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ON SOLVING MULTI OBJECTIVE SET COVERING PROBLEM WITH IMPRECISE LINEAR FRACTIONAL OBJECTIVES

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Abstract. The Set Covering problem is one the of most important NP-complete 0-1 integer programming problems because it serves as a model for many real world problems like the crew scheduling problem, facility location problem, vehicle routing etc. In this paper, an algorithm is suggested to solve a multi objective Set Covering problem with fuzzy linear fractional functionals as the objectives. The algorithm obtains the complete set of efficient cover solutions for this problem. It is based on the cutting plane approach, but employs a more generalized and a much deeper form of the Dantzig cut. The fuzziness in the problem lies in the coefficients of the objective functions. In addition, the ordering between two fuzzy numbers is based on the possibility and necessity indices introduced by Dubois and Prade [Ranking fuzzy numbers in the setting of possibility theory. *Inf. Sci.* **30** (1983) 183–224.]. Our aim is to develop a method which provides the decision maker with a fuzzy solution. An illustrative numerical example is elaborated to clarify the theory and the solution algorithm.

Keywords. Set covering Problem, multi objective linear fractional programming, fuzzy mathematical programming, fuzzy objective function.

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Introduction

Programming problems with linear fractional objectives (*i.e.*, ratio of two affine functions) are useful in production planning, financial and corporate planning, healthcare and hospital planning and so on. They deal with the optimization of ratio between two physical and/or economic quantities. Examples of fractional objectives include inventory/sales, output/employee, earnings and dividends per share etc. Isbell and Marlow [14] first proposed a method involving the solution of a linear fractional programming (LFP) problem. Charnes and Cooper [8] presented the variable transformation method; Bitran and Novaes [6] gave the updated objective function method; and a simplex like iterative procedure has also been developed by Swarup [22] – which has been employed in this paper. In contrast to LFP, only a few approaches have appeared in the literature concerning multi objective linear fractional programming (MLFP) problems. The method of using concepts of fuzzy set theory for solving MLFP has also been explored earlier [7,9,15,18,21]. Li and Chen [17] established a linear fractional programming model with fuzzy coefficients, and Sakawa [20] defined the concept of Pareto optimal solutions for MLFP with fuzzy parameters in order to produce a solution algorithm.

The Set Covering problem (SCP) is one of the most important NP-complete discrete optimization problems, whose applications are seen in the crew scheduling problems in airline, railway and mass transit transportation companies; where a given set of trips has to be covered by a minimum ratio of cost-to-utility set of pairings; a pairing being a sequence of trips that can be performed by a single crew. Other applications include facility location problems, resource allocation, vehicle routing, assembly line balancing etc. The SCP with linear objective function was first studied by Bellmore and Ratliff [5] who developed a cutting plane method for solving this problem. Lemke *et al.* [17] developed an enumerative approach and, Garfinkel and Nemhauser [12] also studied this problem. Arora *et al.* [1] were the first to consider this problem with a linear fractional objective function. In this and subsequent papers they developed various techniques like, the enumerative technique involving the evaluation of extreme points of the problem using branch and bound method [2] and, the cutting plane technique for finding the optimal cover solution [4]. In [3, 13], an iterative cutting plane technique for finding the efficient solutions of the multi objective integer LFP problem and, the SCP with linear fractional objective functions (MOFSCP) respectively, has been developed. This technique employs cuts developed by Verma *et al.* [27], which are deeper than those given by Dantzig.

In the SCP, the parameters in the objective function are supplied according to the decision makers' requirements, who in most cases is unable to provide this information precisely. In the crew scheduling problem for example, all the coefficients of the cost-to-utility ratio may not be available to the decision maker in a crisp form. In such a case, we formulate them as fuzzy numbers; in other words the objective function can be fuzzified and a leverage is provided to the decision maker to operate. The enumerative algorithms for the single objective fuzzy SCP with a linear, a linear fractional and a quadratic fractional objective have been developed [23–25] mainly using the concepts of vector ranking functions and the weighted sums method. The case of the multi objective SCP with fuzzy linear objective functions has also been studied [26] but, to the authors' knowledge, no work has yet been done in developing a solution procedure for solving a multi objective SCP with fuzzy linear fractional objective functions. In this paper we intend to provide an algorithm and a fuzzy solution for the same. Preliminary concepts of fuzzy numbers are studied in Section 1. The problem under discussion is formulated and, related concepts of efficiency are defined in Section 2. The algorithm developed in Section 3 is supported by a numerical example in Section 4.

1. Fuzzy numbers

In this section, we review some fundamentals of fuzzy numbers [19], which will be used through the remainder of this paper.

Fuzzy set: *A fuzzy set A of the referential set X is characterised by a membership function μ_A which is valued in [0,1], that is $\mu_A\colon X \to [0,1]$ with $\mu_A(x)$ representing the degree of membership of $x \in X$ in A.*

Note: Here we consider $X = \mathbb{R}$, the real line.

The concept of fuzzy numbers is defined differently by different authors, but the most common definition is the one given by Dubois and Prade [10].

Fuzzy number: *A fuzzy number $\tilde{a}$ is a convex normalized fuzzy set of the real line $\mathbb{R}$ such that:*

(i) $\exists$ *unique* $x_0 \in \mathbb{R}$ *such that* $\mu_{\tilde{a}}(x_0) = 1$*; and*

(ii) $\mu_{\tilde{a}}$ *is piecewise continuous.*

The *α-cut* of the fuzzy number $\tilde{a}$ is the ordinary set given by

$$\tilde{a}_\alpha = \{x \in \mathbb{R} : \mu_{\tilde{a}}(x) \geq \alpha\}, \qquad \alpha \in]0,1].$$

In other words, for every $\alpha \in]0,1]$ the *α-cut* of $\tilde{a}$ is the finite closed interval $[a_\alpha^L, a_\alpha^R]$ on $\mathbb{R}$, called the *α-interval of confidence* of the fuzzy number $\tilde{a}$. Let us denote the set of fuzzy numbers by $F(\mathbb{R})$. In order to compare two fuzzy numbers, Dubois and Prade [11] defined a suitable ordering in $F(\mathbb{R})$ by introducing the possibility and necessity indices to rank the fuzzy numbers

$$\mathrm{Poss}(\tilde{a} \succ \tilde{b}) = \sup_x\{\inf_y\{\min(\mu_{\tilde{a}}(x), 1-\mu_{\tilde{b}}(y)) : x \leqq y\}\}$$
$$\mathrm{Nece}(\tilde{a} \succcurlyeq \tilde{b}) = \inf_x\{\sup y\{\max(1-\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y),) : x \geqq y\}\}$$

where $\tilde{a}, \tilde{b} \in F(\mathbb{R})$.

