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OPTIMIZATION OF SYSTEM AVAILABILITY FOR A MULTI-STATE PREVENTIVE MAINTENANCE MODEL FROM THE PERSPECTIVE OF A SYSTEM'S COMPONENTS

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Abstract. This study aims at the multi-state degraded system with multi-state components to propose a novel approach of performance evaluation and a preventive maintenance model from the perspective of a system's components. The general non-homogeneous continuous-time Markov model (NHCTMM) and its analogous Markov reward model (NHCTMRM) are used to quantify the intensity of state transitions during the degradation process. Accordingly, the bound approximation approach is applied to solve the established NHCTMMs and NHCTMRMs, thus evaluating system performance including system availability and total maintenance cost to overcome their inherent computational difficulties. Furthermore, this study adopts a genetic algorithm (GA) to optimize a proposed preventive maintenance model. A simulation illustrates the feasibility and practicability of the proposed approach.

Keywords. Multi-state components, preventive maintenance, non-homogeneous continuous-time Markov models, genetic algorithm, bound approximation approach.

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1. Introduction

1.1. Research background and purpose

Modern systems have become large-scale and complex, with components that suffer all kinds of faults and errors including damage, impacts, and aging factors throughout their lifetime. Various systems such as computer server systems, telecommunication systems, and electricity distribution systems, become tolerant to these faults and errors. Even if a fault occurs, these systems still keep on working with an acceptable or degraded performance level. Thus, from being perfectly functioning, systems normally experience multiple intermediate states during the degradation process, before complete failure occurs. Furthermore, with technological advances and developments, each component in a system also becomes fault tolerant; the degraded states of individual components can be monitored through the combined use of online detection equipment and computers. To model such systems, the multi-state system with multi-state components is appropriately established to evaluate the system performance [1,19] such as system availability and maintenance cost, particularly in constructing a preventive maintenance model. Normally, a multi-state system is considered to have completely failed when its performance has deteriorated such that it no longer fulfills its mission requirements [5,7,9].

The non-homogeneous continuous-time Markov model (NHCTMM) is normally used to evaluate the extent to which the failure rate increases with operational time during the degradation process. Solving the NHCTMM can obtain the probability distribution of a multi-state degraded system. Compared to homogeneous continuous-time Markov model (HCTMM), solving the NHCTMM to assess a multi-state degraded system is a much more complex challenge [11,13,16]. Therefore, using common mathematical tools such as MATLAB, MATHCAD, and so on may induce the problem of inaccuracy [15,16]. In order to evaluate the system performance including system availability and total maintenance cost when constructing the preventive maintenance model for multi-state degraded system with multi-state components, the general NHCTMMs and non-homogeneous continuous-time Markov reward models (NHCTMRMs) are established. In addition, this study employs the bound approximation approach [3] to efficiently solve the established NHCTMMs and NHCTMRMs. A preventive maintenance model is constructed from the perspective of a system's components. The purpose of this model is to maximize the minimum system availability during planning horizon subject to allowable total maintenance cost by determining the optimal maintenance activity for degraded states regarding each component. Five maintenance activities including no service or repair, minor service, major service, minor repair, and major repair are exclusively taken into consideration in planning preventive maintenance strategy. In order to efficiently optimize the constructed preventive model, a GA based algorithm is also proposed. A simulation is used to illustrate the efficacy of the proposed approach.

1.2. Literature review

In a system containing degradation components, the gradual decline in performance of each component forms a multi-state degraded system, whose overall performance falls from perfectly functioning to complete failure throughout its lifetime [7]. The system is deemed to have failed when its level of performance cannot meet user demands. The inherent properties of multi-state degradation mean that the mathematical inference involved in a reliability assessment, as well as the optimization of system design and preventive maintenance, is more complex than for a conventional binary-state system. Levitin and Lisnianski [10] and Nahas *et al.* [20] proposed an optimization model for the imperfect preventive maintenance of a multi-state degraded system containing binary-state components. When a system fails or its reliability falls below some threshold level, repairs, or preventive maintenance are implemented immediately. Accordingly, the total maintenance cost is minimized *via* the optimization of a preventive maintenance schedule given the system's minimum reliability requirement. The approach proposed by these two studies is limited to individual components with binary states.

Do Van and Berenguer [4] proposed a condition-based maintenance policy considering aspects of maintenance cost and productivity for a single-unit deteriorating production system whose condition is periodically monitored. Imperfect preventive maintenance actions which restore the production system to better than old are carried out. Different types of imperfect preventive maintenance cost functions are investigated to assess the performance of the proposed maintenance policy. The proposed maintenance model is subject to binary-state assumption for the system rather than a multi-state system. Khatab *et al.* [8] proposed a condition-based maintenance approach for availability optimization problem. The system is subject to stochastic degradations and assumed to be continuously monitored. Imperfect preventive maintenance actions are made on the basis of the hybrid hazard model and the condition to perform a preventive maintenance corresponds to a system reliability threshold. After a number of preventive maintenance cycles, the system is replaced by a new one. The maintenance optimization problem to be solved consists on finding the optimal reliability threshold together with the optimal number of preventive maintenance cycles to maximize the average system availability. The established maintenance model is based on the assumption of a binary-state system.

Platis *et al.* [22] presented the case related to electrical systems using the time non-homogeneous Markov systems in discrete time to evaluate the system performance. Accordingly, some reliability and performance measures are formulated, such as reliability, availability, maintainability and different time variables including new indicators more dedicated to electrical systems like instantaneous expected load curtailed and the expected energy not supplied on a time interval. This study takes into account hazard rate time variation to get more accurate and more instructive indicators that cannot be obtained by classical methods. Chen and Trivedi [2] presented the condition-based maintenance, and derive closed-form expressions of system availability when the device undergoes both deterioration as well as Poisson type failures. These closed-form solutions can be used to find faster algorithms to determine the optimal inspection policy. This inspection policy is established from the perspective of systems. Platis [21] proposed a generalized form of the performance measure of fault tolerant systems. This generalized form takes into account more detailed rewards and can be used in general for maintenance cost analysis as well as in the modeling of the website user's behavior. Different formulations are constructed by means of a homogeneous Markov chain and a cyclic non-homogeneous Markov chain and their asymptotic expression.

Tan and Raghavan [23] proposed a predictive maintenance strategy for a multi-state system in which the maintenance schedule is determined by the state of degradation. Component maintenance is implemented once the system performance cannot meet user requirements. Establishing a maintenance strategy from the overall perspective of the system can prevent the system going offline, because excessive maintenance of components will cause frequent shutdowns. However, component degradation or failure can cause the sudden failure of the system, therefore resulting in even greater system loss. Consequently, if the components can be monitored in real time, establishing a maintenance strategy from the perspective of the components has practical applications. Using a case study, Tan and Raghavan [23] applied the random restore factor technique to describe the extent to which imperfect maintenance can restore system performance in terms of its mean time to failure (MTTF). Huang and Yuan [7] proposed a preventive maintenance strategy for a multi-state degradation system in which periodic examinations are conducted. The proposed strategy determines the optimal time for the maintenance of a multi-state system, as well as the most appropriate maintenance activities to be carried out during each maintenance period. The proposed preventive maintenance model assumes that the failure rate of the system within maintenance intervals remains constant. A discrete-time homogeneous Markov chain is used to describe the system states. By minimizing the total maintenance cost, the optimal preventive maintenance strategy can be determined. However, in practice, the failure rate of a system during the maintenance intervals increases with operation time. Although following this approach can simplify the complexities involved in solving a Markov chain model, the degradation of a multi-state system cannot be evaluated precisely. Therefore, this study aims to determine the optimal preventive maintenance strategy for a multi-state degraded system from the perspective of its components, where the individual components or sub-systems can be monitored in real time. The NHCTMM and NHCTMRM are constructed to evaluate the extent to which the failure rate increases with operational time during the degradation process. Performance indicators are thereby determined using the bound approximation approach [3]. Furthermore, this study utilizes a GA to optimize the constructed preventive maintenance model. The maintenance activities for each component's degradation state are determined to maximize the minimum system availability during planning horizon subject to allowable total maintenance cost.

