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# MULTI-CRITERIA DECISION ANALYSIS FOR EFFICIENT LOCATION-ALLOCATION PROBLEM COMBINING DEA AND GOAL PROGRAMMING

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**Abstract.** Facility location-allocation (FLA) decisions play a significant role in the performance of supply network in many practical applications, such as emergency service system, supply chain system, public service system, *etc.* In this paper, a multi-criteria model (including multi-attribute and multi-objective) for optimal and efficient facility location-allocation patterns was proposed. We first utilize multi attribute decision making (MADM) method–DEA to evaluate the relative efficiency of each potential location, and then combine the efficiency identified from DEA as a goal in a multi objective decision making (MODM) framework by using goal programming. A hypothetical example is presented to illustrate the effectiveness and the efficiency of the proposed model. Results demonstrate that the proposed multi-criteria model is an effective tool for generating a set of more realistic and flexible optimal solution in solving facility location- allocation problems by adjusting the goal priorities with respect to the importance of each objective and the aspiration level with respect to desired target values. The proposed model is also flexible and general enough to consider other specific location decisions such as emergency facilities, undesirable facilities and supply chain design by combing specific location modeling goal with the DEA model.

**Keywords.** Multi-criteria, data envelopment analysis, goal programming, location, allocation.

**Mathematics Subject Classification.** 90C29.

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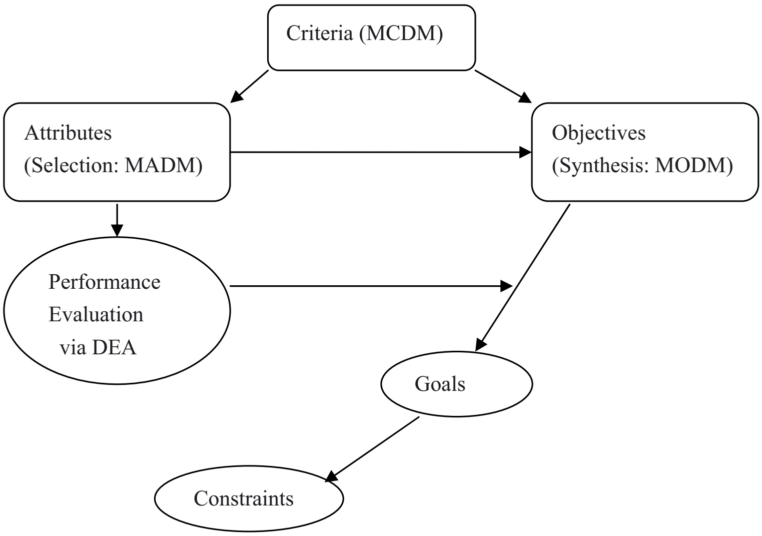


FIGURE 1. The relationship between MADM and MODM.

## 1. Introduction

The facility location-allocation (FLA) problem is widely used in practical life, such as building an emergency service systems and constructing a telecommunication networks. FLA problem was initially studied by Cooper [12], and then Hakimi [18, 19] applied it in network design as a powerful tool [27]. The facility location/allocation models have been developed to answer questions such as how many facilities to establish, where to locate them, and how to distribute the products to the customers in order to satisfy demand and minimize total cost [28].

Traditionally, minimizing cost(or some surrogate for cost such as travel distance) has been the principal objective facility location-allocation decision modeling. However, the facility location- allocation problems often have multiple criteria that conflict with each other in nature. Therefore, the location/allocation decision problem can be viewed as a multiple-criteria decision-making (MCDM) problem [13, 14]. In general terms, MCDM techniques in the presence of multiple, potentially conflicting criteria can be broadly classified as multi attribute decision making (MADM) Models and multi objective decision making (MODM) models. In MADM, the decision maker selects from a set of alternatives that is typically defined explicitly. In MODM, the decision maker must come up with or design the most preferred alternative that is defined implicitly, *e.g.*, by the restrictions of a mathematical program. In a general way, it can be said that MADM selects the best alternative among a finite number of alternatives, unlike MODM where the best alternative is designed with multiple objectives subject to constraints. The following Figure 1 shows the relationship of these two concepts.

