R+b}\left[K+a\left(\mathrm{e}^{bT-1}\right)\right]\right.\right.\\
&+\frac{1}{R+b}\left[a\left(1-\mathrm{e}^{-RT}\right)-K\right]+\frac{1}{R}\left[K\left(1-\mathrm{e}^{-Rt_1}\right)+a\left(\mathrm{e}^{-RT}-1\right)\right]\Big\}\\
&+\frac{KC_r}{T}\left(\frac{1-\mathrm{e}^{-RT}}{R}\right)\left(1-\mathrm{e}^{\left[-\frac{\alpha t_1^{\beta+1}}{\beta+1}\right]}\right)-\frac{sI_e}{T}\left\{\frac{K}{R^2}\left[\mathrm{e}^{-Rt_1}+Rt_1\left(1+\mathrm{e}^{-RT}-\mathrm{e}^{-RM_1}\right)-1\right]\right.\\
&\left.\left.+\frac{at_1}{R+b}\left[1-\frac{K}{a}+\left(\frac{T}{t_1}-1\right)\mathrm{e}^{bT-(R+b)t_1}\right]+\frac{1}{(R+b)^2}\left[(K-a)\ \left(1-\mathrm{e}^{-(R+b)t_1}\right)\ +a(\mathrm{e}^{-RT}-\mathrm{e}^{bT-(R+b)t_1})\right]\right\}\right\}\\
&+\mathrm{e}^{-RT}\left\{\frac{C_1a}{(R+b)}\left(\mathrm{e}^{(R+b)(T-t_1)}-1\right)+KC_r\left(1-\mathrm{e}^{-\frac{\alpha t_1^{\beta+1}}{\beta+1}}\right)\right.\\
&\left.-sI_e\left[-Kt_1+\frac{a}{R+b}\mathrm{e}^{(R+b)(T-t_1)}\left(1+b(T-t_1)\right)\ -\frac{1}{(R+b)^2}\left(aR+b\mathrm{e}^{(R+b)(T-t_1)}\right)\right]\right\}=0 \qquad (4.1)
\end{aligned}
$$

Provided T satisfies the sufficient condition $\frac{\mathrm{d}^2TC_1(T)}{\mathrm{d}T^2}>0$.

By solving equation (4.1) the optimal value of T^* can be obtained and then from equations (3.5) and (3.12), the optimal value of t_1^* and $TC=TC_1^*$ can be found out respectively.

4.2. Case 2: $M_1 \leqslant T \leqslant M_2$.

The necessary and sufficient conditions to minimize $TC_2(T)$ are respectively $\frac{\mathrm{d}TC_2(T)}{\mathrm{d}T}=0$ and $\frac{\mathrm{d}^2TC_2(T)}{\mathrm{d}T^2}>0$. Now $\frac{\mathrm{d}TC_2(T)}{\mathrm{d}T}=0$ gives the following equation in T.

$$
\begin{aligned}
&\frac{-1}{T}\left\{A+\frac{C_1}{b}\left\{\frac{\mathrm{e}^{-(R+b)\ t_1}}{R+b}\ \left[K+a\left(\mathrm{e}^{bT-1}\right)\right]+\frac{1}{R+b}\left[a\left(1-\mathrm{e}^{-RT}\right)-K\right]\right.\right.\\
&+\frac{1}{R}\left[K\left(1-\mathrm{e}^{-Rt_1}\right)+a\left(\mathrm{e}^{-RT}-1\right)\right]\Big\}\\
&+KC_r\left(\frac{1-\mathrm{e}^{-RT}}{R}\right)\left(1-\mathrm{e}^{\left[-\frac{\alpha t_1^{\beta+1}}{\beta+1}\right]}\right)+\frac{pI_{C_1}a}{b}\left[\frac{1}{R+b}\left(\mathrm{e}^{bT-(R+b)M_1}-\mathrm{e}^{-RT}\right)+\frac{1}{R}\left(\mathrm{e}^{-RT}-\mathrm{e}^{-RM_1}\right)\right]\\
&-sI_e\left\{\frac{K}{R^2}\left[\mathrm{e}^{-Rt_1}+Rt_1-1\right]+\frac{at_1}{R+b}\left[1-\frac{K}{a}+\left(\frac{T}{t_1}-1\right)\mathrm{e}^{bT-(R+b)t_1}\right]\right.\\
&\left.\left.+\frac{1}{(R+b)^2}\left[(K-a)(1-\mathrm{e}^{-(R+b)t_1})\quad+a(\mathrm{e}^{-RT}-\mathrm{e}^{bT-(R+b)t_1})\right]\right\}\right\}\\
&+\frac{\mathrm{e}^{-RT}}{(R+b)}\left\{C_1a\left(\mathrm{e}^{(R+b)(T-t_1)}-1\right)+(R+b)\,KC_r\left(1-\mathrm{e}^{\left[-\frac{\alpha t_1^{\beta+1}}{\beta+1}\right]}\right)\right.\\
&\left.+I_{C_1}pa\left(\mathrm{e}^{(R+b)(T-M_1)}-1\right)-sI_e\left[a\mathrm{e}^{(R+b)(T-t_1)}\left(1+b(T-t_1)\right)-\frac{1}{R+b}\left(aR+b\mathrm{e}^{(R+b)(T-t_1)}\right)\right]\right\}=0 \quad (4.2)
\end{aligned}
$$