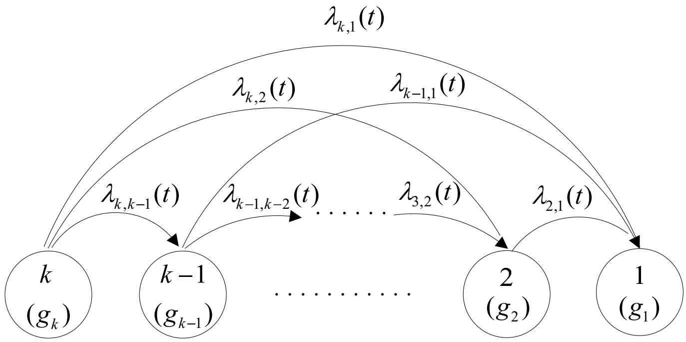

FIGURE 1. State-transition diagram of a component.

2. Background and methods

2.1. Markov model of multi-state degraded components

HCTMMs can only be used to analyze the degradation process of multi-state components without aging [11, 13]. In practice, the degradation process of components is not only related to the immediately preceding state, but also to the age of the components [13, 18]. Considering the aging factors of components, Liu and Huang [15] applied NHCTMM to derive a stochastic process in which the transition intensity between states increases with time.

Figure 1 shows the transition diagram of a non-repairable degrading component with k states moving from higher to lower performance, where $\lambda_{i,j}(t)$ is the failure rate of a component transitioning from state i to j at time t. g_k is the performance level of the component under state k. Figure 1 describes the instantaneous transition intensity of a component from any state i, $i \in \{k, k-1, \ldots, 2\}$ to state j. This value increases monotonically with the age of the component, and can be represented using the following equation:

$$\lambda_{i,j}(t) = \lim_{\Delta t \to 0} \frac{\Pr\{G(t+\Delta t) = j/G(t) = i\}}{\Delta t} \tag{2.1}$$

where $G(t)$ is a random variable representing the component state at time t, and $\Pr\{G(t+\Delta t) = j/G(t) = i\}$ is the conditional probability of a component in state i at time t transitioning to state j during time interval Δt. Solving NHCTMM is more complex than its homogeneous counterpart [11,13,16]. This is especially true when the complexity of the system is enhanced by an increase in the number of components. In this scenario, the number of possible system states is increased, making it extremely difficult to solve the NHCTMM. Considering a non-repairable multi-state system, Liu and Kapur [17] deduced the instantaneous dynamic probability of a multi-state system given the initial state. The instantaneous probability for the best state and subsequent states can be obtained. The method proposed for evaluating a system's reliability using NHCTMM is targeted at non-repairable degraded multi-state systems. As this deductive result cannot be generalized to repairable systems, its practicality is restricted.

2.2. Non-homogeneous markov reward model

The Non-homogeneous Markov reward model [12, 14] can be used to effectively assess the maintenance costs (as well as the reliability and other relevant indicators) of an aging multi-state system (MSS) over its lifetime. This model assumes that, if the process stays in state i during a time period, a certain amount of money r_{ii} should be paid. If the system transitions from state i to state j during this period, then r_{ij} should be paid. Both r_{ii} and r_{ij} are called rewards, and can be either a loss or a gain. Hence, this is known as a Markov process with rewards. Besides the transition matrix, the reward matrix $\mathbf{r} = [r_{ij}]$, $i, j = 1, \ldots, K$ must be constructed according to indicators such as maintenance costs, operational revenue, availability, and time to failure.

A non-homogeneous Poisson process (NHPP) model can be integrated into a Markov model with time varying transition intensity $\alpha_{ij}(t)$. For aging MSSs, transition intensity corresponding to failure of aging components will be increasing function of time $\alpha_{ij}(t)$. Therefore, the system's total expected reward (TER) accumulated over time can be derived using the non-homogeneous Markov reward model. The corresponding Chapman-Kolmogorov differential equations are shown below:

$$\frac{\mathrm{d}V_i(t)}{\mathrm{d}t} = r_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{K} \alpha_{ij}(t) r_{ij} + \sum_{j=1}^{K} \alpha_{ij}(t) V_j(t)\,, \quad i = 1, 2, \ldots, K \tag{2.2}$$

where $V_i(t)$ and $V_j(t)$ are the TER values accumulated until time t while in states i and j, respectively. If we let $u_i(t) = r_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{K} \alpha_{ij}(t) r_{ij}$, (2.2) can be expressed in matrix form as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{V}(\mathbf{t}) = \mathbf{u}(\mathbf{t}) + \alpha(\mathbf{t})\mathbf{V}(\mathbf{t}). \tag{2.3}$$

Substituting the initial value $V_i(0) = 0$ into (2.3), the system's simultaneous differential equations can be solved to derive the TER value of the Non-homogeneous Markov reward model.

2.3. Bound approximation approach for the increasing failure-rate function

Solving the NHCTMM to obtain system performance indicators requires a lot of time. Often, the use of common mathematical tools, such as MATLAB or

MATHCAD and so on, may induce the problem of inaccuracy [15,17]. The bound approximation approach [3] allows the determination of instantaneous state probabilities for a multi-state degraded system. This approach divides the system lifetime into multiple intervals, and sets the failure rate during each interval to be a constant. The HCTMM is then used to find the instantaneous probability at the end of each time interval. This numerical approach initially divides the system lifetime T into N time intervals. The duration of each time interval is $\Delta t = T/N$. Then, two constants λ^{n-1} and λ^{n+} are used to approximate the failure rate $\lambda(t)$ in each time interval $t_n = [\Delta t(n-1), \Delta tn], 1 \leq n \leq N$, using the following equations:

$$\lambda^{n-} = \lambda(\Delta t(n-1)) \tag{2.4}$$

$$\lambda^{n+} = \lambda(\Delta tn) \tag{2.5}$$

where λ^{n-} and λ^{n+} represent the system's failure rate $\lambda(t)$ at the beginning and end of the nth time interval. Equations (2.4) and (2.5) also give the lower and upper bounds of $\lambda(t)$ in the nth time interval. Using λ^{n-1} and λ^{n+1} to solve the system's differential equations, the state probabilities $P_j^{n-}(\Delta t\, n)$ and $P_j^{n+}(\Delta tn)$ can be derived for the time interval $t_n = [\Delta t(n-1), \Delta tn], 1 \leq n \leq N$. These differential equations [3] are:

$$\frac{\mathrm{d}P_j^{n-}(t)}{\mathrm{d}t} = \sum_{\substack{i=1 \\ i \neq j}}^{K} P_i^{n-}(t)\alpha_{ij}^{n-}(t) - P_j^{n-}(t) \sum_{\substack{i=1 \\ i \neq j}}^{K} \alpha_{ji}^{n-}(t) \tag{2.6}$$

$$\frac{\mathrm{d}P_j^{n+}(t)}{\mathrm{d}t} = \sum_{\substack{i=1 \\ i \neq j}}^{K} P_i^{n+}(t)\alpha_{ij}^{n+}(t) - P_j^{n+}(t) \sum_{\substack{i=1 \\ i \neq j}}^{K} \alpha_{ji}^{n+}(t). \tag{2.7}$$