In this paper, we postulate here that locating facilities at different potential sites may affect the system's performance – its ability to transform multiple-cost into useable benefits – especially when site qualitative attributes such as local labor markets, available infrastructure and the receptiveness and perceptions of local populations will heavily modify the acceptability of a siting decision [22]. So we can use a suitable multi attribute decision making (MADM) method to evaluate the performance of each location by considering the feasible alternatives, the related attributes and their weights. There exist many MADM methods such as LINMAP, TOPSIS, ELECTRE, AHP and DEA. For a review of the various MADM methods the reader is referred to Yoon and Hwang [39]. Data envelopment analysis (DEA) [9], which can consider multiple-cost and -benefit measures, is a methodology that can incorporate such concerns into the siting process. Since it was first developed by Charnes *et al.* [9], DEA has been applied to a wide range of problem settings, including health care (hospitals, doctors), education (schools, universities), banks, manufacturing, food restaurants and retail stores [2, 16, 20, 21, 32, 37]. The original focus on this methodology was on the efficiency measurement of organizations and has been recently extended to a more planning orientation for evaluating the efficiency of spatial location patterns [4, 23, 29, 31, 34]. Peijun [31] presented a fuzzy data envelopment analysis (DEA) model for evaluating the efficiencies of objects with fuzzy input and output data which reflect the inherent ambiguity in evaluation problems under uncertainty. Using the proposed fuzzy DEA model, a case study of a Japanese-style rotisserie restaurant location decision is analyzed and determined. Kraiwinee *et al.* [23] used data envelopment analysis to frame their empirical examination of the efficiency of services offshoring to locate offshore facilities. When DEA is used alone to evaluate the relative spatial efficiency of facility location-allocation decisions, the potential location with the highest relative efficiency is selected for implementation. Unfortunately, the selection of location alternatives *via* the DEA – only solution method has not taken into consideration the other constraints (goals) of the problem that include budget restrictions, capacity limitations, and demand requirements [5]. Firstly, when the decision makers are faced with a multiple location problem, extending the traditional DEA method to selecting multiple locations with the highest combined efficiency – score among all the facilities at a time can result in an infeasible selection since possible limiting or constraining resources are not directly considered in the selection process. Secondly, the decision-makers have no direct solutions about how to make allocation decision with each of the location selected. To deal with situations, Klimberg and Ratick [22] formulated a bi-objective model for efficient location/allocation decision where one objective is to maximize the facility efficiencies measured by the resulting DEA efficiency score for the "opened" facilities and the other is optimize the spatial efficiency measured by the least total cost of location and allocation patterns for facilities. In reality, however, Current *et al.*'s [13] review of 45 facility location papers demonstrated that most location/allocation decisions are complex problems and face multiple objectives that often conflict with each other in nature besides the traditional objective of cost minimization.

Recognizing the multiple and conflicting objective nature of the location-allocation problem, many types of facility location/allocation models have been formulated in a multi-objective programming framework eliciting trade-offs among these sometimes conflicting objectives in many practical applications such as emergency service design, undesirable facility location, semi-obnoxious location [1,3,14,15,17,38,40]. Araz *et al.* [3] presented a fuzzy multi-objective covering-based vehicle location model for emergency services which considered three objectives: maximization of the population covered by one vehicle, maximization of the population with backup coverage and increasing the service level by minimizing the total travel distance from locations at a distance bigger than a prespecified distance standard for all zones. Erkut *et al.* [15] proposed a multi-criteria mixed-integer linear programming model, which deals with the location–allocation problem of municipal solid waste management facilities in the Central Macedonia region in North Greece. Yoshiaki and Kazuki [40] presented a bi-criteria location model for the placement of a semi-obnoxious facility with the twin objectives of maximizing the distance to the nearest inhabitant and minimizing the sum of distances to all the users (or the distance to the farthest user) in a unified manner. These multi-objective models can be solved by using mathematical approaches such as weighting method, goal programming (GP) and compromise programming. Goal programming (GP), which allows the decision maker define satisfying levels of the value of each objective and then to find a solution which optimizes unfavorable deviations from those goals, is a most widely used approach within the multi-objective decision-making (MODM) of the facility location-allocation problems. Consequently, Klimberg and Ratick [22] point out that solving for the DEA efficiency measure, simultaneously with other location modeling objectives was an area they want to explore in the future.

To extend the DEA approach to cover the above limitations and simultaneously include other location modeling criteria, a multi-criteria facility-location-allocation model employing DEA and GP approaches (including multi-attribute and multi-objective) is presented. While both of the DEA and GP approaches have been widely in the location literature, we have not found any studies in which these two approaches have been combined to find and evaluate solutions to facility location problems. Furthermore, the facility location selection is a complicated issue because of the large number of criteria to be considered as well as because criteria are both quantitative (*e.g.* the setup cost, the transportation cost, supply capacity) and qualitative (*e.g.* the quality of life, the availability of required technical labor, degree of competition, demographics, the working environment factors). Qualitative criteria can be usually expressed in the form of bounded data, ordinal data, and the ratio bounded data such as high/medium/low or a 5-point scale, which are common for evaluating alternatives in terms of qualitative criteria and easily understood by management. In this paper, we first use DEA to evaluate the criteria with bounded data, ordinal data, and the ratio bounded data to express the efficiency of the locations, and then combining the efficiency identified from DEA as a goal in a multi-objective goal programming framework. Afterwards, we shall demonstrate how the proposed model (including multi-attribute and multi-objective) can be used to aid in optimal spatial and efficient facility location/allocation patterns.

The paper is further organized as follows. Section 2 presents a brief literature review of the existing approaches and the DEA and multi-objective models related to the facility location. In Section 3, we develop and present formulations combing the multi-objective goal programming facility location problem with the DEA problem. Section 4 includes an illustration of the proposed model to demonstrate the effectiveness of the solution approaches used. Finally, concluding remarks are presented in the last section.

## 2. Methodology

### 2.1. Data envelopment analysis (DEA) model

Charnes *et al.* [9] introduce data envelopment analysis (DEA) to assess the relative efficiency of a homogeneous group of decision-making units (DMUs), such as schools, hospitals, or sales outlets. The DMUs usually use a set of resources, referred to as input indices, and transform them into a set of outcomes, referred to as output indices. The more output produced for a given amount of resources, the more efficient (*i.e.*, less wasteful) is the process. Model I shows the CCR (Charnes, Cooper, and Rhodes) model [9].