Provided T satisfies the sufficient condition $\frac{\mathrm{d}^2TC_2(T)}{\mathrm{d}T^2}>0$.

By solving equation (4.2) the optimal value of T^* can be obtained and then from equations (3.5) and (3.15), the optimal value of t_1^* and $TC=TC_2^*$ can be found out respectively.

4.3. Case 3: $M_2 \leqslant T$.

The necessary and sufficient conditions to minimize $TC_3(T)$ are respectively $\frac{\mathrm{d}TC_3(T)}{\mathrm{d}T} = 0$ and $\frac{\mathrm{d}^2TC_3(T)}{\mathrm{d}T^2} > 0$. Now $\frac{\mathrm{d}TC_3(T)}{\mathrm{d}T} = 0$ gives the following equation in T.

$$\begin{aligned}
&\frac{-1}{T}\Bigg\{A+\frac{C_1}{b}\Bigg\{\frac{\mathrm{e}^{-(R+b)\,t_1}}{R+b}\Big[K+a\left(\mathrm{e}^{bT-1}\right)\Big]+\frac{1}{R+b}\Big[a\left(1-\mathrm{e}^{-RT}\right)-K\Big]+\frac{1}{R}\Big[K\left(1-\mathrm{e}^{-R\,t_1}\right)+a\left(\mathrm{e}^{-RT}-1\right)\Big]\Bigg\}\\
&+KC_r\left(\frac{1-\mathrm{e}^{-RT}}{R}\right)\left(1-\mathrm{e}^{\left[-\frac{\alpha\,t_1^{\beta+1}}{\beta+1}\right]}\right)+\frac{pI_{C_2}a}{b}\left[\frac{1}{R+b}\left(\mathrm{e}^{bT-(R+b)M_2}-\mathrm{e}^{-RT}\right)+\frac{1}{R}\left(\mathrm{e}^{-RT}-\mathrm{e}^{-RM_2}\right)\right]\\
&-sI_e\Bigg\{\frac{K}{R^2}\left[\mathrm{e}^{-R\,t_1}+Rt_1-1\right]+\frac{1}{R+b}\Big[(a-K)t_1+a(M_2-T)\,\mathrm{e}^{bT-(R+b)M_2}\\
&+a(T-t_1)\mathrm{e}^{bT-(R+b)t_1}\Big]\quad+\frac{1}{(R+b)^2}\Big[(K-a)(1-\mathrm{e}^{-(R+b)t_1})+a\mathrm{e}^{bT}(\mathrm{e}^{-(R+b)M_2}-\mathrm{e}^{-(R+b)t_1})\Big]\Bigg\}\Bigg\}\\
&+\frac{a\mathrm{e}^{-RT}}{(R+b)}\Bigg\{C_1\left(\mathrm{e}^{(R+b)(T-t_1)}-1\right)+KC_r\left(\frac{R+b}{a}\right)\left(1-\mathrm{e}^{\left[-\frac{\alpha\,t_1^{\beta+1}}{\beta+1}\right]}\right)+I_{C_2}p\left(\mathrm{e}^{(R+b)(T-M_2)}-1\right)\\
&-sI_e\left[\mathrm{e}^{(R+b)(T-M_2)}\left(b(M_2-T)-\frac{R}{R+b}\right)\;+\mathrm{e}^{(R+b)(T-t_1)}\left(b(T-t_1)+\frac{R}{R+b}\right)\right]\Bigg\}=0 \qquad (4.3)
\end{aligned}$$

Provided T satisfies the sufficient condition $\frac{\mathrm{d}^2TC_3(T)}{\mathrm{d}T^2} > 0$.

By solving equation (4.3) the optimal value of T^* can be obtained and then from equations (3.5) and (3.18), the optimal value of t_1^* and $TC = TC_3^*$ can be found out respectively.