At each time interval t_n, the lower bound λ^{n-1} and upper bound λ^{n+} of the failure rate are utilized to represent the intensities $\alpha_{ij}^{n-}(t)$ and $\alpha_{ij}^{n+}(t)$ for transitions from states i to j. During the first time interval, the initial condition of the system is already known. Hence, given the system is in state K at $t = 0$, the initial conditions for (2.6) and (2.7) during the first time interval $n = 1$ are as follows:

$$P_K^{1-}(0) = 1,\ P_{K-1}^{1-}(0) = \ldots P_1^{1-}(0) = 0 \tag{2.8}$$

$$P_K^{1+}(0) = 1,\ P_{K-1}^{1+}(0) = \ldots P_1^{1+}(0) = 0. \tag{2.9}$$

The initial conditions for $t_n, n = 2, 3, \ldots, N$, are defined by the following recurrence relations:

$$P_j^{n-}[\Delta t(n-1)] = P_j^{(n-1)-}[\Delta t(n-1)], j = 1, 2, \ldots, K, n = 1, 2, \ldots, N \tag{2.10}$$

$$P_j^{n+}[\Delta t(n-1)] = P_j^{(n-1)+}[\Delta t(n-1)], j = 1, 2, \ldots, K, n = 1, 2, \ldots, N. \tag{2.11}$$

This means that the initial conditions for the next interval are defined by the solutions at the end of preceding time intervals. By solving the non-homogeneous

Markov reward model (NHMRM) using the bound approximation approach, the lower bound V_i^{n-} and upper bound V_i^{n+} of the TER accumulated at each time interval $[\Delta t\,(n-1)\,,\,\Delta tn]$ can be obtained from any state i, $i = 1, 2, \ldots, K$. The equations for the NHMRM are:

$$\frac{\mathrm{d}V_i^{n-}(t)}{\mathrm{d}t} = r_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{K} \alpha_{ij}^{n-}(t) r_{ij} + \sum_{j=1}^{K} a_{ij}^{n-}(t) V_j^{n-}(t), \qquad i = 1, 2, \ldots, K, n = 1, \ldots, N \quad (2.12)$$

$$\frac{\mathrm{d}V_i^{n+}(t)}{\mathrm{d}t} = r_{ii} + \sum_{\substack{j=1\\ j\neq 1}}^{K} \alpha_{ij}^{n+}(t) r_{ij} + \sum_{j=1}^{K} a_{ij}^{n+}(t) V_j^{n+}(t), \qquad i = 1, 2, \ldots, K, n = 1, \ldots, N. \quad (2.13)$$

For any state during each time interval, the initial reward is 0, that is:

$$V_i^{n-}(0) = V_i^{n+}(0) = 0,\ i = 1, 2, \ldots, K,\ n = 1, \ldots, N. \quad (2.14)$$

Solving (2.12) and (2.13) under the initial condition (2.14) gives the lower and upper bounds of TER accumulated from $t = 0$. Multiplying $V_i^{n-}\,(\Delta t)$ and $V_i^{n+}\,(\Delta t)$ by their corresponding state probabilities $P_i^{n-}\,[\Delta t\,(n-1)]$ and $P_i^{n+}\,[\Delta t\,(n-1)]$ gives the system's upper and lower mean reward values for any state during each time interval. The sum of all mean reward values for all states gives the system's overall lower reward bound V^{n-} and upper reward bound V for any time interval n^{n+}, $n = 1, \ldots, N$. These bounds are calculated as:

$$V^{n-} = \sum_{i=1}^{K} V_i^{n-}[\Delta t] P_i^{n-}[\Delta t(n-1)], n = 1, \ldots, N \quad (2.15)$$

$$V^{n+} = \sum_{i=1}^{K} V_i^{n+}[\Delta t] P_i^{n+}[\Delta t(n-1)], n = 1, \ldots, N. \quad (2.16)$$

Finally, summing the TER over N time intervals gives the lower and upper bounds of TER over the system's lifetime:

$$TER^{-} = \sum_{n=1}^{N} V_i^{n-} \quad (2.17)$$

$$TER^{+} = \sum_{n=1}^{N} V_i^{n+}. \quad (2.18)$$

The exact TER value falls somewhere between the lower and upper bound, *i.e.*, $TER \leq TER \leq TER$. A more accurate TER value can be obtained by dividing T into smaller intervals.

3. Proposed approach

3.1. Model assumptions

The proposed preventive maintenance model for a multi-state degraded system makes the following assumptions:

(1) Real-time monitoring of the system can identify the performance of individual components within the system.
(2) The components of the system degrade from perfectly functioning to complete failure over multiple states of degradation.
(3) The components of the system degrade randomly over time to a state of lower performance.
(4) Components at a particular degradation state can be restored to a previous, better state by appropriate maintenance.
(5) There are five maintenance alternatives representing the extent to which implement maintenance restores the component to a better condition:
    (i) No service or repair.
    (ii) Minor service: enables restoration to state $j+1$ from state j, such as simple maintenance and cleaning, lubrication, alignment, adjustment, consumable materials, maintenance and inspection of the supplementary work.
    (iii) Major service: enables restoration to state $j+2$ from state j such as dismantling equipment, assembly, functional testing, spare parts and fuel replacement.
    (iv) Minor repair: enables restoration to state $j+3$ from state j, such as repair when the device is abnormal or not operating properly.
    (v) Major repair: enables restoration to state $j+4$ from state j, such as repair when equipment failures, including fault detection, diagnosis, disassembly, assembly and functional testing.
(6) The failure rate of an individual component is an increasing function of time.
(7) The maintenance time of an individual component is distributed exponentially, that is, the maintenance rate is assumed to be constant.

3.2. Construction of the model

The preventive maintenance model of a multi-state degraded system is constructed so as to maximize the minimum system availability during planning horizon. The maximum allowable total maintenance cost during its planning horizon is used as a constraint. Thus, mathematically, the problem can be formulated as follows:

Objective:

$$\text{Max Min}\, A(t) = \sum_{i=1}^{K} p_i(t) \cdot 1(g_i \geq w). \tag{3.1}$$

Constraints:

$$\sum_{l=1}^{m} C_{pm,l} \leq C_s, \tag{3.2}$$

where $A(t)$ is the system availability at time t; $p_i(t)$ is the probability of state i occurring at time t; g_i is the system performance at state i; K indicates the best performance state of the system; $1(g_i \geq w)$ is a unit function that takes a value of 1 when g_i is greater than or equal to w, and a value of 0 otherwise; w is the user demand; m is the number of components constituting the system; $\sum_{l=1}^{m} C_{pm,l}$ is the total cost for implementing preventive maintenance; C_s is the maximum allowable system maintenance cost.

3.3. Simulation case

A simulation case derived from Huang and Wang [6] is used to examine the proposed approach. The simulated system contains three components. Components 1 and 2 are connected in parallel; both are connected to component 3 in series. Each component has five states possessing different output performance, with state 5 being perfectly functioning and state 1 being complete failure. For example, the output performance of five states for component 1 in descending order are 150, 100, 80, 50, and 0, respectively. Appropriate maintenance can restore components to previous, better states. The planning horizon of preventive maintenance is set as one year. The minimum acceptable performance (user demand w) is set as 120, while the maximum allowable total maintenance cost is set as 50. Each individual component is initially in a perfectly functioning state. Hence, the initial probability of all the states is 0, except for state 5, which has a probability of 1. Figure 2 shows the system configuration, where $g_{l,i}$ is the performance of component l at state i, and $\lambda_{i,j}^{l}(t)$ is the transition intensity of component l when degraded from state i to j at time t. $\mu_{j,i}^{l}$ is the transition intensity of component l when restored from state j back to state i following maintenance. Tables 1 and 2 present the transition intensity parameters of each component. Table 3 shows the cost parameters relating to various maintenance activities.