The condition that the degrees of these indices are equal to or greater than α can be equivalently expressed as the usual constraints by using the α-cut sets of $\tilde{a}$ and $\tilde{b}$.

Proposition 1.1.

(1) $\mathrm{Poss}(\tilde{a} \succ \tilde{b}) \geqq \alpha$ *if and only if* $a^R_\alpha \geqq b^R_{1-\alpha}$.
(2) $\mathrm{Nece}(\tilde{a} \succcurlyeq \tilde{b}) \geqq \alpha$ *if and only if* $a^L_{1-\alpha} \geqq b^L_\alpha$.

Throughout this paper, we shall take the ordering between two fuzzy numbers $\tilde{a}, \tilde{b} \in F(\mathbb{R})$ as follows

$$\tilde{a} \succcurlyeq_\alpha \tilde{b} \text{ if and only if } \mathrm{Poss}(\tilde{a} \succ \tilde{b}) \geqq \alpha \text{ and , } \mathrm{Nece}(\tilde{a} \succcurlyeq \tilde{b}) \geqq \alpha$$

Proposition 1.2.
$\tilde{a} \succcurlyeq_\alpha \tilde{b}$ *if and only if* $a^L_\alpha \geqq b^L_\alpha$ *and* $a^R_\alpha \geqq b^R_\alpha \quad \forall \alpha \in [0.5, 1]$.

Similarily, $\tilde{a} \succ_\alpha \tilde{b}$ if and only if $a^L_\alpha > b^L_\alpha$ and $a^R_\alpha > b^R_\alpha \forall \alpha \in [0.5, 1]$. It will be understood that $\tilde{a} \preccurlyeq_\alpha \tilde{b}$ if and only if $\tilde{b} \succcurlyeq_\alpha \tilde{a}$; and that $\tilde{a} \prec_\alpha \tilde{b}$ if and only if $\tilde{b} \succ_\alpha \tilde{a}$. The order relation between two fuzzy vectors $\tilde{a}, \tilde{b} \in F^n(\mathbb{R})$ is defined as follows

$$\tilde{a} \preccurlyeq_\alpha \tilde{b} \text{ if and only if } a^L_{i\alpha} \leqq b^L_{i\alpha} \text{ and } a^R_{i\alpha} \leqq b^R_{i\alpha}\ \forall i = 1, \ldots, n \quad \forall \alpha \in [0.5, 1].$$

A fuzzy number $\tilde{a}$ is called a *nonnegative fuzzy number i.e.* $\tilde{a} \in F(\mathbb{R}_+)$ denoted by $\tilde{a} \succcurlyeq_\alpha 0$, if for every $\alpha \in [0.5, 1]$ we have, $[a^L_\alpha, a^R_\alpha] \subset \mathbb{R}_+ -$ the nonnegative real orthant. Also, a fuzzy number $\tilde{a}$ is called a *positive fuzzy number i.e.* $\tilde{a} \in F(\mathbb{R}_{++})$ denoted by $\tilde{a} \succ_\alpha 0$, if for every $\alpha \in [0.5, 1]$ we have, $[a^L_\alpha, a^R_\alpha] \subset \mathbb{R}_+ \setminus \{0\} \equiv \mathbb{R}_{++}$.

Some basic operations using interval arithmetic for fuzzy numbers $\tilde{a}, \tilde{b} \in F(\mathbb{R})$ are:

(i) Addition: $(\tilde{a} + \tilde{b})_\alpha = [a^L_\alpha + b^L_\alpha, a^R_\alpha + b^R_\alpha]$.
(ii) Scalar Multiplication:
$$(k.\tilde{a})_\alpha = \begin{cases} [ka^L_\alpha, ka^R_\alpha] & \text{if, } k > 0 \\ [ka^R_\alpha, ka^L_\alpha] & \text{if, } k < 0 \end{cases}.$$
(iii) Division: If $\tilde{a} \succcurlyeq_\alpha 0$ and $\tilde{b} \succ_\alpha 0$, then $\left(\frac{\tilde{a}}{\tilde{b}}\right)_\alpha = \left[\frac{a^L_\alpha}{b^L_\alpha}, \frac{a^R_\alpha}{b^R_\alpha}\right] \qquad \forall \alpha \in [0.5, 1]$.

2. Theoretical development

In this section, we first present the concepts related to the multi objective Set Covering problem with fuzzy linear fractional objectives, and later we formulate this problem.

2.1. Fuzzy fractional programming problem

Consider the following linear fractional programming problem (FFP) with fuzzy parameters in the objective function

$$\textbf{(FFP)} \qquad \text{Min } \tilde{z}(x) = \frac{\tilde{c}x + \tilde{p}}{\tilde{d}x + \tilde{q}}$$

$$\text{subject to } X \in S$$

$$\text{where, } S = \{X \in \mathbb{R}^n : \sum_{j=1}^{n} a_{ij}x_j \geq 1, x_j \geq 0, i \in I, j \in J\}.$$

Based on the concept of α-cut sets, the optimal solution of FFP is shown to be equivalent to the optimal solution of a corresponding crisp linear fractional programming problem by Mehra *et al.* [19]. Let $\alpha \in [0, 1]$ be the grade of satisfaction associated with the fuzzy objective function of FFP. Then the corresponding optimal solution is defined as

α-***Optimal solution:*** *A vector $x^*_\alpha \in S$ is said to be an α-optimal solution of FFP if and only if there does not exist any $x \in S$ such that $\tilde{z}(x^*_\alpha) \succ_\alpha \tilde{z}(x)$.*
In accordance with the operations defined in Section 1, we assume that $\alpha \in [0.5, 1]$. Also, let

$$\tilde{c}x + \tilde{p} \in F(\mathbb{R}_+) \text{ and } \tilde{d}x + \tilde{q} \in F(\mathbb{R}_{++}) \quad \forall x \in S.$$