Model I

$$\begin{aligned}
&\max \sum_{r=1}^{s} \mu_r y_{rp} \\
&\text{s.t.} \sum_{i=1}^{m} v_i x_{ip} = 1 \\
&\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0; \quad j = 1, \ldots, n \\
&\mu_r \geqslant 0\,, v_i \geqslant 0 \quad r = 1, \ldots, s; \quad i = 1, \ldots, m.
\end{aligned}$$

In the basic DEA model (CCR), the DEA procedure finds the set of weights that makes the efficiency of that DMU as large as possible for each DMU, but at the same time the efficiencies of all the units in the set when evaluated with these weights is prevented from exceeding a value of 1. The procedure is repeated for all other DMUs to obtain their weights and associated relative efficiency score: where $p$ is the decision making unit being evaluated, $s$ represents the number of outputs, $m$ represents the number of inputs, $y_{rj}$ the amount of output $r$ provided by DMU $j$, $x_{ij}$ the amount of input $i$used by DMU $j$, and $\mu_r$ and $v_i$ are the weights given to output $r$ and input $i$, respectively.

The CCR assumes that data on the inputs and outputs are known exactly. However, this assumption may not be true. For example, some outputs and inputs

Table 1. Data for.

| DMU | Outputs | | Inputs | |
|---|---|---|---|---|
| $j$ | $y_{1j}^{a}$ | $y_{2j}^{b}$ | $x_{1j}$ | $x_{2j}^{c}$ |
| 1 | 4 | 2 | 3 | [80,85] |
| 2 | 2 | 1 | 4 | [85,90] |
| 3 | 6 | 3 | 2 | [75,80] |
| 4 | 7 | 1 | 6 | 100 |
| 5 | 3 | 2 | 7 | [75,80] |
| 6 | 5 | 3 | 1 | [95,100] |
| 7 | 1 | 2 | 5 | [90,95] |

[a] Ordinal ranks (7 = the best; 1 = the worst), [b] three ordinal ranks (3 = the best; 1 = the worst), [c] ratio bound based on the reference DMU4 (*e.g.* $0.80x_{24} \leqslant x_{21} \leqslant 0.85x_{24}$).

may be only known as in the forms of bounded data, ordinal data, and ratio bounded data. If we incorporate such imprecise data information into the standard CCR model, we have:

Model II

$$\begin{aligned}
&\max \quad \sum_{r=1}^{s} \mu_r y_{rp} \\
&\text{s.t.} \; \sum_{i=1}^{m} v_i x_{ip} = 1 \\
&\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0; \quad j = 1, \ldots, n \\
&(x_{ij}) \in D_i^-; \qquad\qquad\qquad i = 1, \ldots, m \\
&(y_{rj}) \in D_r^+; \qquad\qquad\qquad r = 1, \ldots, s \\
&\mu_r \geqslant 0, \quad v_i \geqslant 0 \quad r = 1, \ldots, s; \quad i = 1, \ldots, m
\end{aligned}$$

where $(x_{ij}) \in D_i^-$ and $(y_{rj}) \in D_r^+$ represent any of the forms of bounded data, ordinal data, and ratio bounded data.

To provide an example of this model, we turn to Table 1, which presents the data where the inputs and the outputs are in ordinal and ratio bounded forms.

Based on scale transformation and variable alterations, the nonlinear model (2) can be transformed into a linear programming problem.

In recent efforts, there has been a significant interest in evaluating the efficiency of spatial location patterns both by practitioners and academics. The first of these applications using DEA was by Shroff *et al.* [34]. Based on the DEA methodology as originally proposed by Charnes *et al.* [9], they described their problem of locating long-term care facilities in the Northern Virginia region as one of "locational benchmarking" and used DEA as a locational benchmarking tool to measure the relative efficiencies of potential geographical regions to support the siting decision for a long term health care facility.

Thomas *et al.* [36] developed a modified version of the DEA model called the multi-alternative DEA model. This multi-alternative DEA model simultaneously solves the DEA model in one linear programming for picking the most efficient $p$ obnoxious-facility locations based on the DEA score. Finally, a model combined location and DEA models into one single-objective that maximizes the efficiencies of $p$ facilities to be opened.

The research of Klimberg and Ratick [22] appeared as a pioneering attempt that applied bi-objective programming to location problems by first formulating the simultaneous DEA linear program, and then combining that formulation, in a multi-objective framework, with both the uncapacitated and capacitated fixed charge facility location problem. Their model formulation simultaneously considers the interaction of spatial efficiencies of different location patterns through the use of least cost objectives (facility and transport cost), and the facility efficiencies at those sites through the use of the DEA objective as follows:

$$\text{Max} \quad Z_1 = \sum_{k=1}^{K} \sum_{l=1}^{L} (1 - d_{kl})$$

$$\text{Min} \quad Z_2 = \sum_{k=1}^{K} \sum_{l=1}^{L} c_{kl} \text{dem}_l x_{kl} + \sum_{k=1}^{K} F_k y_k.$$

The first objective function maximizes the facility efficiencies serving the demand as measured by the sum of the efficiencies for all facility ($k$) and demand ($l$) combinations that may operate in the optimal solution. The second objective function minimizes the spatial efficiency as measured by the total cost (transportation and fixed opening costs) of supplying the demand in the system. In conclusion, they also point out that similar promising results would obtain if this approach were used with location models formulated with other criteria such as profit, access, or capacity limitations; an area they hope to explore in the future.

In conclusion, when DEA is used alone to evaluate the relative spatial efficiency of facility location-allocation decisions, the potential location with the highest relative efficiency is selected for implementation, which does not take the multiple objectives nature of the location- allocation problem into account and can sometimes result in an infeasible solution. In this paper, to overcome the limitations of DEA approach and simultaneously consider other location modeling criteria that Klimberg and Ratick [22] addressed in their paper, a combined DEA and goal-programming approach is proposed in the next section.