3.4. Using a GA to solve the simulated case

This study adopts a GA to optimize the proposed preventive maintenance model. For the simulated case, the most appropriate maintenance activities for all three components are determined at all degradation states to minimize the total maintenance cost. Using this GA to solve the simulated case involves two stages.

Stage I: Establish the initial chromosome population

Step 1: Encode chromosomes.

The encoded chromosomes consist of 15 genes because the simulated case contains three components, each of which has five possible states. The genes are coded as integers; each code corresponds to preventive maintenance activity for the individual state of the three components. Therefore, different combinations

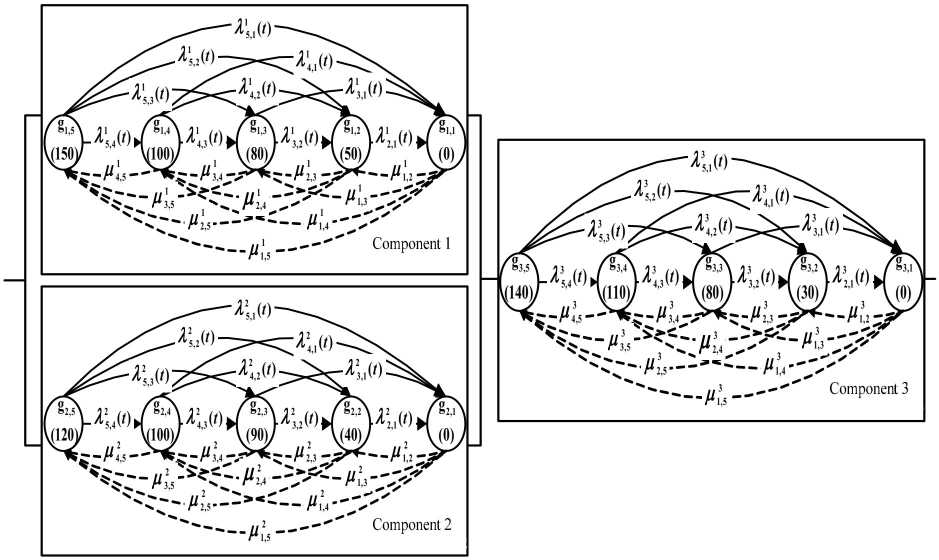

Figure 2. System configuration of the simulated case.

Table 1. Failure-rate function of each component between states.

Failure rates \ Components	1	2	3
$\lambda_{5,4}\ (t)$	$0.24 + 0.07t$	$0.24 + 0.07t$	$0.34 + 0.14t$
$\lambda_{5,3}\ (t)$	$0.18 + 0.04t$	$0.18 + 0.04t$	$0.28 + 0.08t$
$\lambda_{5,2}\ (t)$	$0.14 + 0.02t$	$0.14 + 0.02t$	$0.24 + 0.04t$
$\lambda_{5,1}(t)$	$0.12 + 0.01t$	$0.12 + 0.01t$	$0.22 + 0.02t$
$\lambda_{4,3}\ (t)$	$0.26 + 0.08t$	$0.26 + 0.08t$	$0.36 + 0.16t$
$\lambda_{4,2}\ (t)$	$0.20 + 0.05t$	$0.20 + 0.05t$	$0.30 + 0.1t$
$\lambda_{4,1}(t)$	$0.16 + 0.03t$	$0.16 + 0.03t$	$0.26 + 0.06t$
$\lambda_{3,2}\ (t)$	$0.28 + 0.09t$	$0.28 + 0.09t$	$0.38 + 0.18t$
$\lambda_{3,1}(t)$	$0.22 + 0.06t$	$0.22 + 0.06t$	$0.32 + 0.12t$
$\lambda_{2,1}(t)$	$0.30 + 0.1t$	$0.30 + 0.1t$	$0.4 + 0.2t$

Note: λ_{ij} (t) is the failure rate at time t of each component from state i to state j.

Table 2. Repair rate of each component between states.

Components \ Repair rates	$\mu_{1,5}$	$\mu_{1,4}$	$\mu_{2,5}$	$\mu_{1,3}$	$\mu_{2,4}$	$\mu_{3,5}$	$\mu_{1,2}$	$\mu_{2,3}$	$\mu_{3,4}$	$\mu_{4,5}$
1	0.125	0.320	0.335	0.410	0.425	0.440	0.455	0.470	0.485	0.500
2	0.80	0.245	0.260	0.275	0.290	0.305	0.350	0.365	0.380	0.395
3	0.65	0.95	0.110	0.140	0.155	0.170	0.185	0.200	0.215	0.230

Note: $\mu_{j,i}$ is the repair rate of each component from state j to state i.

TABLE 3. Cost parameters of maintenance activities.

Coded values	Maintenance activities	Cost		
		Component 1	Component 2	Component 3
0	no service or repair	0	0	0
1	minor service	54	72	120
2	major service	72	96	160
3	minor repair	90	120	200
4	major repair	180	240	400

of maintenance codes represent a distinct solution for the proposed preventive maintenance model.

Step 2: Produce the initial chromosome population.

Different combinations of maintenance codes are obtained at random to form a gene sequence constituting a chromosome. Under the maximum total maintenance cost constraint of 50, 30 chromosomes are produced to form the initial chromosome population. The GA search mechanism is conducted on the basis of 30 chromosomes.

Stage II: Execute the GA search mechanism

Step 1: Conduct chromosome crossover.

A multi-point crossover method is used to conduct chromosome crossover. The crossover rate and mask number are set to 0.6 and 8, respectively. Initially, 18 chromosomes are selected at random from the chromosome population and placed into a crossover pool. Then, paired chromosomes are randomly selected from the crossover pool and subjected to the crossover procedure under the predetermined number of masks.

Step 2: Conduct chromosome mutation

The mutation procedure is conducted on the chromosome population obtained from step 1. The mutation rate and mask number are set to 0.4 and 2, respectively. Accordingly, 12 chromosomes are randomly selected for mutation, whereby two masks of each chromosome are randomly mutated.

Step 3: Determine the fitness value of chromosomes.

The maximum system availability is defined as the fitness value of the chromosomes, as the proposed preventive maintenance model aims to maximize the minimum system availability during planning horizon. Each chromosome corresponds to a maintenance solution of this simulated case. According to the solutions obtained, the constructed NHCTMM and NHCTMRM are used to derive the instantaneous system state probabilities, and thereby calculate the system availability and total maintenance cost. The procedure for solving the NHCTMM and NHCTMRM using the bound approximation approach [16, 20] relating to each chromosome has two parts.