Now the FFP can be written as

$$(\alpha\textbf{-FFP}) \qquad \underset{X \in S}{\text{Min}}\ z_\alpha(x) = \frac{\sum_{j=1}^{n} [c^L_{j\alpha}, c^R_{j\alpha}]x_j + [p^L_\alpha, p^R_\alpha]}{\sum_{j=1}^{n} [d^L_{j\alpha}, d^R_{j\alpha}]x_j + [q^L_\alpha, q^R_\alpha]}$$

where, $z_\alpha(x) = [z^L_\alpha(x), z^R_\alpha(x)]$ is the α-cut set of $\tilde{z}(x)$. Using the division operation of fuzzy numbers, the objective of α-FFP transforms into the following

$$\text{Min}[z^L_\alpha(x), z^R_\alpha(x)] = \left[\frac{\sum_{j=1}^{n} c^L_{j\alpha}x_j + p^L_\alpha}{\sum_{j=1}^{n} d^R_{j\alpha}x_j + q^R_\alpha}, \frac{\sum_{j=1}^{n} c^R_{j\alpha}x_j + p^R_\alpha}{\sum_{j=1}^{n} d^L_{j\alpha}x_j + q^L_\alpha} \right].$$

We further reduce it to an equivalent bi-objective programming problem

$$(\alpha\textbf{-BOP}) \qquad \underset{X \in S}{\text{Min}}(z^L_\alpha(x), z^R_\alpha(x)) = \left(\frac{\sum_{j=1}^{n} c^L_{j\alpha}x_j + p^L_\alpha}{\sum_{j=1}^{n} d^R_{j\alpha}x_j + q^R_\alpha}, \frac{\sum_{j=1}^{n} c^R_{j\alpha}x_j + p^R_\alpha}{\sum_{j=1}^{n} d^L_{j\alpha}x_j + q^L_\alpha} \right).$$

For $\alpha \in [0.5, 1]$, x^*_α is said to be an α-optimal solution of FFP if and only if there does not exist any $x \in S$ such that $\tilde{z}(x^*_\alpha) \succ_\alpha \tilde{z}(x)$ or,

$$z^L_\alpha(x) < z^L_\alpha(x^*_\alpha) \text{ and } z^R_\alpha(x) < z^R_\alpha(x^*_\alpha) \text{ or,}$$

$$(z^L_\alpha(x), z^R_\alpha(x)) < (z^L_\alpha(x^*_\alpha), z^R_\alpha(x^*_\alpha))$$

i.e. x^*_α is a weakly efficient solution of the bi-objective programming problem α-BOP.

Now for every $\alpha \in [0.5, 1]$, two (crisp) linear fractional programming problems namely, α-LFP and α-RFP are associated with α-FFP.

$$\textbf{(α-LFP)} \qquad \underset{X \in S}{\text{Min}}\ z^L_\alpha(x) = \frac{\sum_{j=1}^{n} c^L_{j\alpha} x_j + p^L_\alpha}{\sum_{j=1}^{n} d^R_{j\alpha} x_j + q^R_\alpha}$$

$$\textbf{(α-RFP)} \qquad \underset{X \in S}{\text{Min}}\ z^R_\alpha(x) = \frac{\sum_{j=1}^{n} c^R_{j\alpha} x_j + p^R_\alpha}{\sum_{j=1}^{n} d^L_{j\alpha} x_j + q^L_\alpha}.$$

Let x^L_α and x^R_α be the optimal solutions of α-LFP and α-RFP respectively. Then they are both weakly efficient solutions of α-BOP. Moreover, we also have

$$z^L_\alpha(x^L_\alpha) \leqq z^L_\alpha(x^R_\alpha) \leqq z^R_\alpha(x^R_\alpha) \leqq z^R_\alpha(x^L_\alpha).$$

Since the α-interval of confidence of FFP evaluated at x^R_α, *i.e.* $[z^L_\alpha(x^R_\alpha), z^R_\alpha(x^R_\alpha)]$, is a subset of the α-interval of confidence of FFP evaluated at x^L_α, *i.e.* $[z^L_\alpha(x^L_\alpha), z^R_\alpha(x^L_\alpha)]$, we solve the problem α-LFP in order to provide the decision maker with more flexibility. Thus, in order to obtain an α-acceptable solution of FFP, we need only solve the problem α-LFP and obtain x^L_α.

2.2. Problem formulation

Consider a set $I = \{1, 2, \ldots, m\}$ and a set $P = \{P_1, P_2, \ldots P_n\}$, where $P_j \subseteq I, j \in J = \{1, 2, \ldots n\}$. A subset J^* of J is said to be a *cover* of I if $\bigcup_{j \in J^*} P_j = I$. The multi objective Set Covering problem having fuzzy linear fractional functionals as objective functions is then formulated below as

$$\textbf{(FMCP)} \qquad \text{Minimize } (\tilde{Z}_1, \tilde{Z}_2, \ldots, \tilde{Z}_p)$$

$$\text{subject to} \qquad \sum_{j=1}^{n} a_{ij} x_j \geq 1, \qquad i \in I \tag{2.1}$$

$$x_j = 0 \ \text{ or } 1, \qquad j \in J \tag{2.2}$$

where, $x_j = \begin{cases} 1, & \text{if j is in the cover} \\ 0, & \text{otherwise} \end{cases}$ and $a_{ij} = \begin{cases} 1, & \text{if i} \in P_j \\ 0, & \text{otherwise} \end{cases}$.