### 2.2. Goal programming

Goal programming (GP), proposed by Charnes and Cooper [8], is most widely used approach within the multi-criteria decision-making (MCDM) field. The GP approach of multi-criteria problems has received increasing interest due to its modeling flexibility and conceptual simplicity. Unlike linear programming, the GP model does not optimize (maximize/minimize) the objectives directly. Instead, GP solution technique focuses on the minimization of the deviations from each goal, subject to the goal constraints and system constraints. Also, these goals must be prioritized in a hierarchy of importance. The over and under achievements of goals is measured in GP using the so called deviation variables. A commonly used generalized model for goal programming is as follows [24]:

Model III

$$\begin{aligned} \text{Min} \quad & \sum_{i=1}^{k} P_i \left( w_i^+ d_i^+ + w_i^- d_i^- \right) \\ \text{s.t} \quad & C^1 X + d_1^+ - d_1^- = t_1 \\ & \quad \vdots \\ & C^k X + d_k^+ - d_k^- = t_k \\ & X \in S \\ & d_i^-, d_i^+ \geqslant 0 \quad i = 1, 2, \ldots, k \end{aligned}$$

in which $S$ is the feasible region; $P_i$ is the preemptive factor/priority level assigned to each relevant goal in rank order (*i.e.* $P_1 > P_2 > \ldots > P_k$), $C^i X$ is the $i$th goal criterion function, and $t_i$ are the target values of the $k$ goal criteria. The variables $d_i^+$ and $d_i^-$ are the deviation variables, which measure achievements below and above $i$th goal. The $w_i^+$ and $w_i^-$ are relative importance weights attached to the underachievement and overachievement deviational variables.

GP model consists of two sets of constraints, *i.e.*, system constraints and goal constraints. System constraints are formulated following the concept of LP that cannot be violated, whilst goal constraints are taken as the auxiliary constraints which utilize deviation variables (typically both positive and negative deviation variables) that measure the difference between the desired value (or goal) and the predicted model value. The GP model used in this paper is called as preemptive GP, in which the unwanted deviations are minimized hierarchically according to the priority levels of the goals so that the goals of primary importance can receive first-priority attention, those of second importance can receive second-priority attention, and so forth. The preemptive GP model accepts implicitly infinite tradeoffs among goals placed in different priority levels [26].

It is noted that, in optimization formulation, we have $d_i^- \cdot d_i^+ = 0$ for all $k$. Moreover, in the achievement function, $w_i^- = 0$ if $d_i^-$ is not an unwanted deviation and $w_i^+ = 0$ if $d_i^+$ is not an unwanted deviation. It implies that only unwanted deviations are included into the achievement function.

Since it was initially introduced by Charnes and Cooper [8], goal programming has been widely applied to solve different real-world problems which involves multiple objectives such as healthcare planning, engineer design, resource allocation, quality control [7,11,30,33]. In recent years, there was a rich literature on the use of goal programming in location science research and several models have been formulated and applied for location problems.

The research of Charnes and Storbeck [10] appeared as a pioneering attempt that applied goal programming to location problems. They developed a goal programming model for the siting of multilevel EMS systems. By applying location covering techniques within a goal programming framework, this study develops a method for the siting of multilevel EMS systems so that (1) each service level maximizes coverage of its own demand population and (2) "back-up" coordination between levels is assured.

Sydney and Lisa [35] developed a modeling framework for hospital location and service allocation for the supply and demand matching of public hospital beds in Hong Kong. It addresses the planning issues of hospital locations and service allocations, which include new services distribution as well as existing services redistribution. A goal programming model to solve the problem is developed and a small example is presented as an illustration for its intended useful purposes.

Recognizing the multiple and conflicting objective nature of the location-allocation problem in an international setting, Badri [5] proposes the use of the analytic hierarchy process and multi-objective goal-programming methodology as aids in making location-allocation decisions. The analysis of the application of these methodologies in a real life problem verified that the methodology presented can help facility planning authorities to formulate viable location strategies in the volatile and complex global decision environment.

Alsalloum and Rand [1] proposed a goal programming model for identifying the optimal locations of a pre-specified number of emergency medical service stations. Two objectives are considered. The first goal is to locate these stations so that the maximum expected demand can be reached within a pre-specified target time. The second goal is to ensure that any demand arising located within the service area of the station will find at least one vehicle, such as an ambulance, available. The model developed was used to evaluate locations for the Saudi Arabian Red Crescent Society, Riyadh City, Saudi Arabia.

## 3. Combined DEA and goal programming model for the FLA problem

In this section, two methodologies are combined for the facility location/allocation problem. DEA is first presented as a stand-alone methodology and then a combined DEA and GP model is presented as an extension to consider additional criteria in decision making process.

### 3.1. Notation

#### *3.1.1. Parameters*

$i$ index of demand areas
$j$ index of potential facility sites

$a_i$ demand in area $i$
$f_j$ fixed cost of opening facility $j$
$h_j$ the relative efficiency scores of the facility $j$
$h_{\text{ave}}$ the average system efficiency score that needs to be maintained for open facilities
$c_{ij}$ cost of shipping one unit of demand from facility $j$ to demand $i$
$S$ the target value for the total fixed cost
$O$ the target value for the total transportation cost $l$
$T$ the target value for the total cost
$Q$ the target value for the quality of life index
$L$ the targeted number of facilities required.