Part 1: Determine the system availability

1. Determine the performance in all possible states for the simulated case.

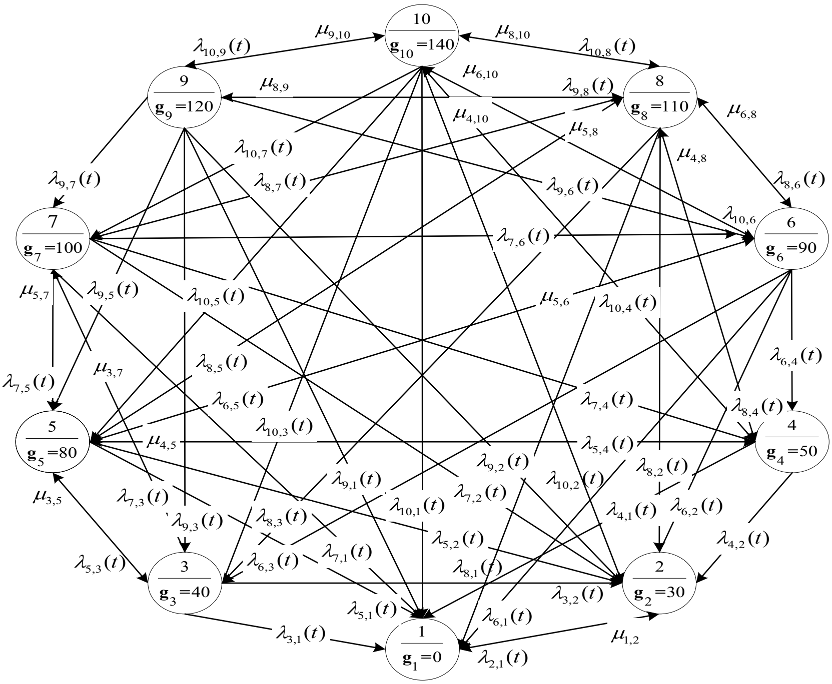

FIGURE 3. Reduced state-transition diagram of the system.

Because the series-parallel system under consideration consists of three components, each involving five states, a total of 125 ($5 \times 5 \times 5$) possible states and their corresponding performances are calculated.

2. Reduce the system states.
   System states with identical performance are united into one state. Similarly, the failure-rate function and repair rate corresponding to these united states are also combined to reduce a system complexity. This lessens the computational complexity of determining the system availability and total maintenance cost. In total, ten system states possessing different output performances from 0 to 140 are obtained. Figure 3 shows the reduced transition diagram for this case. The addition of failure rate function of components related $\lambda_{i,j}^{l}(t)$ constitutes the failure rate function of the system $\lambda_{i,j}(t)$. Similarly, the addition of repair rate of components related $\mu_{i,j}^{l}$ constitutes the repair rate of the system $\mu_{j,i}$.
3. Construct the Chapman–Kolmogorov equation of the NHCTMM

The ten state transitions are used to construct the Chapman–Kolmogorov equation of the NHCTMM as follows:

$$\frac{\mathrm{d}P_j(t)}{\mathrm{d}t} = \sum_{\substack{i=1 \\ i \neq j}}^{10} P_i(t)\alpha_{ij}(t) - P_j(t) \sum_{\substack{i=1 \\ i \neq j}}^{10} \alpha_{ji}(t),\ j = 1, 2, \ldots, 10 \tag{3.3}$$

where $\alpha_{ij}(t)$ and $\alpha_{ji}(t)$ represent the intensities for transition from states i to j and states j to i, respectively, which are the addition of failure rates at time and repair rates regarding reduced system states. Then, the bound approximation approach is employed to calculate instantaneous state probabilities and thereby obtain the system availability. The calculation steps are described below:

(1) Define the length of each time interval
Initially, the one-year planning horizon is divided into 200 time intervals. The length of each time interval is $\Delta t = T/N = 0.002$, and boundary of each interval is given by $t_{n+1} = t_n + 0.002, n = 1, \ldots, N-1$.
(2) Calculate values of λ^{n-} and λ^{n+}
λ^{n-} and λ^{n+} are obtained using (2.10) and (2.11).
(3) Find the instantaneous state probability
P_i^{n-} and $P_i^{n+}, j = 1, \ldots, 10$, at the end point of each time interval are obtained using (2.14)–(3.1).
(4) Determine the lower and upper bounds of system availability
The probabilities of those states fulfilling a minimum performance level of 120 are summed to determine the lower and upper bounds of system availability at $t_{n+1} = t_n + 0.002, n = 1, \ldots, N-1$.

Part 2: Determine the total maintenance cost

The Chapman–Kolmogorov equations of the NHCTMM and NHCTMRM for all components are constructed according to the state-transition diagrams shown in Figure 2. For details of the constructed models refer to Appendixes A and B. The bound approximation approach is used to solve these simultaneous differential equations. Hence, the instantaneous state probabilities, maintenance cost for each component, and total maintenance cost for the system are obtained. Details of these calculations are given below:

1. Define the length of each time interval
As for stage 1, the one-year planning horizon is divided into 200 intervals.
2. Calculate the λ^{n-} and λ^{n+} values
Values of λ^{n-} and λ^{n+} are again obtained using (2.4) and (2.5).
3. Find the instantaneous state probability
P_j^{n-} and P_j^{n+}, $j = 1, \ldots, 5$, are obtained at the end point of each time interval for five states of each component using (2.6)–(2.11).
4. Determine the maintenance cost of each degraded state
The lower and upper bounds of the maintenance cost, V_i^{n-} and V_i^{n+}, $i = 1, \ldots, 5$, are determined for five states of each component during each time interval using (2.12)–(2.14).
5. Determine total expected maintenance cost during each time interval
The lower and upper bounds of the total expected maintenance cost, V_i^{n-} and V_i^{n+}, are determined for each component during each time interval using (2.15) and (2.16)

TABLE 4. Optimal maintenance activities of three components.

Components	1				2				3			
Degraded states	4	3	2	1	4	3	2	1	4	3	2	1
Coded values of maintenance activities	1	0	3	1	1	1	2	1	0	1	1	0

6. Determine the total expected maintenance cost during the system planning horizon
   The lower and upper bounds of the total expected maintenance cost, TER^{-} and TER^{+} are determined for each component over the system planning horizon using (2.17) and (2.18).
7. Find total expected maintenance cost for the system.

Summing the total expected maintenance cost for each component gives the total maintenance cost for the entire system.
Step 4: Determine the termination condition.

The GA will terminate when the optimized total maintenance cost does not improve for 100 iterations. The optimal preventive maintenance strategy is then presented and this will include the most appropriate maintenance activities for each component and degradation state. This study employs a MATLAB program to perform the mathematical computations of the proposed approach.

4. RESULTS

The proposed preventive maintenance model, solved through the GA described in the previous section, enabled the optimal preventive strategy to be derived. This gives the maintenance activities necessary for each component's degradation state in the system. Table 4 summarizes the results. Figures 4–6 show the state-transition diagrams for the three components. Minor services are implemented in component 1 when this reaches states 4 and 1; minor repair is implemented when this reaches states 2. However, no maintenance activity is implemented in states 3. In component 2, minor services are undertaken in states 4, 3, and 1; major service is implemented when this reaches states 2. For component 3, no maintenance activity is performed until state 3 and 2 are reached, at which points minor services are implemented. The optimal system availability is 0.948. Figure 7 shows the trend in system availability with time over the one-year planning horizon. The total maintenance cost is 41.73 which less of maximum allowable constraint of 50. Figure 8 plots the convergence of the maximum system availability while conducting the GA to optimize the proposed maintenance model.