Here, p is the number of objectives and

$$\tilde{Z}_r = \frac{\tilde{C}_r X + \tilde{\alpha_r}}{\tilde{D}_r X + \tilde{\beta_r}}, \qquad r \in R = \{1, 2, \ldots, p\}$$

where, $\tilde{C}_r = \{\tilde{c}_{r1}, \tilde{c}_{r2}, \ldots, \tilde{c}_{rn}\} \in F(\mathbb{R}_+)$ and $\tilde{D}_r = \{\tilde{d}_{r1}, \tilde{d}_{r2}, \ldots, \tilde{d}_{rn}\}$ $(r \in R)$, are $nx1$ vectors with entries from the set $F(\mathbb{R})$, and $\tilde{\alpha_r}, \tilde{\beta_r} \in F(\mathbb{R})$ $(r \in R)$. It is further assumed that the numerator $\tilde{C}_r X + \tilde{\alpha_r}$ is a nonnegative fuzzy number and that the denominator $\tilde{D}_r X + \tilde{\beta_r}$ is a positive fuzzy number for each $r \in R$ in the feasible region of the problem. In other words, $\tilde{C}_r X + \tilde{\alpha_r} \in F(\mathbb{R}_+)$ and $\tilde{D}_r X + \tilde{\beta_r} \in F(\mathbb{R}_{++})$ $(r \in R)$, for all X in the feasible region.

Let $\gamma \in [0, 1]$ be the grade of satisfaction associated with each of the fuzzy objective function $\tilde{Z}_r$. For consistency with the ranking relations defined in Section 1, we assume that the grade of satisfaction $\gamma \in [0.5, 1]$. Then we have,

$$(C_r)^L_\gamma X + (\alpha_r)^L_\gamma \geqq 0, (D_r)^L_\gamma X + (\beta_r)^L_\gamma > 0, \forall \gamma \in [0.5, 1], r \in R.$$

A few definitions in this respect are:

Cover solution: *A solution of FMCP which satisfies both (1) and (2) is called a cover solution.*

Redundant column: *A column corresponding to j belonging to the cover J^* is said to be a redundant column if $J^* - \{j\}$ is also a cover. Then the cover J^* is termed as a redundant cover.*

Prime cover: *A cover J^* is said to be a prime cover if none of the columns $j \in J^*$ is redundant.*

γ-Efficient cover solution: [20] *A solution X^* of FMCP is said to be a γ-efficient cover solution if and only if there does not exist another cover solution X such that $\tilde{Z}_i(X) \precsim_\gamma \tilde{Z}_i(X^*)$ $\forall i \in I$ and $\tilde{Z}_k(X) \prec_\gamma \tilde{Z}_k(X^*)$ for at least one $k \in I$. Also, the corresponding nondominated criterion vector, or objective function value is $(\tilde{Z}_1(X^*), \tilde{Z}_2(X^*), \ldots, \tilde{Z}_p(X^*))$.*

The relaxed multi objective fractional program with fuzzy objectives, associated with FMCP is obtained by replacing (2.2) with $x_j \geq 0,\ j \in J$. We thus study the following program and obtain its γ-efficient cover solutions of the form 0-1 [3].

(FMFP) $\qquad \text{Min } (\tilde{Z}_1, \tilde{Z}_2, ..., \tilde{Z}_p)$

$$\text{subject to } X \in S$$

$$\text{where } S = \{X \in \mathbb{R}^n : \sum_{j=1}^{n} a_{ij} x_j \geq 1, x_j \geq 0, i \in I, j \in J\}.$$

Consider the problem FMFP above with the first objective function as

$$\textbf{(FMFP-}\tilde{Z}_1\textbf{)} \qquad \underset{X\in S}{\text{Min}}\ \tilde{Z}_1 = \frac{\tilde{C}_1 X + \tilde{\alpha}_1}{\tilde{D}_1 X + \tilde{\beta}_1}\cdot$$

Solve FMFP-$\tilde{Z}_1$ by solving the crisp problem γ-LFP, and let $X = \{x_j\}$ be its γ-acceptable optimal solution with the corresponding objective value of $\tilde{Z}_1$ as $\tilde{Z}_1(X) = [(Z_1)^L_\gamma(X), (Z_1)^R_\gamma(X)]$. If $\{x_j\}$ is of the form 0-1, take $X = X^1$ and find the value of the other objective functions at X^1. If $\{x_j\}$ is not of the form 0-1, then apply Gomory cut to find an integer solution of 0-1 form; let it be X^1. Find the value of the other objective functions at X^1. The vector $(\tilde{Z}^1_1, \tilde{Z}^1_2, \ldots, \tilde{Z}^1_p)$ is then a criterion vector corresponding to the first efficient cover solution of FMCP, where $\tilde{Z}^1_r = \tilde{Z}_r(X^1), r \in R$.

Before we proceed with the solution procedure, we note a few concepts regarding the linear fractional set covering problem (LFCP) equivalent to γ-LFP.

Remark 2.1. Consider the following linear fractional set covering problem

$$\textbf{(LFCP)} \qquad \underset{X\in S}{\text{Min}}\ Z(X) = \frac{\displaystyle\sum_{j\in J} c_j x_j + \alpha}{\displaystyle\sum_{j\in J} d_j x_j + \beta}\cdot$$

It is required to find the optimal cover J^* which minimizes the ratio

$$\frac{\displaystyle\sum_{j\in J^*} c_j + \alpha}{\displaystyle\sum_{j\in J^*} d_j + \beta},$$

where $J^* \subseteq I$ and $\displaystyle\bigcup_{j\in J^*} P_j = I$.

Theorem 2.2 ([3])**.** *Every optimal cover is a prime cover, if either*

(a) *all d_j's are negative; or*
(b) *any ratio of the partial sums of c_j's and d_j's is greater than the value of the objective function at the cover solution and, the latter partial sum is positive.*

Given the first efficient solution X^1, the other integer feasible solutions of the crisp linear fractional program γ-LFP are obtained and ranked in an increasing order of the values of its objective function. In order to do so, a cut – which is a more generalized form of the Dantzig cut – is introduced. But first, various notations employed in defining the special cut are given below:

X^k: The kth efficient solution of the 0-1 form obtained after applying the cut

$$\sum_{j \in N^{k-1} \setminus \{j_{k-1}\}} x_j \geq 1, \; j_{k-1} \in T^{k-1}$$

B^k: Basis corresponding to the solution X^k

a_j^k: The activity vector of $X^k, \; j \in J$

I^k: $\{j \in J : a_j^k \in B^k\}$

N^k: $\{j \in J : a_j^k \notin B^k\}$

U_{rj}^k: $(C_{B_r}^k)^T y_j^k$, $j \in J$, $r \in R$, where $C_{B_r}^k$ is the vector having its components as the coefficients associated with the basic variables in the numerator of the rth objective function.