Since $TC_i\,(T)$ for $i = \{1,2,3\}$ are very complicated functions due to non-linear and high-power expression of the exponential function, it is unlikely to show analytically the validity of the above sufficient conditions. However, it can be assessed numerically in the following illustrative examples. By examining the second order sufficient condition for above three cases, it can be verified that the total cost TC_i for $i = \{1,2,3\}$ are convex functions with respect to T. The convexity of the total cost function is graphically illustrated through the following numerical examples.

5. Numerical examples

To illustrate the solution procedure, let us solve the following numerical examples and the results can be found by applying the Matlab 6.5. The numerical examples given below cover all the three cases that arise in the model.

Example 1 (Case 1).

Consider an inventory system with the following data: $A = 200$; s $= 20$; $a = 15$; $b = 0.5$; $K = 500$; $M_1 = 1.4$; $I_\mathrm{e} = 0.15$; $I_{\mathrm{c}1} = 0.18$; $I_{\mathrm{c}2} = 0.20$; $M_2 = 1.65$; $C_1 = 15$; $p = 10$; $\alpha = 0.01$; $\beta = 0.053$; $C_\mathrm{r} = 1.5$; $R = 0.1$ in appropriate units.

Then we get the optimal values as $t_1^* = 0.0449$, $T^* = 1.1270$, $TC_1^* = 272.9799$ in appropriate units.

The graph (Fig. 5) shows that the function TC_1 is convex with respect T. The graph (Fig. 6) shows that the function TC_1 is convex with respect to t_1 and T.

Example 2 (Case 2).

Consider an inventory system with the following data: $A = 200$; $s = 20$; $a = 15$; $b = 0.5$; $K = 500$; $M_1 = 1.2$; $\mathrm{I_e} = 0.15$; $\mathrm{Ic}_1 = 0.18$; $\mathrm{Ic}_2 = 0.20$; $M_2 = 1.65$; $C_1 = 15$; $p = 10$; $\alpha = 0.01$; $\beta = 0.053$; $\mathrm{C_r} = 1.5$; $R = 0.1$ in appropriate units.

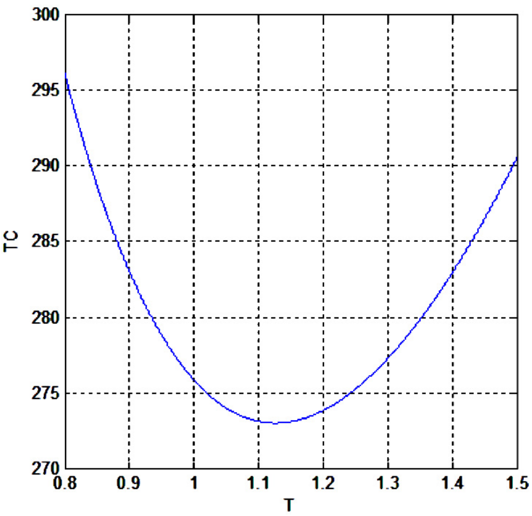

FIGURE 5. The total cost (Example 1) with respect to T.

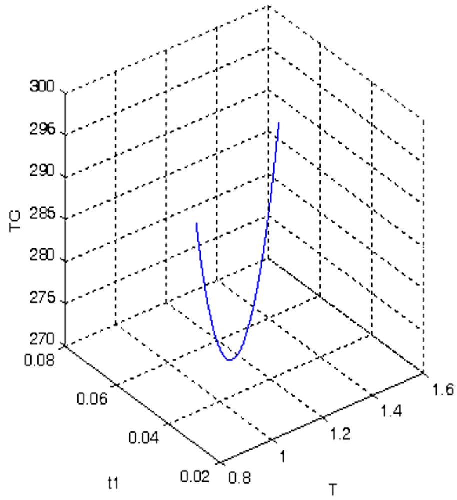

FIGURE 6. The total cost (Example 1) with respect to T and t_1.

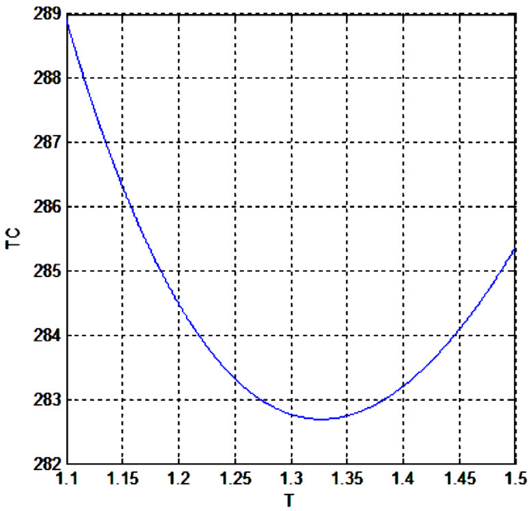

FIGURE 7. The total cost (Example 2) with to T.