In a reverse optimization problem, we minimized the total maintenance cost while simultaneously satisfying a user-defined availability 0.7 during planning horizon one year. From the result of Huang and Wang [6], Table 5 gives the maintenance activities necessary for each component and each degraded state in the

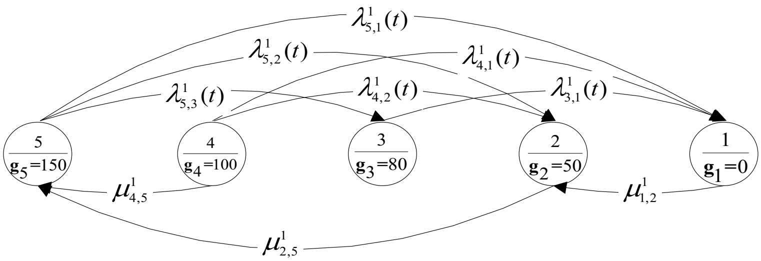

FIGURE 4. State-transition diagram of component 1.

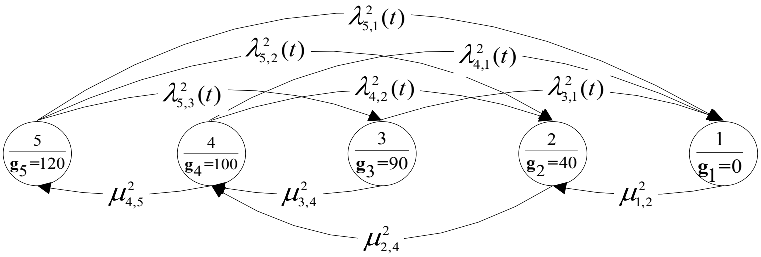

FIGURE 5. State-transition diagram of component 2.

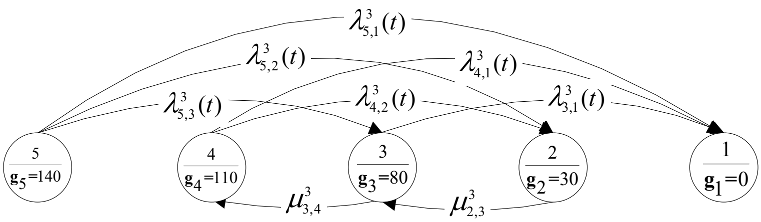

FIGURE 6. State-transition diagram of component 3.

system. The optimal total maintenance cost is 48. Figure 9 shows the trend in system availability with time. Figure 10 plots the convergence of the total maintenance cost while conducting the GA to optimize the proposed maintenance model.

5. Conclusions

Based on the findings of this study, the following conclusions and suggestions can be made:

(1) The proposed approach enables engineers to determine the appropriate activity needed to maximize the minimum system availability during planning horizon given limited resources, total maintenance cost. From this optimized case, it is clear that maintenance need only be implemented for certain states of the

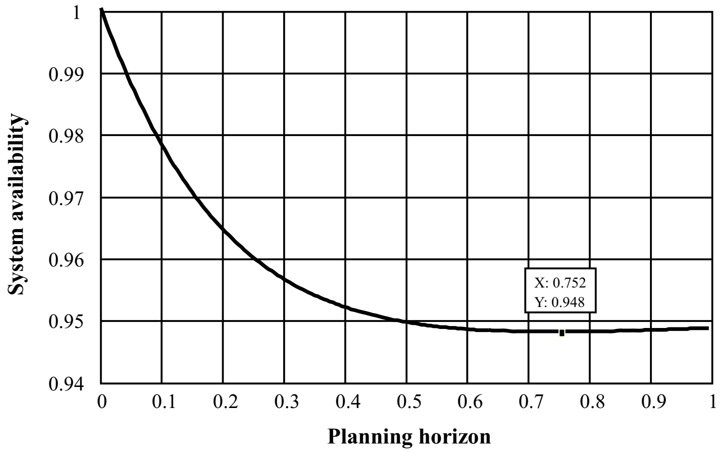

FIGURE 7. Diagram of system availability with time for optimal maintenance strategy.

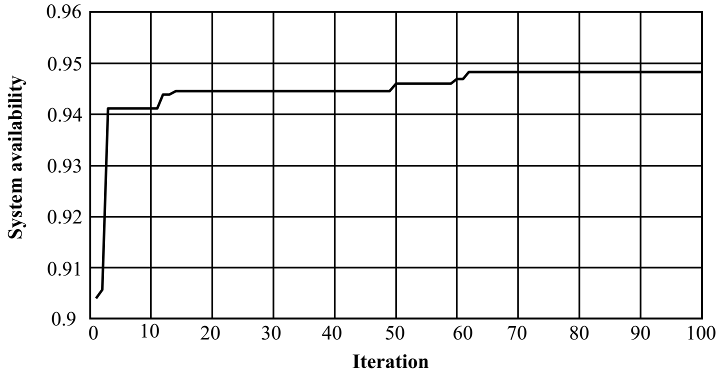

FIGURE 8. GA convergence diagram of system availability for optimal maintenance strategy.

TABLE 5. Optimal maintenance activities of three components in a reverse optimization problem.

Components	1				2				3			
Degraded states	4	3	2	1	4	3	2	1	4	3	2	1
Coded values of maintenance activities	0	1	3	0	0	2	0	0	1	1	0	1

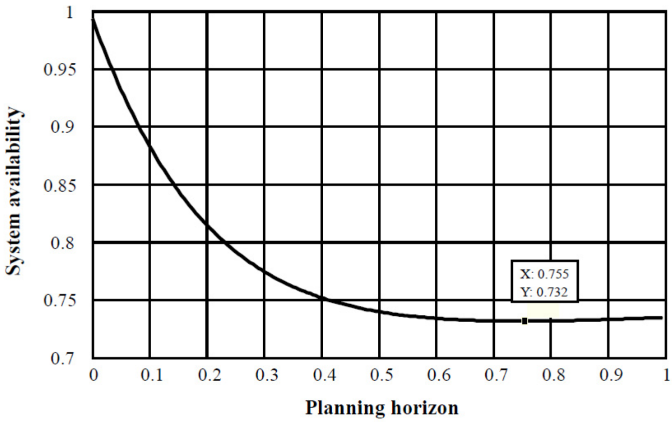

FIGURE 9. Diagram of system availability with time for optimal maintenance strategy.

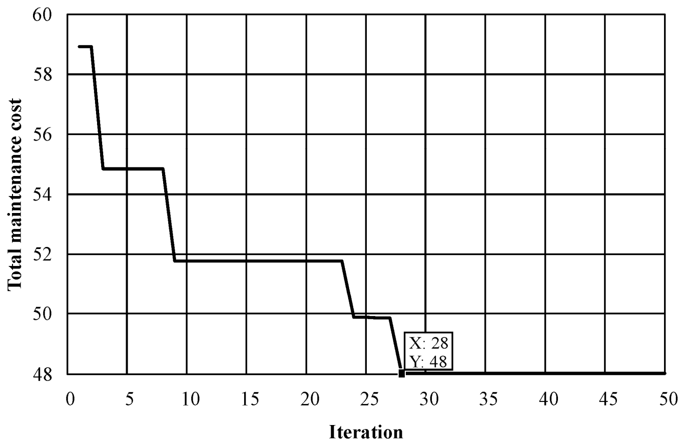

FIGURE 10. GA convergence diagram with total maintenance cost for optimal maintenance strategy.

three system components. Therefore, the proposed approach can considerably reduce the frequency of system shutdowns due to maintenance significantly decreasing system loss. This advantage makes the planning of a maintenance strategy from the perspective of components more practical.

(2) The proposed model provides engineers to gain further insight into the impacts on system availability while implementing different maintenance activities for each component's degradation state in the system.