L_{rj}^k: $(D_{B_r}^k)^T y_j^k$, $j \in J$, $r \in R$, where $D_{B_r}^k$ is the vector having its components as the coefficients associated with the basic variables in the denominator of the rth objective function.

y_j^k: $(B^k)^{-1} a_j^k, \; j \in J$

u_r^k: $C_r^k X^k + \alpha_r, \; r \in R$

l_r^k: $D_r^k X^k + \beta_r, \; r \in R$

δ_{rj}^k: $u_r^k (L_{rj}^k - d_{rj}^k) - l_r^k (U_{rj}^k - c_{rj}^k)$

T^k: $\{j \in J : j \in N^k, \; \delta_{rj}^k \geq 0$ and $\delta_{rj}^k < 0$ for at least one $r \in R'\}$,

where $R' = \{2, 3, ..., p\}$

J^k: $\{j \in J : j \in N^k, \; \delta_{rj}^k \geq 0, \; r \in R\}$.

Edge $E_{j_k}^k$ incident at solution X^k for $j_k \in T^k$ is defined as:

$$E_{j_k}^k = \left\{ X = (x_1, x_2, \ldots, x_n) : \begin{array}{ll} x_i & = x_i^k - \phi_{j_k} y_{ij_k}^k, \; i \in I_k \\ x_{j_k} & = \phi_{j_k} \\ x_\nu & = 0, \; \nu \in N^k \setminus j_k \end{array} \right.$$

where $\phi_{j_k} \leq \min\limits_{i \in I^k} \left\{ \frac{x_i^k}{y_{ij_k}^k}, \; y_{ij_k}^k > 0 \right\}$.

The following theorem justifies the cut which is used to find the efficient cover solutions of FMCP.

Theorem 2.3 ([13])**.** *An efficient solution of FMFP not on an edge* $E^k_{j_k},\ j_k \in T_k$ *through* X_k *in the truncated region of S lies in the closed half space* $\sum_{j \in N^k \setminus \{j_k\}} x_j \geq 1$*.*

Proof. Let $\bar{X} = (\bar{x}_1, \bar{x}_2, ..., \bar{x}_n)$ be an efficient solution of FMFP not on the edge $E^k_{j_k}$ through X_k , and let if possible

$$\sum_{j \in N^k \setminus \{j_k\}} \bar{x}_j < 1$$

$\Rightarrow$ $\quad \bar{x}_j = 0$ for all $j \in N^k \setminus \{j_k\}$

i.e. $\quad \bar{x}_j = 0$ for all $j \in N^k,\ j \neq \{j_k\}$.

Now either $\quad \bar{x}_{j_k} = 0$ or $\bar{x}_{j_k} = 1$

Case (i): $\bar{x}_{j_k} = 0$

Since $\bar{x}_j = 0$ for all $j \in N^k \setminus \{j_k\}$, therefore $\bar{x}_j = 0$ for all $j \in N^k$

Case (ii): $\bar{x}_{j_k} = 1$

such that $\quad \bar{x}_{j_k} \leq \min_{i \in I^k} \left\{ \frac{x^k_i}{y^k_{ij_k}},\ y^k_{ij_k} > 0 \right\}$

then $\bar{X}$ lies on the edge $E^k_{j_k}$, which is a contradiction. Therefore, $\bar{x}_j = 1$ for at least one $j \in N^k \setminus \{j_k\}$.

Hence in both the cases, $\bar{x}_j = 1$ for at least one $j \in N^k \setminus \{j_k\}$. Hence $\sum_{j \in N^k \setminus \{j_k\}} \bar{x}_j \geq 1$ for $\bar{X} = (\bar{x}_1, \bar{x}_2, ..., \bar{x}_n)$ which is an efficient solution of FMFP. □

Note: The algorithm terminates after a finite number of steps because the feasible region is truncated at each step by repeated application of the cuts of the form $\sum_{j \in N^k \setminus \{j_k\}} x_j \geq 1$. An edge or a point once deleted cannot reappear. In turn, the entire set of efficient p-tuples is obtained.

3. Algorithm

Step 1. Given the fuzzy multi objective linear fractional Set Covering problem FMCP, relax the conditions on the decision variables to obtain an equivalent fuzzy multi objective linear fractional programming problem FMFP. Then solve FMFP-$\tilde{Z}_1$ and, consequently solve the crisp linear fractional problem by the simplex method of Swarup [22].

Step 2. If the solution X^1 is of the 0-1 form, then it is the first efficient cover solution of FMCP otherwise, apply Gomory cut to obtain X^1 in 0-1 form. Form the corresponding sets I^1, N^1, T^1 and $F^1 = (\tilde{Z}_1(X^1), \tilde{Z}_2(X^1), \ldots, \tilde{Z}_p(X^1))$.

Step 3. Choose $j^1 \in T^1$, find the corresponding minimum ratio ϕ_{j_1}.

(a) If $\phi_{j_1} = 0$, no alternate efficient solution on edge $E^1_{j_1}$ exists. Choose another $j^1 \in T^1$ and find ϕ_{j_1}.
(b) If $\phi_{j_1} = 1$, determine the alternate solution X^1_1 along the edge $E^1_{j_1}$. Introduce the corresponding p-tuple in F^1. Remove all dominated p-tuples from F^1.

Step 4. Truncate the edge $E^1_{j_1}$ by the cut $\sum_{j \in N^{k-1} \setminus \{j_{k-1}\}} x_j \geq 1, \; j_{k-1} \in T^{k-1}$. Solve the problem as mentioned above in Step 1 and use the Dual Simplex method and Gomory cut (if necessary), to get a second efficient cover solution X^2 of 0-1 form in the truncated region.

Step 5. If the corresponding p-tuple is nondominated, then augment the p-tuple to F^1 and name it F^2. Find I^2, N^2, T^2 and go to Step 3. The process terminates after the sth stage when either

(a) $T^s = \phi$; or
(b) $T^s \neq \phi$; with
  (i) any $j_s \in T^s$ yields dominated edge only; or
  (ii) the application of the cut $\sum_{j \in N^s \setminus \{j_s\}} x_j \geq 1$ leads to an infeasible solution in the truncated region for some $j_s \in N^s$.