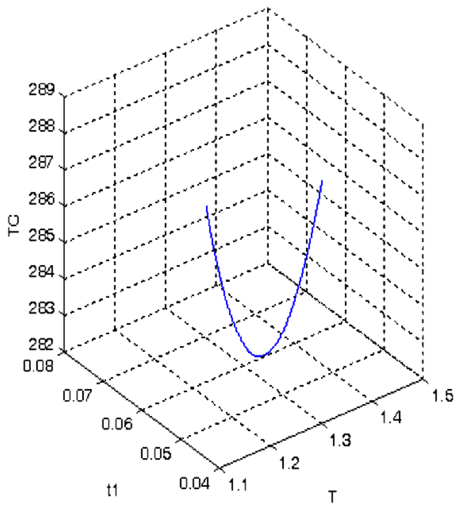

FIGURE 8. The total cost (Example 2) with respect to T and t_1.

Then we get the optimal values as $t_1^* = 0.0557$, $T^* = 1.3274$, $TC_2^* = 282.6910$ in appropriate units. The graph (Fig. 7) shows that the function TC_2 is convex with respect to T. The graph (Fig. 8) shows that the function TC_2 is convex with respect to t_1 and T.

Example 3 (Case 3).
Consider an inventory system with the following data: $A = 200$; $s = 20$; $a = 15$; $b = 0.5$; $K = 500$; $M_1 = 1.1$; $I = 0.15$; $Ic_{c1} = 0.18$; $I_{c2} = 0.20$; $M_2 = 1.3$; $C_1 = 15$; $p = 10$; $\alpha = 0.01$; $\beta = 0.053$; $C_r = 1.5$; $R = 0.1$ in appropriate units.

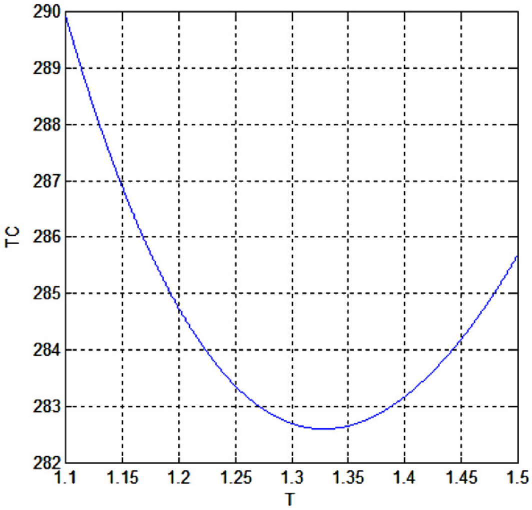

FIGURE 9. The total cost (Example 3) with respect to T.

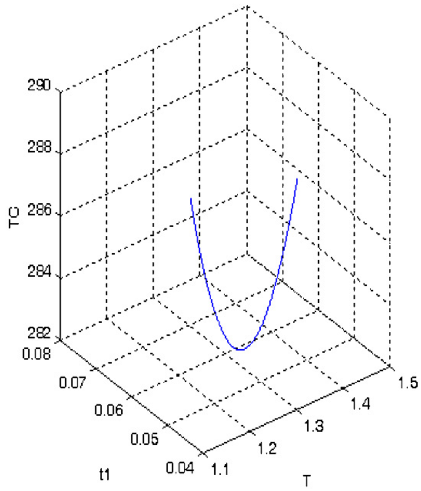

FIGURE 10. The total cost (Example 3) with respect to T and t_1.

Then we get the optimal values as $t_1^* = 0.0558$, $T^* = 1.3290$, $TC_3^* = 282.5954$ in appropriate units.

The graph (Fig. 9) shows that the function TC_3 is convex with respect to T. The graph (Fig. 10) shows that the function TC_3 is convex with respect to t_1 and T.

When the supplier does not provide a credit period (*i.e.*, $M_1 = 0$ and $M_2 = 0$), the optimal retailer's total cost can be found as $TC^* = 319.7550$. It can be seen that optimal total cost increases. So, retailers should try to get credit periods for their payments and if they wish to decrease their total cost.

6. Sensitivity analysis

The change in the values of parameters may happen due to uncertainties in any decision- making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision-making. We now study the effects of changes in the values of the system parameters a, b, α, β, R, C_{r}, A and M_1 on the optimal length of production period, t_1^*, the optimal production cycle T^* and the optimal total cost TC^*. The sensitivity analysis is performed by changing each of the parameters by −50%, −25%, +25%, +50%, taking one parameter at a time and keeping the remaining parameters unchanged. The analysis is based on the Example 1 and the results are shown in Table 1.