(3) Furthermore, the maintenance rate was held constant in the constructed multi-state system. In future work, we will consider the maintenance rate as a

reduced function of time, making the findings more compatible with actual maintenance situations. However, the computational difficulties of evaluating performance indicators such as maintenance cost, system availability, and MTTF, will present a considerable challenge.

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Appendix A: NHCTMM Chapman–Kolmogorov equations of each component

$$\text{Component 1}\begin{cases} \mathrm{d}P_5(t)/\mathrm{d}t = -(\lambda^1_{5,4}(t)+\lambda^1_{5,3}(t)+\lambda^1_{5,2}(t)+\lambda^1_{5,1}(t))P_5(t) \\ \qquad +\mu^1_{4,5}P_4(t)+\mu^1_{3,5}P_3(t)+\mu^1_{2,5}P_2(t)+\mu^1_{1,5}P_1(t) \\ \mathrm{d}P_4(t)/\mathrm{d}t = \lambda^1_{5,4}(t)P_5(t)-(\mu^1_{4,5}+\lambda^1_{4,3}(t)+\lambda^1_{4,2}(t) \\ \qquad +\lambda^1_{4,1}(t))P_4(t)+\mu^1_{3,4}P_3(t)+\mu^1_{2,4}P_2(t)+\mu^1_{1,4}P_1(t) \\ \mathrm{d}P_3(t)/\mathrm{d}t = \lambda^1_{5,3}(t)P_5(t)+\lambda^1_{4,3}(t)P_4(t)-(\mu^1_{3,5}+\mu^1_{3,4} \\ \qquad +\lambda^1_{3,2}(t)+\lambda^1_{3,1}(t))P_3(t)+\mu^1_{2,3}P_2(t)+\mu^1_{1,3}P_1(t) \\ \mathrm{d}P_2(t)/\mathrm{d}t = \lambda^1_{5,2}(t)P_5(t)+\lambda^1_{4,2}(t)P_4(t)+\lambda^1_{3,2}(t)P_3(t) \\ \qquad -(\mu^1_{2,5}+\mu^1_{2,4}+\mu^1_{2,3}+\lambda^1_{2,1}(t))P_2(t)+\mu^1_{1,2}P_1(t) \\ \mathrm{d}P_1(t)/\mathrm{d}t = \lambda^1_{5,1}(t)P_5(t)+\lambda^1_{4,1}(t)P_4(t)+\lambda^1_{3,1}(t)P_3(t) \\ \qquad +\lambda^1_{2,1}(t)P_2(t)-(\mu^1_{1,5}+\mu^1_{1,4}+\mu^1_{1,3}+\mu^1_{1,2})P_1(t) \end{cases}$$

$$\text{Component 2}\begin{cases} \mathrm{d}P_5(t)/\mathrm{d}t = -(\lambda^2_{5,4}(t)+\lambda^2_{5,3}(t)+\lambda^2_{5,2}(t)+\lambda^2_{5,1}(t))P_5(t) \\ \qquad +\mu^2_{4,5}P_4(t)+\mu^2_{3,5}P_3(t)+\mu^2_{2,5}P_2(t)+\mu^2_{1,5}P_1(t) \\ \mathrm{d}P_4(t)/\mathrm{d}t = \lambda^2_{5,4}(t)P_5(t)-(\mu^2_{4,5}+\lambda^2_{4,3}(t)+\lambda^2_{4,2}(t) \\ \qquad +\lambda^2_{4,1}(t))P_4(t)+\mu^2_{3,4}P_3(t)+\mu^2_{2,4}P_2(t)+\mu^2_{1,4}P_1(t) \\ \mathrm{d}P_3(t)/\mathrm{d}t = \lambda^2_{5,3}(t)P_5(t)+\lambda^2_{4,3}(t)P_4(t)-(\mu^2_{3,5}+\mu^2_{3,4}+\lambda^2_{3,2}(t) \\ \qquad +\lambda^2_{3,1}(t))P_3(t)+\mu^2_{2,3}P_2(t)+\mu^2_{1,3}P_1(t) \\ \mathrm{d}P_2(t)/\mathrm{d}t = \lambda^2_{5,2}(t)P_5(t)+\lambda^2_{4,2}(t)P_4(t)+\lambda^2_{3,2}(t)P_3(t) \\ \qquad -(\mu^2_{2,5}+\mu^2_{2,4}+\mu^2_{2,3}+\lambda^2_{2,1}(t))P_2(t)+\mu^2_{1,2}P_1(t) \\ \mathrm{d}P_1(t)/\mathrm{d}t = \lambda^2_{5,1}(t)P_5(t)+\lambda^2_{4,1}(t)P_4(t)+\lambda^2_{3,1}(t)P_3(t) \\ \qquad +\lambda^2_{2,1}(t)P_2(t)-(\mu^2_{1,5}+\mu^2_{1,4}+\mu^2_{1,3}+\mu^2_{1,2})P_1(t) \end{cases}$$

$$
\text{Component 3}\begin{cases}
\mathrm{d}P_5(t)/\mathrm{d}t = -(\lambda^3_{5,4}(t) + \lambda^3_{5,3}(t) + \lambda^3_{5,2}(t) + \lambda^3_{5,1}(t))P_5(t) \\
\qquad + \mu^3_{4,5}P_4(t) + \mu^3_{3,5}P_3(t) + \mu^3_{2,5}P_2(t) + \mu^3_{1,5}P_1(t) \\
\mathrm{d}P_4(t)/\mathrm{d}t = \lambda^3_{5,4}(t)P_5(t) - (\mu^3_{4,5} + \lambda^3_{4,3}(t) + \lambda^3_{4,2}(t) \\
\qquad + \lambda^3_{4,1}(t))P_4(t) + \mu^3_{3,4}P_3(t) + \mu^3_{2,4}P_2(t) + \mu^3_{1,4}P_1(t) \\
\mathrm{d}P_3(t)/\mathrm{d}t = \lambda^3_{5,3}(t)P_5(t) + \lambda^3_{4,3}(t)P_4(t) - (\mu^3_{3,5} + \mu^3_{3,4} + \lambda^3_{3,2}(t) \\
\qquad + \lambda^3_{3,1}(t))P_3(t) + \mu^3_{2,3}P_2(t) + \mu^3_{1,3}P_1(t) \\
\mathrm{d}P_2(t)/\mathrm{d}t = \lambda^3_{5,2}(t)P_5(t) + \lambda^3_{4,2}(t)P_4(t) + \lambda^3_{3,2}(t)P_3(t) \\
\qquad - (\mu^3_{2,5} + \mu^3_{2,4} + \mu^3_{2,3} + \lambda^3_{2,1}(t))P_2(t) + \mu^3_{1,2}P_1(t) \\
\mathrm{d}P_1(t)/\mathrm{d}t = \lambda^3_{5,1}(t)P_5(t) + \lambda^3_{4,1}(t)P_4(t) + \lambda^3_{3,1}(t)P_3(t) \\
\qquad + \lambda^3_{2,1}(t)P_2(t) - (\mu^3_{1,5} + \mu^3_{1,4} + \mu^3_{1,3} + \mu^3_{1,2})P_1(t)
\end{cases}
$$