4. Numerical example

Consider the following FMCP

$$\textbf{(FMCP)} \qquad \text{Min } \tilde{Z}_1(X) = \frac{\tilde{c}_{11}x_1 + \tilde{c}_{12}x_2 + \tilde{c}_{13}x_3}{\tilde{d}_{11}x_1 + \tilde{d}_{12}x_2 + \tilde{d}_{13}x_3 + \tilde{\beta}_1}$$

$$\text{Min } \tilde{Z}_2(X) = \frac{\tilde{c}_{21}x_1 + \tilde{c}_{22}x_2 + \tilde{c}_{23}x_3}{\tilde{d}_{21}x_1 + \tilde{d}_{22}x_2 + \tilde{d}_{23}x_3 + \tilde{\beta}_2}$$

$$\text{subject to} \qquad \sum_{j=1}^{3} a_{ij}x_j \geq 1, \qquad i \in I \tag{4.1}$$

$$x_j = 0 \;\text{ or } 1, \qquad j \in J \tag{4.2}$$

$$\text{where} \qquad x_j = \begin{cases} 1, & \text{if j is in the cover} \\ 0, & \text{otherwise} \end{cases} \qquad \text{and} \qquad a_{ij} = \begin{cases} 1, & \text{if i} \in P_j \\ 0, & \text{otherwise} \end{cases}$$

where the membership functions of the fuzzy numbers in the objective are

$$\mu_{\tilde{c}_{11}} = \max\left\{1 - \frac{|x-13.5|}{3}, 0\right\}, \qquad \mu_{\tilde{c}_{12}} = \max\left\{1 - \frac{|x-3|}{2}, 0\right\}$$

$$\mu_{\tilde{c}_{13}} = \max\left\{1 - \frac{|x-6|}{2}, 0\right\}, \qquad \mu_{\tilde{c}_{21}} = \max\{1 - |x-9.5|, 0\}$$

$$\mu_{\tilde{c}_{22}} = \begin{cases} 0, & x < 10 \\ \dfrac{x-10}{6}, & 10 \le x < 16 \\ \dfrac{23-x}{7}, & 16 \le x < 23 \\ 0, & x \ge 23 \end{cases}, \qquad \mu_{\tilde{c}_{23}} = \begin{cases} 0, & x < 0 \\ \dfrac{x}{2}, & 0 \le x < 2 \\ \dfrac{5-x}{3}, & 2 \le x < 5 \\ 0, & x \ge 5 \end{cases}$$

$$\mu_{\tilde{d}_{11}} = \begin{cases} 0, & x < -6 \\ \dfrac{216+x^3}{7}, & -6 \le x < -3 \\ 1, & -3 \le x < -2 \\ \dfrac{x^2}{4}, & -2 \le x < 0 \\ 0, & x \ge 0 \end{cases}, \qquad \mu_{\tilde{d}_{12}} = \begin{cases} 0, & x < -14 \\ 1 - \left(\dfrac{x+14}{3}\right)^2, & -14 \le x < -11 \\ 1, & -11 \le x < -5 \\ 1 - \left(\dfrac{x+5}{2}\right)^2, & -5 \le x < -3 \\ 0, & x \ge -3 \end{cases}$$

$$\mu_{\tilde{d}_{13}} = \begin{cases} 0, & x < -6 \\ \dfrac{x+6}{3}, & -6 \le x < -3 \\ 1, & -3 \le x < -2 \\ -1-x, & -2 \le x < -1 \\ 0, & x \ge -1 \end{cases}, \qquad \mu_{\tilde{d}_{21}} = \begin{cases} 0, & x < -13 \\ \dfrac{x+13}{7}, & -13 \le x < -9 \\ 1, & -9 \le x < -7 \\ -5-x, & -7 \le x < -5 \\ 0, & x \ge -5 \end{cases}$$

$$\mu_{\tilde{\beta}_1} = \begin{cases} 0, & x \le 1 \\ x-1, & 1 < x \le 2 \\ 3-x, & 2 < x \le 3 \\ 0, & x > 3 \end{cases}, \qquad \mu_{\tilde{\beta}_2} = \begin{cases} 0, & x \le 14 \\ x-14, & 14 < x \le 15 \\ 16-x, & 15 < x \le 16 \\ 0, & x > 16. \end{cases}$$

On converting the problem FMCP into FMFP by relaxing the condition on the decision variables we have the following problem

(FMFP)
$$\underset{X\in S}{\text{Min}}\ \tilde{Z}_1(X) = \frac{\tilde{c}_{11}x_1 + \tilde{c}_{12}x_2 + \tilde{c}_{13}x_3}{\tilde{d}_{11}x_1 + \tilde{d}_{12}x_2 + \tilde{d}_{13}x_3 + \tilde{\beta}_1}$$

$$\underset{X\in S}{\text{Min}}\ \tilde{Z}_2(X) = \frac{\tilde{c}_{21}x_1 + \tilde{c}_{22}x_2 + \tilde{c}_{23}x_3}{\tilde{d}_{21}x_1 + \tilde{d}_{22}x_2 + \tilde{d}_{23}x_3 + \tilde{\beta}_2}$$

where $S = \{X \in \mathbb{R}^3 : \sum_{j=1}^{3} a_{ij}x_j \ge 1,\ x_j \ge 0,\ i = 1,2,3,\ j = 1,2,3\}$.