Based on our numerical results, we obtain the following managerial phenomena:

1) The optimal length of production cycle (T^*) decreases while the duration of replenishment (t_1^*) and the total cost of the system (TC^*) increase with increase in the value of the parameter a. This fact may occur in real life business, because, if the buyer's demand increases, then it will increase the order quantity, so that the total cost automatically will be increased.
2) The optimal length of production cycle (T^*) decreases while the duration of replenishment (t_1^*) and the total cost of the system (TC^*) increase with increase in the value of the parameter b.
3) The duration of replenishment (t_1^*) and the optimal length of production cycle (T^*) decrease while the total cost of the system (TC^*) increases with increase in the value of the parameter α That is, minimum deterioration rate of the products will minimize the deterioration cost of the items for the retailer. Therefore, if the retailer minimizes the deterioration rate of the item, then he/she can reduce the total cost.
4) The duration of replenishment (t_1^*) and the optimal length of production cycle (T^*) increase while the total cost of the system (TC^*) decreases with increase in the value of the parameter β.

Table 1. Sensitivity analysis for various inventory parameters.

Parameter	% change	t_1^*	T^*	TC^*
a	−50	0.03334	1.50323	213.2491
	−25	0.03960	1.27186	247.0668
	+25	0.04959	1.02506	247.0668
	−50	0.05385	0.94816	311.0198
b	−50	0.04167	1.19789	266.7873
	−25	0.04326	1.16037	270.0085
	+25	0.04660	1.09716	275.7138
	+50	0.04837	1.07026	278.2203
α	−50	0.04492	1.12748	272.8516
	−25	0.04491	1.12725	272.9157
	+25	0.04488	1.12679	273.0440
	+50	0.04487	1.12656	273.1081
β	−50	0.044891	1.12694	273.0090
	−25	0.044893	1.12698	272.9940
	+25	0.044893	1.12698	272.9665
	+50	0.044905	1.12709	272.9538
R	−50	0.04389	1.10734	274.0200
	−25	0.04439	1.11707	273.5168
	+25	0.04542	1.13719	272.4099
	+50	0.04596	1.14760	271.8074
C_r	−50	0.04492	1.12748	272.8516
	−25	0.04491	1.12725	272.9157
	+25	0.04488	1.12679	273.0440
	+50	0.04487	1.12655	273.1081
A	−50	0.03055	0.82832	170.9929
	−25	0.03821	0.99319	225.8402
	+25	0.05095	1.24126	315.1896
	+50	0.05655	1.34175	353.8941
M_1	−50	0.04340	1.09772	310.5057
	−25	0.04414	1.11231	291.4502
	+25	0.04566	1.14184	255.0761
	+50	0.04644	1.15679	237.7208

5) The duration of replenishment (t_1^*) and the optimal length of production cycle (T^*) increase while the total cost of the system (TC^*) decreases with increase in the value of the net discount rate of inflation R. That is, for higher values of the net discount rate of inflation, the total cost of the retailer will be minimized.
6) The duration of replenishment (t_1^*) and the optimal length of production cycle (T^*) decrease while the total cost of the system (TC^*) increases with increase in the value of the rework cost for the defective item C_{r}.That is, the minimum cost for reworking the defective items will minimize the total cost of the retailer.
7) The duration of replenishment (t_1^*) and the optimal length of production cycle (T^*) and the total cost of the system (TC^*) increase with increase in the value of the ordering cost A. That is, minimum ordering cost will minimize the total cost of the retailer.
8) The duration of replenishment (t_1^*) and the optimal length of production cycle (T^*) increase while the total cost of the system (TC^*) decreases with increase in the value of the first offered trade credit period without any charge M_1. That is, when the permissible delay period increasing, the total cost TC decreasing. This is expected because when the trade credit period goes up, the customer orders a large quantity. This large quantity will increase the profit of the retailer. Therefore, increasing the trade credit will decrease the total cost of the retailer.

When the setup cost (A) and the parameter a are increase, the total cost of the system (TC) highly increases, whereas the permissible delay period (M_1) increases, the total cost of the system (TC) highly decreases.

6.1. Some special cases

1) When $b = 0$, the demand of the inventory system will be constant.
2) When $\beta = 0$, the deterioration rate of the item will be constant.
3) When $\alpha = 0$, there is no deterioration in this model.

7. Conclusion

In the marketing management policy, display stock level plays a very important role in different sectors. Thus, it is very clear that the demand rate increased rapidly if the stored amount is high and vice-versa. In the present situation, inflation and time value of money are also main factors. In keeping with this reality, these factors are incorporated in the present model. In the business transactions, the supplier usually offers a permissible delay in payment to his/her retailer to attract more sales. Based on the above phenomena, in this paper, a model for imperfect items is developed where delay in payment is allowed. Finite replenishment rate, stock dependent demand pattern and Weibull parameter deterioration rate are considered in this model and the imperfect items are reworked at a cost to become the perfect one. The aim of this paper is to obtain the optimal solution of cycle length, time intervals and order quantity simultaneously with the objective of minimizing total cost of the retailer.