Appendix B: NHCTMRM Chapman–Kolmogorov equations of each component

$$
\text{Component 1}\begin{cases}
\mathrm{d}V_5(t)/\mathrm{d}t = -(\lambda^1_{5,4}(t)+\lambda^1_{5,3}(t)+\lambda^1_{5,2}(t)+\lambda^1_{5,1}(t))V_5(t) \\
\qquad +\lambda^1_{5,4}(t)V_4(t)+\lambda^1_{5,3}(t)V_3(t)+\lambda^1_{5,2}(t)V_2(t)+\lambda^1_{5,1}(t)V_1(t) \\
\mathrm{d}V_4(t)/\mathrm{d}t = 54\mu^1_{4,5}+\mu^1_{4,5}V_5(t) - (\mu^1_{4,5}+\lambda^1_{4,3}(t)+\lambda^1_{4,2}(t) \\
\qquad +\lambda^1_{4,1}(t))V_4(t)+\lambda^1_{4,3}(t)V_3(t)+\lambda^1_{4,2}(t)V_2(t)+\lambda^1_{4,1}(t)V_1(t) \\
\mathrm{d}V_3(t)/\mathrm{d}t = 72\mu^1_{3,5}+54\mu^1_{3,4}+\mu^1_{3,5}V_5(t) \\
\qquad +\mu^1_{3,4}V_4(t) - (\mu^1_{3,5}+\mu^1_{3,4}+\lambda^1_{3,2}(t)+\lambda^1_{3,1}(t))V_3(t) \\
\qquad +\lambda^1_{3,2}(t)V_2(t)+\lambda^1_{3,1}(t)V_1(t) \\
\mathrm{d}V_2(t)/\mathrm{d}t = 90\mu^1_{2,5}+72\mu^1_{2,4}+54\mu^1_{2,3} \\
\qquad +\mu^1_{2,5}V_5(t)+\mu^1_{2,4}V_4(t)+\mu^1_{2,3}V_3(t) - (\mu^1_{2,5} \\
\qquad +\mu^1_{2,4}+\mu^1_{2,3}+\lambda^1_{2,1}(t))V_2(t)+\lambda^1_{2,1}(t)V_1(t) \\
\mathrm{d}V_1(t)/\mathrm{d}t = 180\mu^1_{1,5}+90\mu^1_{1,4}+72\mu^1_{1,3}+54\mu^1_{1,2}+\mu^1_{1,5}V_5(t)+\mu^1_{1,4}V_4(t) \\
\qquad +\mu^1_{1,3}V_3(t)+\mu^1_{1,2}V_2(t) - (\mu^1_{1,5}+\mu^1_{1,4}+\mu^1_{1,3}+\mu^1_{1,2})V_1(t)
\end{cases}
$$

$$
\text{Component 2}\begin{cases}
\mathrm{d}V_5(t)/\mathrm{d}t = -(\lambda_{5,4}^2(t)+\lambda_{5,3}^2(t)+\lambda_{5,2}^2(t)+\lambda_{5,1}^2(t))V_5(t)+\lambda_{5,4}^2(t)V_4(t)\\
\qquad +\lambda_{5,3}^2(t)V_3(t)+\lambda_{5,2}^2(t)V_2(t)+\lambda_{5,1}^2(t)V_1(t)\\
\mathrm{d}V_4(t)/\mathrm{d}t = 72\mu_{4,5}^2+\mu_{4,5}^2V_5(t)-(\mu_{4,5}^2+\lambda_{4,3}^2(t)+\lambda_{4,2}^2(t)\\
\qquad +\lambda_{4,1}^2(t))V_4(t)+\lambda_{4,3}^2(t)V_3(t)+\lambda_{4,2}^2(t)V_2(t)+\lambda_{4,1}^2(t)V_1(t)\\
\mathrm{d}V_3(t)/\mathrm{d}t = 96\mu_{3,5}^2+72\mu_{3,4}^2+\mu_{3,5}^2V_5(t)+\mu_{3,4}^2V_4(t)\\
\qquad -(\mu_{3,5}^2+\mu_{3,4}^2+\lambda_{3,2}^2(t)+\lambda_{3,1}^2(t))V_3(t)\\
\qquad +\lambda_{3,2}^2(t)V_2(t)+\lambda_{3,1}^2(t)V_1(t)\\
\mathrm{d}V_2(t)/\mathrm{d}t = 120\mu_{2,5}^2+96\mu_{2,4}^2+72\mu_{2,3}^2+\mu_{2,5}^2V_5(t)+\mu_{2,4}^2V_4(t)\\
\qquad +\mu_{2,3}^2V_3(t)-(\mu_{2,5}^2+\mu_{2,4}^2\\
\qquad +\mu_{2,3}^2+\lambda_{2,1}^2(t))V_2(t)+\lambda_{2,1}^2V_1(t)\\
\mathrm{d}V_1(t)/\mathrm{d}t = 240\mu_{1,5}^2+120\mu_{1,4}^2+96\mu_{1,3}^2+72\mu_{1,2}^2\\
\qquad +\mu_{1,5}^2V_5(t)+\mu_{1,4}^2V_4(t)+\mu_{1,3}^2V_3(t)\\
\qquad +\mu_{1,2}^2V_2(t)-(\mu_{1,5}^2+\mu_{1,4}^2+\mu_{1,3}^2+\mu_{1,2}^2)V_1(t)
\end{cases}
$$

$$
\text{Component 3}\begin{cases}
\mathrm{d}V_5(t)/\mathrm{d}t = -(\lambda_{5,4}^3(t)+\lambda_{5,3}^3(t)+\lambda_{5,2}^3(t)+\lambda_{5,1}^3(t))V_5(t)\\
\qquad +\lambda_{5,4}^3(t)V_4(t)+\lambda_{5,3}^3(t)V_3(t)\\
\qquad +\lambda_{5,2}^3(t)V_2(t)+\lambda_{5,1}^3(t)V_1(t)\\
\mathrm{d}V_4(t)/\mathrm{d}t = 120\mu_{4,5}^3+\mu_{4,5}^3V_5(t)-(\mu_{4,5}^3+\lambda_{4,3}^3(t)\\
\qquad +\lambda_{4,2}^3(t)+\lambda_{4,1}^3(t))V_4(t)+\lambda_{4,3}^3(t)V_3(t)\\
\qquad +\lambda_{4,2}^3(t)V_2(t)+\lambda_{4,1}^3(t)V_1(t)\\
\mathrm{d}V_3(t)/\mathrm{d}t = 160\mu_{3,5}^3+120\mu_{3,4}^3+\mu_{3,5}^3V_5(t)+\mu_{3,4}^3V_4(t)\\
\qquad -(\mu_{3,5}^3+\mu_{3,4}^3+\lambda_{3,2}^3(t)+\lambda_{3,1}^3(t))V_3(t)\\
\qquad +\lambda_{3,2}^3V_2(t)+\lambda_{3,1}^3(t)V_1(t)\\
\mathrm{d}V_2(t)/\mathrm{d}t = 200\mu_{2,5}^3+160\mu_{2,4}^3+120\mu_{2,3}^3+\mu_{2,5}^3V_5(t)\\
\qquad +\mu_{2,4}^3V_4(t)+\mu_{2,3}^3V_3(t)-(\mu_{2,5}^3+\mu_{2,4}^3\\
\qquad +\mu_{2,3}^3+\lambda_{2,1}^3(t))V_2(t)+\lambda_{2,1}^3V_1(t)\\
\mathrm{d}V_1(t)/\mathrm{d}t = 240\mu_{1,5}^3+200\mu_{1,4}^3+160\mu_{1,3}^3+120\mu_{1,2}^3\\
\qquad +\mu_{1,5}^3V_5(t)+\mu_{1,4}^3V_4(t)+\mu_{1,3}^3V_3(t)\\
\qquad +\mu_{1,2}^3V_2(t)-(\mu_{1,5}^3+\mu_{1,4}^3+\mu_{1,3}^3+\mu_{1,2}^3)V_1(t)
\end{cases}
$$

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