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AN EFFICIENT DECISION MAKING PROCESS FOR VEHICLES OPERATIONS IN UNDERGROUND MINING

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Abstract. Day by day, the LHD (load-haul-dump) vehicle operator addresses the routing problems at the production level of mine, whose solution impacts on the performance of all the production chain. Unfortunately, the operator's goal of minimizing the makespan of his workload is not necessarily optimal as one needs to take into account the coordination with next operation levels. In this paper, we stated the problem to determine the working path of LHD vehicle for minimizing the makespan subject to the coordination between the production level and next the operation level, so called the reduction level. We prove that the problem is NP-hard in the strong sense, propose an exact formulation by a mixed integer linear programming (MIP) model and generate an approximation algorithm. From a real implementation point of view, we developed a simple-to-execute decision-making process (DMP) for the LHD vehicle operator based on the generated approximation algorithm. Finally, we study DMP performance by a numerical analysis based on data from the Chilean underground copper mine, called *El Teniente*. The results show that the approximation ratio in practice is only 1.08.

Keywords. Operation underground mining, decision making process, mining vehicles management.

Mathematics Subject Classification. 90B50, 68M20.

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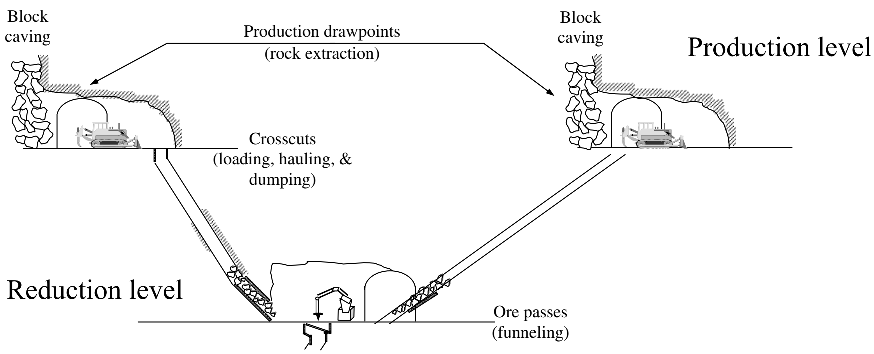


FIGURE 1. Operation levels vertically positioned of an underground mine (source: [4]).

1. Introduction

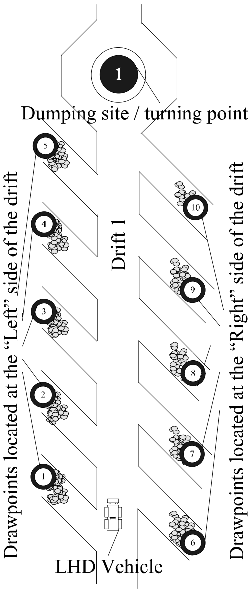
Operations research (OR) is becoming increasingly prevalent in the natural resource sector, especially in mining. Many authors use case studies to demonstrate the advantages of OR, which has played a particularly important industry role in the strategic and tactical levels [4, 7].

However, little published work has been carried out at the operation level. This leads to several questions such as a lack of trust and/or ability regarding the use of optimization models for short-term decisions and perhaps due to a lack of implementation of already-published work (see [1]).

In this context, the study of simple-to-execute decision-making process (DMP) with a good performance guaranteed would be a very interesting subject for OR approaching to real implementation in mining operation levels. An interesting case in underground mine is the DMP of LHD (load-haul-dump) vehicles operators. Daily, they address the routing problems at the production level of mine, whose solution impacts on the performance of other operations levels vertically positioned in the production chain.

At the production level, the ore is extracted from the drawpoints, loaded, transported and unloaded at dumping sites/turning points by LHD vehicles according to the established program of the fleet management. Whereas at the next level, so called the reduction level, the ore from dumping sites falls into a chopping/crushing chamber where a RBH (rock breaker hammer) reduces its granularity and then the material continues its gravitational movement. Figure 1 shows the operation levels vertically positioned of an underground mine.

In practice, both operation levels are pushed top-down by a plan-driven strategy elaborated by higher-level management, where the production goal for the production level is computed as a number of ore bucketfuls to extract from a set of drawpoints within a drift for a working shift [2]. Figure 2 shows an example of a plan-driven strategy data for a haulage network within a drift.



Date	1/10/2007				
Drift	1				
Working shift	2				
LHD vehicle number	1				
	Left side				
Drawpoint	**1**	**2**	**3**	**4**	**5**
Bucketfuls	10	13	13	11	9
	Right side				
Drawpoint	**6**	**7**	**8**	**9**	**10**
Bucketfuls	7	10	13	14	10
Dumping site /turning point	1				

FIGURE 2. Example of plan-driven strategy data of a drift for a working shift (source: [10]).