The necessary and sufficient conditions of the existence and uniqueness of the optimal solutions are also provided. The proposed solution procedure in this model is simple and does not require tedious computation effort. Numerical examples and sensitivity analysis are given to illustrate the application and the performance of the proposed methodology. The graphical illustrations are also given to analyze the efficiency of the model clearly. The results show that, when A and a increase, the total cost highly increases, whereas M_1 increases, the total cost is highly decreases. When the delay in payment is allowed, the total cost for the retailer become decreases. To the authors' best of knowledge, this type of model has not yet been considered by any of the researchers in inventory literature. Therefore, this model has a new managerial insight that helps the retailer to reduce the total inventory cost.

The proposed model incorporates some realistic features that are likely to be associated with some kinds of inventory. This model can be adopted in inventory control of the system such as food industries, other manufacturing companies, domestic goods *etc.*, There are several extensions of this work that could constitute future research related to this field. One immediate probable extension could be to discuss the effect of shortages. Another possible extension of this work may be conducted by considering multi item. Furthermore, some major parameters of the model such as demand rate, supply rate, holding cost, and ordering cost may be fuzzy variables.

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References

[1] H.K. Alfares, Inventory model with stock-level dependent demand rate and variable holding cost. *Int. J. Prod. Econ.* **108** (2007) 259–265.
[2] R.C. Baker and T.L. Urban, A deterministic inventory system with an inventory-level-dependent demand rate. *J. Oper. Res. Soc.* **39** (1988) 823–831.
[3] H. Bierman and J. Thomas, Inventory decisions under inflationary conditions. *Decis. Sci.* **8** (1977) 151–155.
[4] J.A. Buzacott, Economic order quantities with inflation. *Oper. Res. Quarterly* **26** (1975) 553–558.

[5] L.E. Cardenas-Barron, Observation on: economic production quantity model for items with imperfect quality. *Int. J. Prod. Econ.* **64** (2000) 59–64.

[6] L.E. Cardenas-Barron, K.J. Chung and G. Treviño-Garza, Celebrating a century of the economic order quantity model in honor of Ford Whitman Harris. *Int. J. Prod. Econ.* **155** (2014) 1–7.

[7] S.C. Chen, L.E. Cardenas-Barron and J.T. Teng, Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity. *Int. J. Prod. Econ.* **155** (2014) 284–291.

[8] S.C. Chen and J.T. Teng, Retailer's optimal ordering policy for deteriorating items with maximum lifetime under supplier's trade credit financing. *Appl. Math. Model.* **38** (2014) 4049–4061.

[9] K.D. Choudhury, B. Karmakar, M. Das and T.K. Datta, An inventory model for deteriorating items with stock-dependent demand, time-varying holding cost and shortages. *Opsearch* **52** (2013) 55–74.

[10] K.J. Chung, Optimal ordering time interval taking account of time value. *Prod. Plann. Control* **7** (1996) 264–267.

[11] K.J. Chung and L.E. Cardenas-Barron, The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade credit in the supply chain management. *Appl. Math. Model.* **37** (2013) 4653–4660.

[12] K.J. Chung, L.E. Cardenas-Barron, P.S. Ting, An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit. *Int. J. Prod. Econ.* **155** (2014) 310–317

[13] D. Das and A. Roy, Samarjit Improving production policy for a deteriorating item under permissible delay in payments with stock-dependent demand rate. *Comput. Math. Appl.* **60** (2010) 1973–1985.

[14] T.K. Datta and A.K. Pal, Deterministic inventory systems for deteriorating items with inventory level dependent demand rate and shortages. *J. Oper. Res. Soc.* **27** (1990) 213–224.

[15] T.K. Datta and A.K. Pal, Effects of inflation and time value of money on an inventory model with linear time dependent demand rate and shortages. *Eur. J. Oper. Res.* **52** (1991) 326–333.

[16] Y. Duan, G. Li, J.M. Tien and J. Huo, Inventory models for perishable items with inventory level dependent demand rate. *Appl. Math. Model.* **36** (2012) 5015–5028.

[17] C.Y. Dye and T.P. Hsieh, Deterministic ordering policy with price- and stock-dependent demand under fluctuating cost and limited capacity. *Expert Syst. Appl.* **38** (2011) 14976–14983.

[18] B.H. Gilding, Inflation and the optimal inventory replenishment schedule within a finite planning horizon. *Eur. J. Oper. Res.* **234** (2014) 683–693.