*3.1.2. Decision variables*

$$Y_j = \begin{cases} 1 & \text{if a facility is located at site } j \\ 0 & \text{otherwise} \end{cases}$$

$X_{ij}$ amounts of units shipped from facility $j$ to demand $i$.

*3.1.3. Auxiliary variables*

$d_e^-$ deviation of underachievement of $\sum_{j=1}^{n} Y_j h_{\text{ave}}$
$d_e^+$ deviation of overachievement of $\sum_{j=1}^{n} Y_j h_{\text{ave}}$
$d_s^-$ deviation of underachievement of $S$
$d_s^+$ deviation of overachievement of $S$
$d_o^-$ deviation of underachievement of $O$
$d_o^+$ deviation of overachievement of $O$
$d_t^-$ deviation of underachievement of $T$
$d_t^+$ deviation of overachievement of $T$
$d_q^-$ deviation of underachievement of $Q$
$d_q^+$ deviation of overachievement of $Q$
$d_l^-$ deviation of underachievement of $L$
$d_l^+$ deviation of overachievement of $L$

## 3.2. Goal programming model incorporating the relative efficiency by DEA

Extending the use of the DEA methodology to consider resource limitations and other goals, we incorporates the DEA relative efficiencies as one of its goals as in model IV in a multi-objective goal programming framework. Hence, given the above-defined goals and variables, the facility location-allocation problem was reduced to the problem of minimizing the sum of goal deviational variables subject to the goal constraints and system constraints giving due considerations to the priority factors. Of course, the priorities given to each goal will attempt to reflect the decision making criteria of decision-makers. Therefore, the proposed GP model incorporating the relative efficiency goal can be formulated as follows:

Model IV

$$\min Z = P_1 d_s^+ + P_2 d_o^+ + P_3 d_{\text{total}}^+ + P_4 d_q^- + P_5 d_e^- + P_6 (d_l^- + d_l^+)$$

$$\text{s.t.} \sum_{j=1}^{n} Y_j h_j + d_e^- - d_e^+ = \sum_{j=1}^{n} Y_j h_{\text{ave}} \tag{3.1}$$

$$\sum_{j=1}^{n} Y_j f_j + d_s^- - d_s^+ = S \tag{3.2}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} X_{ij} + d_o^- - d_o^+ = O \tag{3.3}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} X_{ij} + \sum_{j=1}^{n} Y_j f + d_t^- - d_t^+ = T \tag{3.4}$$

$$\sum_{j=1}^{n} q_i Y_i + d_q^- - d_q^+ = Q \tag{3.5}$$

$$\sum_{j=1}^{n} Y_j + d_l^- - d_l^+ = L \tag{3.6}$$

$$\sum_{j=1}^{n} X_{ij} - M Y_i \leqslant 0 \tag{3.7}$$

$$X_{ij} \geqslant 0. \tag{3.8}$$

### 3.3. Goal constraints

Suppose that $n$potential sites are being considered as locations for new facilities. Let $y_{rj}$ and $x_{ij}$ denote the $r$th benefit measure and $i$th cost measure of the $j$th alternative. Let $u_r$ denote the weight placed on the $r$th benefit of the $j$th alternative and $v_i$ the $i$th cost of the $j$th alternative. We utilized the ratio DEA model, shown as Model II, in evaluating the relative efficiency scores $h_j$ of the facility $j$. The greater the relative efficiency $h_j$, the greater the preference for the specific$j$th location. Extending the use of the DEA methodology to consider resource limitations and other goals, we propose a GP model, which incorporates the DEA relative efficiencies as one of its goals as in equation (3.1)

Goal 1: Maximize the sum of the DEA efficiencies of all potential facilities

As Klimberg and Ratick [22] noted, the first objective function is to maximize the facility efficiencies serving the demand as measured by the sum of the efficiencies for all facility and demand combinations that may operate in the optimal solution. In this paper we modified and redefined this objective function expressed as a goal constraint (3.1) in a goal programming formulation.

$\sum_{j=1}^{n} Y_j h_j$ represents the sum of the efficiency scores for all potential locations. $h_{\text{ave}}$ represents the average system efficiency score that needs to be maintained for open facilities. The corresponding goal is stated as: minimize the negative deviation from the planned budget $(d_e^-)$ (Eq. (3.1)).

In addition to the DEA relative efficiency goal, multiple as well as conflicting goals are present in the facility location-allocation problems. In our approach we adopt the goals that are the mostly used goals in the location literature [3,5,6,13,17,22,25]. These are explained below:

Goal 2: Minimize the setup cost

One common objective used in many facility- location related studies is to minimize the fixed cost associated with locating the new facility. The total fixed costs of newly locating emergency facilities may include land acquisition cost, facilities construction, operating and maintenance cost. Usually, the goal is to have fixed costs not exceeding a given budgeted amount [5]. The corresponding goal is stated as: minimize the positive deviation from the planned budget ($d_s^+$) (Eq. (3.2)).

In equation (3.2), $S$ is the total fixed cost targeted $n$ is the number of potential locations, and $f_j$ is the fixed cost associated of opening a facility at candidate location $j$.

Goal 3: Minimize the transportation cost

In equation (3.3), $c_{ij}$ cost of shipping one unit of demand from facility $j$ to demand $i$. $m$ is the number of demand areas. $O$ is the target value for the total transportation cost goal. The corresponding goal is stated as: minimize the positive deviation from the planned budget ($d_o^+$) (Eq. (3.3)).