Let $\gamma \in [0.5, 1]$ be the specified grade of satisfaction associated with the objective functions of FMFP. The γ-cut sets of the corresponding fuzzy coefficients are given as follows

$$
\begin{aligned}
&[c_{11}]_\gamma = [10.5 + 3\gamma, 16.5 - 3\gamma], && [c_{12}]_\gamma = [1 + 2\gamma, 5 - 2\gamma],\\
&[c_{13}]_\gamma = [4 + 2\gamma, 8 - 2\gamma], && [c_{21}]_\gamma = [8.5 + \gamma, 10.5 - \gamma],\\
&[c_{22}]_\gamma = [10 + 6\gamma, 23 - 7\gamma], && [c_{23}]_\gamma = [2\gamma, 5 - 3\gamma],\\
&[d_{11}]_\gamma = [-3 + \gamma, -2\gamma^{\frac{1}{2}}], && [d_{12}]_\gamma = [3(1-\gamma)^{\frac{1}{2}} - 6, 2(1-\gamma)^{\frac{1}{2}} - 5],\\
&[d_{13}]_\gamma = [\gamma - 3, -1 + \gamma], && [d_{21}]_\gamma = [7\gamma - 13, -5 - \gamma],\\
&[d_{22}]_\gamma = [13\gamma - 27, -7\gamma^{\frac{1}{2}}], && [d_{23}]_\gamma = [17\gamma - 40, -(128\gamma + 16)^{\frac{1}{2}}],\\
&[\beta_1]_\gamma = [9 + \gamma, 11 - \gamma], && [\beta_2]_\gamma = [74 + \gamma, 76 - \gamma].
\end{aligned}
$$

Consider the objective function $\tilde{Z}_1$, then the corresponding problem FMFP-$\tilde{Z}_1$ is

$$
\textbf{(FMFP-}\tilde{Z}_1\textbf{)} \qquad \underset{X \in S}{\text{Min}}\ \tilde{Z}_1(X) = \frac{\tilde{c}_{11}x_1 + \tilde{c}_{12}x_2 + \tilde{c}_{13}x_3}{\tilde{d}_{11}x_1 + \tilde{d}_{12}x_2 + \tilde{d}_{13}x_3 + \tilde{\beta}_1}.
$$

In order to find a γ-acceptable solution of FMFP-$\tilde{Z}_1$, we need to solve the following crisp linear fractional program

$$
\begin{aligned}
\gamma\textbf{-(LFP-}\tilde{Z}_1\textbf{)} \qquad \underset{X \in S}{\text{Min}}(Z_1)^L_\gamma(X) &= \frac{(c_{11})^L_\gamma x_1 + (c_{12})^L_\gamma x_2 + (c_{13})^L_\gamma x_3}{(d_{11})^R_\gamma x_1 + (d_{12})^R_\gamma x_2 + (d_{13})^R_\gamma x_3 + (\beta_1)^R_\gamma}\\
&= \frac{(10.5 + 3\gamma)x_1 + (1 + 2\gamma)x_2 + (4 + 2\gamma)x_3}{-2\gamma^{\frac{1}{2}}x_1 + (2(1-\gamma)^{\frac{1}{2}} - 5)x_2 + (-1 + \gamma)x_3 + (11 - \gamma)}.
\end{aligned}
$$

The objective functions corresponding to the problems γ-(RFP-$\tilde{Z}_1$), γ-(LFP-$\tilde{Z}_2$), and γ-(RFP-$\tilde{Z}_2$) are as follows:

$$
\begin{aligned}
\text{Min } (Z_1)^R_\gamma(X) &= \frac{(16.5 - 3\gamma)x_1 + (5 - 2\gamma)x_2 + (8 - 2\gamma)x_3}{(\gamma - 3)x_1 + (3(1-\gamma)^{\frac{1}{2}} - 6)x_2 + (\gamma - 3)x_3 + (9 + \gamma)}\\
\text{Min } (Z_2)^L_\gamma(X) &= \frac{(8.5 + \gamma)x_1 + (10 + 6\gamma)x_2 + 2\gamma x_3}{(-5 - \gamma)x_1 - 7\gamma^{\frac{1}{2}}x_2 - (128\gamma + 16)^{\frac{1}{2}}x_2 + (76 - \gamma)}\\
\text{Min } (Z_2)^R_\gamma(X) &= \frac{(10.5 - \gamma)x_1 + (23 - 7\gamma)x_2 + (5 - 3\gamma)x_3}{(7\gamma - 13)x_1 + (13\gamma - 27)x_2 + (17\gamma - 40)x_3 + (74 + \gamma)}.
\end{aligned}
$$

For $\gamma = 0.5$, the problem γ-(LFP-$\tilde{Z}_1$) reduces to

$$
\underset{X \in S}{\text{Min}}\ (Z_1)^L_{0.5}(X) = \frac{12x_1 + 2x_2 + 5x_3}{-1.4x_1 - 3.6x_2 - 0.5x_3 + 10.5}.
$$

The initial basic feasible solution obtained is used to solve the above problem by simplex method [22], the optimal solution of which is given by Table 1. Note that, here $D^1_{B_2}$ and $C^1_{B_2}$ correspond to the objective function $(Z_2)^L_\gamma(X)$.

Thus, $X^1 = (0, 1, 1)$ is an optimal 0-1 integer solution of the problem 0.5-(LFP-$\tilde{Z}_1$), and hence the first efficient cover solution of FMFP, with the criterion vector $(\tilde{Z}_1(X^1), \tilde{Z}_2(X^1))$ as $([\frac{7}{5.4}, \frac{11}{3.1}], [\frac{14}{61.6}, \frac{23}{22.5}])$.

TABLE 1.

$D^1_{B_2}$	$C^1_{B_2}$	$D^1_{B_1}$	$C^1_{B_1}$	X_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
0	0	0	0	$x_6 = 1$	2	0	0	−1	−1	1	0	0	0
−9	−1	−1.5	−5	$x_3 = 1$	1	0	1	0	−1	0	0	0	0
−4.9	−13	−3.6	−2	$x_2 = 1$	1	1	0	−1	0	0	0	0	0
0	0	0	0	$x_7 = 1$	1	0	0	0	0	0	1	0	0
0	0	0	0	$x_8 = 0$	−1	0	0	1	0	0	0	1	0
0	0	0	0	$x_9 = 0$	−1	0	0	0	1	0	0	0	1
$l^1_1 = 5.4$		$u^1_1 = -7$		$U^1_{1j} - c^1_j =$	5	0	0	2	5	0	0	0	0
				$L^1_{1j} - d^1_j =$	−3.7	0	0	3.6	1.5	0	0	0	0
				$\delta^1_{1j} =$	−1.1	0	0	−36	−7.5	0	0	0	0
$l^1_2 = 61.6$		$u^1_2 = -14$		$U^1_{2j} - c^2_j =$	−5	0	0	13	1	0	0	0	0
				$L^1_{2j} - d^2_j =$	−8.4	0	0	4.9	9	0	0	0	0
				$\delta^1_{2j} =$	425.6	0	0	−869.4	−187.6	0	0	0	0

TABLE 2.