[19] S.K. Goyal, Economic order quantity under conditions of permissible delay in payments. *J. Oper. Res. Soc.* **36** (1985) 335–338.

[20] S.K. Goyal, Cardenas-Barron LE Note on economic production quantity model for items with imperfect quality a practical approach. *Int. J. Prod. Econ.* **77** (2002) 85–87.

[21] S.K. Goyal, C.H. Huang and K.C. Chang, Note on economic production quantity model for items with imperfect quality a practical approach. *Prod. Plann. Control* **14** (2003) 596–602.

[22] A. Guria, B. Das, S. Mondal and M.Maiti, Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. *Appl. Math. Model.* **37** (2013) 240–257.

[23] L.M. Haidar, M. Salameh and W. Nasr, An instantaneous replenishment model under the effect of a sampling policy for defective items. *Appl. Math. Model.* **37** (2013) 719–727.

[24] Hariga M Effects of inflation and time value of money on an inventory model with time dependent demand rate and shortages. *Eur. J. Oper. Res.* **81** (1995) 512–520.

[25] C.K. Jaggi, V.S.S. Yadavalli, M. Verma, A.Sharma, An EOQ model with allowable shortage under trade credit in different scenario. *Appl. Math. Comput.* **252** (2015) 541–551.

[26] R.F. Jolai, T. Moghaddam, M. Rabbani, M.R. Sadoughian, An economic production lot size model with deteriorating items, stock-dependent demand, inflation, and partial backlogging. *Appl. Math. Comput.* **181** (2006) 380–389

[27] I.P. Krommyda, K. Skouri, I. Konstantaras, Optimal ordering quantities for substitutable products with stock-dependent demand. *Appl. Math. Model.* **39** (2015) 147–164.

[28] Y.P. Lee, C.Y. Dye, An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. *Comput. Ind. Eng.* **63** (2012) 474–482.

[29] H.C. Liao, CH. Tsai and C.T. Su, An inventory model with deteriorating items under inflation when a delay in payment is permissible. *Int. J. Prod. Econ.* **63** (2000) 207–214.

[30] J.J. Liao, K.J. Chung and K.N. Huang, A deterministic inventory model for deteriorating items with two warehouses and trade credit in a supply chain system. *Int. J. Prod. Econ.* **146** (2013) 557–565

[31] Y.J. Lin, L.Y. Ouyang and Y.F. Dang, A joint optimal ordering and delivery policy for an integrated supplier–retailer inventory model with trade credit and defective items. *Appl. Math. Comput.* **218** (2012) 7498–7514.

[32] S.T. Lo, H.M. Wee and W.C. Huang, An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation. *Int. J. Prod. Econ.* **106** (2007) 248–260

[33] B. Maddah, L. Moussawi and M.Y. Jaber, Lot sizing with a Markov production process and imperfect items scrapped. *Int. J. Prod. Econ.* **124** (2010) 340–347.

[34] R. Maihami and I.N.K. Abadi, Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging. *Math. Comput. Model.* **55** (2012) 1722–1733.

[35] B.N. Mandal and S. Phaujdar, An inventory model for deteriorating items and stock dependent consumption rate. *J. Oper. Res. Soc.* **40** (1989) 483–488