Goal 4: Minimize the total cost

In equation (3.4), the first term is the total transportation cost and the second term the fixed cost of opening the facilities. $T$ is the target value for the total cost goal. The corresponding goal is stated as: minimize the positive deviation from the planned budget ($d_t^+$) (Eq. (3.4)).

Goal 5: Maximize the quality of life index

Management desires to locate facilities at sites where there is satisfactory level of quality of life index weights. By using the $Q$ as the target value, it would allow the model to select facilities in such a way as to maximize the quality of life index ($d_q^-$) [25].

Goal 6: Attain targeted number of facilities required

The goal for attaining the targeted number of facilities, given by equation (3.6), represents the desired expansion rate reflecting forecasted demand for services [6].

In equation (3.6), $L$ is the targeted number of facilities required. We should mention that if the main objective of the exercise is to determine the number of fire stations, one should formulate equation (3.6) as a regular constraint. In that case, the right-hand side will be a large number and the equality sign will be switched to a less than or equal (or greater than or equal depending on the objectives).

### 3.4. System constraints

System constraints may be necessary to force the $Y_i$'s to be 1 if $X_{ij} \neq 0$ for that location. This procedure, represented by equation (3.7), has been used in many other applications [6]. $M$ is an arbitrary large number.

TABLE 2. Input and output data for each facility.

| Facility | Outputs | | | Inputs | | | Efficiency |
|---|---|---|---|---|---|---|---|
| | $y_{1j}$ | $y_{2j}$ | $y_{3j}$ | $x_{1j}$ | $x_{2j}$ | $x_{3j}$ | |
| F1 | 92 | 50 | 3 | 65 | 53 | 100 | 0.785 |
| F2 | 78 | 25 | 1 | 74 | 52 | [95,100] | 0.737 |
| F3 | 71 | 73 | 2 | 76 | 75 | [85,90] | 0.823 |
| F4 | 74 | 52 | 7 | 86 | 59 | [85,90] | 0.823 |
| F5 | 35 | 55 | 8 | 40 | 30 | [85,90] | 0.823 |
| F6 | 82 | 97 | 10 | 30 | 24 | [65,70] | 1 |
| F7 | 42 | 75 | 9 | 80 | 91 | [65,70] | 1 |
| F8 | 75 | 74 | 5 | 53 | 97 | [80,85] | 0.875 |
| F9 | 76 | 65 | 4 | 75 | 25 | [80,85] | 0.960 |
| F10 | 48 | 69 | 6 | 45 | 62 | [75,80] | 0.933 |

## 4. EXAMPLE

In this section, we give an illustrative example to show how the proposed novel models can be used to optimize the facility locations for public services. We first use DEA model to optimize the performance-based configuration of facility network, which can maintain a high degree of customer satisfaction and convenience. We also compare the solution obtained by the proposed novel model with the solution to the DEA-only location problem, thereby assessing the advantages of the combined model (if any).

In this example, a total of ten potential facilities serving 12 demand zones, each with three inputs and three outputs, were identified. Table 2 contains the input-output vectors for each potential facility and their corresponding DEA scores solved by model (2). Table 3 lists the fixed cost for each potential facility and the unit transportation cost from facility to demand.

Given the DEA relative efficiencies in Table 2 and the data in Table 3, a model employing DEA and goal programming approaches is formulated. The model (3) is solved by LINGO 8.0 on a PC 2.20 GHz Intel Core Dual E2200 with 1.98 GB of RAM. CPU times were under 1 second for all scenarios. The resulting solutions are presented in Tables 4 and 5.

A comparison between the DEA-only solutions and the combined DEA-GP solutions reveals the potential superiority of the combined model. If we run the DEA-only model to select the four highest DEA relative efficiency locations in Table 2 by setting the associated $Y_i$'s to be 1 (*i.e.* F6, F7, F9, F10, respectively), we can see that this selection decision will exceed the budgeted fixed cost (*i.e.*, by 40 000) and most importantly increase the budgeted transportation cost (*i.e.*, by 10 700) and the total cost (*i.e.*, by 50 700). If we run the DEA-GP model by assuming $P_1 > P_2 > P_3, > P_4 > P_5$, the selected facilities are F1, F6, F7, F9. In this case, the negative deviation variable $d_s^-$ and $d_t^-$ with the selected facilities (F1, F6, F7, F9) are 5000 and 4550, respectively. In other words, the DEA-only solution of selecting the four highest relative efficiencies constitutes an infeasible solution since insufficient resources exist to support that selection. Thus, the solution provided by the

TABLE 3. Resource data for the proposed model.