$D^1_{B_1}$	$C^1_{B_1}$	X_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
0	0	$x_6 = 1$	2	0	0	−1	−1	1	0	0	0	0
−1.5	−5	$x_3 = 1$	1	0	1	0	−1	0	0	0	0	0
−3.6	−2	$x_2 = 1$	1	1	0	−1	0	0	0	0	0	0
0	0	$x_7 = 1$	1	0	0	0	0	0	1	0	0	0
0	0	$x_8 = 0$	−1	0	0	1	0	0	0	1	0	0
0	0	$x_9 = 0$	−1	0	0	0	1	0	0	0	1	0
0	0	$x_{10} = -1$	0	0	0	−1	−1	0	0	0	0	1
$l^1_1 = 5.4$	$u^1_1 = -7$	$U^1_{1j} - c^1_j =$	5	0	0	2	5	0	0	0	0	0
		$L^1_{1j} - d^1_j =$	−3.7	0	0	3.6	1.5	0	0	0	0	0
		$\delta^1_{1j} =$	−1.1	0	0	−36	−7.5	0	0	0	0	0

Now using Table 1, we have $F^1 = \{([\frac{7}{5.4}, \frac{11}{3.1}], [\frac{14}{61.6}, \frac{23}{22.5}])\}$, $I^1 = \{2, 3, 6, 7, 8, 9\}$, $N^1 = \{1, 4, 5\}$, $T^1 = \{1\}$.

For efficient solutions of FMFP on the edge $E^1_{j_1}$, take $j^1 = 1 \in T^1$, so edge E^1_1 is to be scanned. Here $0 < \theta_1 \leq \frac{1}{2}$, *i.e.* $\theta_1 < 1$. Thus no integer feasible solution can be obtained on edge E^1_1. Next, truncate the edge E^1_1 by introducing the generalized cut

$$\sum_{j \in N^1 \setminus \{j_1\}} x_j \geq 1, \; i.e. x_4 + x_5 \geq 1, \text{ or } -x_4 - x_5 + x_{10} = -1.$$

This cut appended to Table 1 gives us Table 2.

As $x_{10} = -1$ the solution is infeasible and, by then applying the dual simplex algorithm to Table 2 we get the next efficient cover solution $X^2 = (1, 0, 1)$ which

TABLE 3.

$D^2_{B_2}$	$C^2_{B_2}$	$D^2_{B_1}$	$C^2_{B_1}$	X_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
0	0	0	0	$x_8 = 1$	0	0	0	0	0	1	0	1	1	0
-9	-1	-1.5	-5	$x_3 = 1$	0	0	1	0	0	0	0	0	1	0
-4.9	-13	-3.6	-2	$x_2 = 0$	0	1	0	0	0	-1	0	0	-1	0
0	0	0	0	$x_7 = 0$	0	0	0	0	0	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$
0	0	0	0	$x_5 = 1$	0	0	0	0	1	$\frac{1}{2}$	0	0	1	$-\frac{1}{2}$
-5.5	-9	-1.4	-12	$x_1 = 1$	1	0	0	0	0	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$
0	0	0	0	$x_4 = 0$	0	0	0	1	0	$-\frac{1}{2}$	0	0	-1	$-\frac{1}{2}$
$l^2_1 = 7.6$		$u^2_1 = -17$		$U^2_{1j} - c^2_j =$	0	0	0	0	0	-4	0	0	-3	6
				$L^2_{1j} - d^2_j =$	0	0	0	0	0	2.9	0	0	2.9	0.7
				$\delta^2_{1j} =$	0	0	0	0	0	-18.9	0	0	-12.9	-57.5
$l^2_2 = 61$		$u^2_2 = -10$		$U^2_{2j} - c^2_j =$	0	0	0	0	0	8.5	0	0	12	4.5
				$L^2_{2j} - d^2_j =$	0	0	0	0	0	2.15	0	0	-4.1	2.75
				$\delta^2_{2j} =$	0	0	0	0	0	-540	0	0	-727.9	-302

is given by Table 3. Here, $X^2 = (1,0,1)$ is an efficient cover solution of FMFP, with the criterion vector $(\tilde{Z}_1(X^2), \tilde{Z}_2(X^2))$ as $\left(\left[\frac{17}{7.6}, \frac{22}{4.5}\right], \left[\frac{10}{61}, \frac{13.5}{33.5}\right]\right)$.

From Table 3 we have, $F^2 = \left\{\left(\left[\frac{7}{5.4}, \frac{11}{3.1}\right], \left[\frac{14}{61.6}, \frac{23}{22.5}\right]\right), \left(\left[\frac{17}{7.6}, \frac{22}{4.5}\right], \left[\frac{10}{61}, \frac{13.5}{33.5}\right]\right)\right\}$, $I^2 = \{1,2,3,5,7,8,9\}, N^2 = \{6,9,10\}, T^2 = \phi$.

Thus the algorithm terminates here, giving the decision maker various efficient covers $X^1 = (0,1,1)$ and $X^2 = (1,0,1)$ of the given problem MCP and corresponding fuzzy non dominated 2-tuple F^2.

5. Conclusion

The method proposed in this paper attempts to not only solve the multi objective Set Covering problem with fuzzy linear fractional objective functions, but also to provide the decision maker with a fuzzy solution for the same. This algorithm obtains the complete set of efficient cover solutions of the problem. The generalized cut employed here is better than the Dantzig cut in the sense that the former cuts off an entire edge, whereas the latter only cuts off a point. Fuzzy solutions of the problem offer a set of good alternatives and encompass the more precise solutions obtained using other methods. In future, we intend to look into the complexity and performance of this algorithm in terms of the processing time, and possibly study the scenario when the size of the problem becomes very large, in which case this algorithm will be less suited.

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