[36] A. Musa and B. Sani, Inventory ordering policies of delayed deteriorating items under permissible delay in payments. *Int. J. Prod. Econ.* **136** (2012) 75–83.
[37] H.S. Nita and L.E. Cardenas-Barron, Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount. *Appl. Math. Comput.* **259** (2015) 569–578.
[38] L.Y. Ouyang, C.T. Yang, Y.L. Chan and L.E. Cardenas-Barron, A comprehensive extension of the optimal replenishment decisions under two levels of trade credit policy depending on the order quantity. *Appl. Math. Comput.* **224** (2013) 268–277.
[39] L.Y. Ouyang, CH. Ho, CH. Su and C.T. Yang, An integrated inventory model with capacity constraint and order-size dependent trade credit. *Comput. Ind. Engrg.* **84** (2015) 133–143.
[40] S. Pal, A. Goswami and K.S. Chaudhuri, A deterministic inventory model for deteriorating items with stock dependent rate. *Int. J. Prod. Econ.* **32** (1993) 291–299.
[41] M. Palanivel and R. Uthayakumar, An EPQ model with variable production, probabilistic deterioration and partial backlogging under inflation. *J. Manag. Anal.* **1** (2014) 200–223.
[42] M. Palanivel and R. Uthayakumar, An EPQ model for deteriorating items with variable production cost, time dependent holding cost and partial backlogging under inflation. *Opsearch* **52** (2015) 1–17.
[43] S.K. Patra and P.C. Ratha, An inventory replenishment policy for deteriorating items under inflation in a stock dependent consumption market with shortage. *Int. J. Transdiscip. Res.* **6** (2012) 1–23.
[44] J. Ray and K.S. Chaudhuri, An EOQ model with stock-dependent demand, shortage, inflation and time discounting. *Int. J. Prod. Econ.* **53** (1997) 171–180.
[45] G.S.N. Reddy and K.V.S. Sarma, A periodic review inventory problem with variable stock dependent demand. *Opsearch* **38** (2001) 332–341
[46] J. Rezaei and N. Salimi, Economic order quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. *Int. J. Prod. Econ.* **137** (2012) 11–18.
[47] M.K. Salameh and M.Y. Jaber, Economic production quantity model for items with imperfect quality. *Int. J. Prod. Econ.* **26** (2000) 59–64.
[48] M.K. Salameh, M.Y. Jaber, N. Noueihed, Effect of deteriorating items on the instantaneous replenishment model. *Prod. Plann. Control* **10** (1999) 175–180
[49] S. Sana, S.K. Goyal and K.S. Chaudhuri, A production-inventory model for a deteriorating item with trended demand and shortages. *Eur. J. Oper. Res.* **157** (2004) 357–371.
[50] B. Sarkar, An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production. *Appl. Math. Comput.* **218** (2012a) 8295–8308.
[51] B. Sarkar, An EOQ model with delay in payments and time varying deterioration rate. *Math. Comput. Model.* **55** (2012b) 367–377.
[52] B. Sarkar, An inventory model with reliability in an imperfect production process. *Appl. Math. Comput.* **218** (2012c) 4881–4891.
[53] B. Sarkar and I.K. Moon, An EPQ model with inflation in an imperfect production system. *Appl. Math. Comput.* **217** (2011) 6159–6167
[54] B. Sarkar, S.S. Sana and K.S. Chaudhuri, An imperfect production process for time varying demand with inflation and time value of money – An EMQ model. *Expert Syst. Appl.* **38** (2011) 13543–13548.
[55] B. Sarkar, L.E. Cárdenas-Barrón, M. Sarkar and M.L. Singgih, An economic production quantity model with random defective rate, rework process and backorders for a single stage production system. *J. Manuf. Syst.* **33** (2014) 423–435.
[56] B. Sarkar, S. Saren and L.E. Cardenas-Barron, An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. *Ann. Oper. Res.* **229** (2015) 677-702
[57] A.A. Taleizadeh, L.E. Cárdenas-Barrón, B.A. Mohammadi, deterministic multi product single machine EPQ model with backordering, scraped products, rework and interruption in manufacturing process. *Int. J. Prod. Econ.* **150** (2014) 9–27.
[58] J.T. Teng, On the economic order quantity under conditions of permissible delay in payments. *J. Operat. Res. Soc.* **53** (2002) 915–918.
[59] A. Thangam and R. Uthayakumar, An inventory model for deteriorating items with inflation induced demand and exponential partial backorders–a discounted cash flow approach. *Int. J. Manage. Sci. Eng. Manage* **5** (2010) 170–174.
[60] J.T. Tolgari, A. Mirzazadeh and A. Jolai, An inventory model for imperfect items under inflationary conditions with considering inspection errors. *Comput. Math. Appl.* **63** (2012) 1007–1019.
[61] R. Uthayakumar and M. Palanivel, An inventory model for defective items with trade credit and Inflation. *Prod. Manuf. Res.* **2** (2014) 355–379
[62] M. Valliathal and R. Uthayakumar, The Production – Inventory Problem for Ameliorating/Deteriorating Items with Non-Linear Shortage Cost under Inflation and Time Discounting. *Appl. Math. Sci.* **4** (2010) 289–304
[63] H.M. Wee, J. Yu and M.C. Chen, Optimal inventory model for items with imperfect quality and shortage backordering. *OMEGA* **35** (2007) 7–11.
[64] G.A. Widyadana and H.M. Wee, An economic production quantity model for deteriorating items with multiple production setups and rework. *Int. J. Prod. Econ.* **138** (2012) 62–67.
[65] J. Wu, L.Y. Ouyang, L.E. Cardenas-Barron and S.K. Goyal, Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. *Eur. J. Oper. Res.* **237** (2014) 898–908.
[66] H.L. Yang, J.T. Teng and M.S. Chern, An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. *Int. J. Prod. Econ.* **123** (2010) 8–19.
[67] C.T. Yang, An inventory model with both stock-dependent demand rate and stock-dependent holding cost rate. *Int. J. Prod. Econ.* **155** (2014) 214–221.