| Facility | Maximum production | Fixed cost ($000) | Unit transportation cost from facility to demand | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | D1 | D2 | D3 | D4 | D5 | D6 | D7 | D8 | D9 | D10 | D11 | D12 |
| F1 | 180 | 33 | 35 | 30 | 50 | 45 | 58 | 65 | 25 | 78 | 89 | 150 | 65 | 123 |
| F2 | 300 | 67 | 48 | 75 | 46 | 58 | 59 | 63 | 78 | 95 | 110 | 98 | 42 | 36 |
| F3 | 160 | 70 | 78 | 25 | 24 | 62 | 68 | 75 | 71 | 69 | 35 | 25 | 21 | 33 |
| F4 | 180 | 86 | 24 | 56 | 45 | 28 | 89 | 98 | 97 | 95 | 86 | 65 | 52 | 53 |
| F5 | 510 | 50 | 75 | 76 | 64 | 62 | 63 | 68 | 85 | 24 | 23 | 56 | 59 | 58 |
| F6 | 810 | 54 | 68 | 61 | 23 | 13 | 34 | 36 | 63 | 68 | 27 | 85 | 65 | 66 |
| F7 | 660 | 46 | 75 | 71 | 52 | 56 | 65 | 85 | 120 | 36 | 35 | 19 | 43 | 65 |
| F8 | 350 | 37 | 68 | 84 | 41 | 51 | 65 | 85 | 95 | 45 | 12 | 14 | 81 | 62 |
| F9 | 350 | 59 | 52 | 51 | 85 | 82 | 42 | 83 | 87 | 65 | 69 | 91 | 26 | 27 |
| F10 | 220 | 85 | 75 | 76 | 58 | 54 | 35 | 36 | 60 | 45 | 89 | 88 | 61 | 65 |
| Annual Demand | | | 230 | 140 | 450 | 320 | 180 | 90 | 80 | 150 | 70 | 70 | 90 | 100 |

TABLE 4. Comparison of the DEA-GP model solution and DEA-only solution (decision variables).

| Location alternative | The combined model selection solution | DEA-only selection decision |
|---|---|---|
| F1 | Yes | No (0.785) |
| F2 | No | No (0.737) |
| F3 | No | N0 (0.823) |
| F4 | No | No (0.823) |
| F5 | No | N0 (0.823) |
| F6 | Yes | Yes (1.0) |
| F7 | Yes | Yes (1.0) |
| F8 | No | No (0.875) |
| F9 | Yes | Yes (0.960) |
| F10 | No | Yes (0.933) |

TABLE 5. Comparison of the DEA-GP model solution and DEA-only solution (deviation from goals).

| Resource | Targeted goals | DEA-GP model deviation | DEA-only model deviation |
|---|---|---|---|
| Total fixed costs | 200 000 | –5000 | 40 000 |
| Total transportation costs | 55 000 | 450 | 10 700 |
| Total costs | 255 000 | –4550 | 50 700 |
| Desired expansion rate | 4 | 4 | 4 |
| Quality of life | 400 | –74 | –77 |
| DEA relative efficiency | 4 | –0.255 | –0.107 |

combined DEA-GP model is realistic and feasible since it takes into consideration resource limitation. We also notice that the negative deviation associated with quality of life by the combined model is (–74) compared to the DEA-only result of (–77). However, the negative deviation by the DEA-GP model associated with the DEA relative efficiency is –0.255 compared to the DEA-only result of −0.107.

In order to investigate the effect on the optimal solution of different priority structure of the goals and different target values. Sensitivity analysis was carried out by changing the priority order and the target value. Optimum locations, givens the different priority weights of the goals and different target values, are presented in Table 6. The second column shows that the objectives were sought. The third and remaining columns show the result when applying the proposed novel model when the level of the priorities and the aspiration values are changed accordingly.

It can be argued that these targets have been selected somewhat arbitrarily. However, we had to make these assumptions since this is a hypothetical example. In a real life application the decision maker (government, local authority, *etc.*) would set these targets, possibly based on any international standard(s) relevant for such situations.

Scenario 1, 3 and 4 result in the same resolution. These outcomes reflect the high priority that is assigned to the fixed costs and the total costs. These scenarios attempt to minimize the fixed costs and the total costs. As a result, the negative

TABLE 6. Model solutions using different suggested scenarios.

| Scenario number | Suggested priorities | Model solution | Fixed cost deviation | Transportation cost deviation | Total cost deviation | Quality of life deviation | DEA score deviation |
|---|---|---|---|---|---|---|---|
| 1 | $P_1 > P_2 > P_3 > P_4 > P_5$ | F1 F6 F7 F9 | –5000 | 450 | –4550 | –74 | –0.255 |
| 2 | $P_2 > P_1 > P_3 > P_4 > P_5$ | F1 F3 F6 F7 | 10 000 | –960 | 9040 | –72 | –0.392 |
| 3 | $P_3 > P_1 > P_2 > P_4 > P_5$ | F1 F6 F7 F9 | –5000 | 450 | –4550 | –74 | –0.255 |
| 4 | $P_3 > P_2 > P_1 > P_4 > P_5$ | F1 F6 F7 F9 | –5000 | 450 | –4550 | –74 | –0.255 |
| 5 | $P_4 > P_1 > P_2 > P_3 > P_5$ | F1 F6 F7 F8 | 2000 | 4950 | 6950 | –62 | –0.340 |
| 6 | $P_4 > P_2 > P_1 > P_3 > P_5$ | F1 F3 F6 F8 | 26 000 | –1870 | 24 130 | –62 | –0.517 |
| 7 | $P_5 > P_1 > P_2 > P_3 > P_4$ | F6 F7 F9 F10 | 40 000 | 10 700 | 50 700 | –77 | –0.107 |

deviation variable $d_s^-$ and $d_t^-$ with the selected facilities (F1, F6, F7, F9) are 5000 and 4550, reduced by 4% and around 2% respectively. But the positive deviation variable of the transportation costs $d_o^+$ is 450.

In Scenario 2, the transportation cost is set as the first priority. As a result, the negative devastation variable $d_o^-$ with the selected facilities (F1, F3, F6, F7) is 960. The second-priority goal is to minimize the fixed cost. The solution presented in this scenario has three alternatives that were also chosen by the above three scenarios (F1, F6, F7).

Scenario 5 deals with the minimization of the quality of life. The negative deviation variable $d_q^-$ is –62, which is the highest level of the quality of life. We should point out that the right-hand side of the constraints associated with the quality of life was set as 400, this goal would never be attained fully. The second-priority goal is to minimize the fixed costs. Thus, the positive deviation variables $d_s^+$ is 2000, which is the lowest except scenario 1 that assigns the highest priority to the minimization of the fixed costs. This scenario provides a selection list that has three alternatives that were chosen by the scenario 1 that targeted the minimization of the fixed costs (F1, F6, F7).

Scenario 6 also deals with the minimization of the quality of life. In this scenario, the quality of life is the same as that in scenario 5. Compared with Scenario 5, the next highest priority goal is associated with the minimization of the transportation costs. As a result, the negative deviation variable $d_o^-$ is 1870, which is the lowest level of the transportation costs. This scenario provides a selection list that has three alternatives that were chosen by the scenario 2 that targeted the minimization of the transportation costs (F1, F3, F6).

Scenario 7 assigns the highest priority goal to the DEA relative efficiency. As a result, the facilities with the four highest DEA score (F6, F7, F9, F10) were selected. The negative deviation variable $d_e^-$ is –0.107. Scenario 7 provides the highest relative efficiency score. The positive deviation variables $d_o^+$ and $d_s^+$ are 10 700 and 40 000, respectively. This shows that the transportation costs and the fixed costs are extremely high. It is obvious to state that this solution is not acceptable.

In summary, the selected choice is dependent on the priority structure and the aspiration levels (as follows). The analysis shows that the proposed novel model can be studied by using the interactive procedure by changing the priority structure and the aspiration levels (as follows) to reach out to a solution which is more suitable to the decision maker.

To analyze the inter-relationships among the various goals of facility location-allocation system, the sensitivity analysis of the effect of change in the DEA relative efficiency target on the fixed costs and the total costs is made. It is observed from Figure 2 that prior to 15% decrease in DEA score, substantial deviations from the fixed costs target and the transportation costs target are observed. With a minimum of 15% decrease in the DEA score target, the fixed costs and the transportation costs targets can be overachieved.

From the analysis, it reveals that when applying DEA to find optimal and efficient facility location-allocation patterns, it should be combined simultaneously

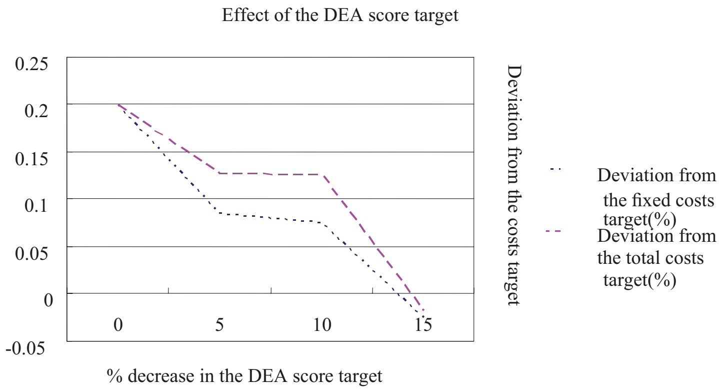


FIGURE 2. Effect of change in DEA score target on the fixed costs and the total costs.

with other location modeling objectives. It also coincides with the conclusion drawn by Narasimhan *et al.* [29]. Narasimhan *et al.* [25] employ data envelopment analysis (DEA) and mixed integer programming models to design efficient service location in government services. A series of experiments are conducted with the proposed novel model to trade off the cost (in the form of the minimization of the total number of facilities located) and the DEA efficiency by varying the levels of system-wide efficiency using the data of real-life169 branch offices of the agency located in the State of Michigan. The analysis shows that reasonable minimum efficiency levels (not the highest level of efficiency score) can be set simultaneously with other location modeling objectives for feasible solutions to facility location system when applying DEA to find efficient facility location-allocation decisions.

As mentioned above, target value and priority structure play an important role in finding the optimal solution in the preemptive goal programming formulation. The analysis of the priority structures and the aspiration levels will assist policy makers to understand the effect of the target values and the priority structure of individual goals on the system behavior and also guide the managers in deciding the best priority structure and aspiration level for the final selection list of facilities under the given condition. A similar analysis can be performed by varying the target levels of the other goals as well.

## 5. Conclusion

In this paper a DEA and GP combined multi-objective model for facility location-allocation problem is presented. This is accomplished by first using DEA to evaluate the relative efficiency of each potential location, and then combining that formulation in a multi-objective goal programming framework. The applicability of the proposed model has been demonstrated, through a numerical example. Firstly, a comparison of the DEA-only and the combined DEA and GP solutions reveals the potential superiority of the combined solution when making the facility location-allocation decisions. Secondly, further experiments are conducted with the proposed model by adjusting the goal priorities with respect to the importance of each objective and the aspiration level with respect to desired target values. The results obtained demonstrate that the proposed model combining DEA and GP is a viable tool and can be used to assist decision-makers in making appropriate decisions regarding the facility location-allocation problems.

Further studies are strongly recommended that include the DEA detailed cost-benefit analysis of potential locations regarding to various scenarios such as emergency system design, supply chain network design and undesirable facilities location.

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