At the beginning of each working shift, the supervisor allocates operators for the LHD's and drifts, taking into account the current resources status and the plan-driven strategy data. Thus, the execution of plan starts when the operators begin his work path. In general, the operator does not have any kind of decision support system for the routing and he uses rules of thumb according to his experience to decide a sequence of LHD work to minimize the makespan [8].

In a former work paper [10], we studied the routing problem subject to the production level constraints. We proposed an optimal resolution algorithm in polynomial time, which is incorporated into a DMP for LHD operators. Unfortunately, minimizing the makespan in the production level will be not necessarily optimal when taking into account that the optimal sequence of LHD work can block the fall from dumping site towards to RHB by the ore excess deposited in a time window.

In this paper, we focus on the operations coordination problem between production and reduction levels in underground mining. Our goal is to propose a simple and efficient DMP for LHD operator, which guarantees certain performance in an integrated operation system.

The paper is organized as follows: in Section 2, the problem is stated; in Section 3, the NP-hardness of the problem is proved; in Section 4, an exact formulation by a mixed integer linear programming (MIP) model is proposed; in Section 5 an approximation algorithm is developed and a simple-to-execute DMP is generated from above algorithm, whose performance is studied by a numerical analysis based on data from *El Teniente* underground copper mine in Chile; finally, in Section 6, the conclusions and directions for further research are given.

2. Problem statement

We are given a set of drawpoints $\mathcal{P}$ to work on during the working shift, a LHD vehicle and a RBH. The drawpoints are located in both side of the drift. We denote $\mathcal{L}, \mathcal{R} \subset \mathcal{P}$ the subsets of drawpoints located to left side and right side of the drift, respectively as shown in Figure 2. Each drawpoint i has a number of ore bucketfuls to be extracted B_i, a LHD transfer time I_i between drawpoint i and the initial point and a LHD transfer time D_i between drawpoint i and the dumping site/turning point.

LHD vehicle takes a turning time T to pass vehicle to the other side of the drift and takes a loading time L and unloading time U for each ore bucketful. The RBH reduces ore bucketfuls at constant rate R and the ore-pass from the dumping site to the RBH has a ore maximum capacity of bucketfuls $C \in [1, 2)$ that must not be overseed.

We denote $\sigma = (\sigma(j))_{j=1}^{|\mathcal{P}|}$ the sequence of the working path of LHD vehicle, where $\sigma(j) \in \mathcal{P}$ is the drawpoint in the position j. Our coordination problem is to determine the working path σ^* for minimizing the makespan subject to the following two operational constraints from production and reduction levels:

LHD can turn to the opposite side of the drift only once: Considering the topology of the mine, which determines that LHD must enter to drawpoints with its shovel in front position; then travel to a turning point and return, it moves back and forth keeping the direction of vehicle while remaining on the same side. But when it goes to a drawpoint on the opposite side, a slow, difficult and risky maneuver must be done due to narrow angle of 30 $^\circ$ (see Fig. 2). Therefore, it is convenient to change side only one time and with the bucketful of LHD empty by taking into consideration the LHD operators safety and the maneuver time. Currently, the long time of the maneuver allows to begin any sequence on any side of the drift with an accumulated ore to be processed by RHB equal than zero. On now we assume $T/C \geq R$ to capture this long time of the maneuver.

LHD can dump the ore bucketful only if the ore-pass capacity is not overseed: It considers the production system of the mine, in which the ore to be processed by RBH is accumulated in the ore-pass (see Fig. 1). This constraint precludes the ore-pass blockage by the ore excess deposited within a time window, which perturbs the established working program, breaking off the work of LHD vehicle and RBH until the intervention of specialized equipment which unblocks the ore-pass. In practice, the LHD must wait whenever the ore bucketful to dump plus the accumulated ore to be processed by RBH exceeds the maximum capacity of the ore-pass between the production and reduction levels. We consider the initial accumulated ore to be processed by RBH equal to zero.

Note that the LHD cannot dump the ore bucketful for $C < 1$ and then our problem is infeasible. On the other hand, our problem reduces to the routing problem in [10] for $C \geq 2$, where the constraint of ore-pass is inactive.

3. Complexity

Theorem 3.1. *The coordination problem is NP-hard in the strong sense.*

Proof. We consider 3-PARTITION problem, which is NP strongly hard [5]. Given an instance of 3-PARTITION, we construct a problem instance $\mathcal{I}$ as follows: Let $\mathcal{P}$ be a finite set of $3a + 4$ drawpoints located in $\mathcal{R}$, the first one with the minimum value $I_i - D_i$. We have $B_i = 1$ for all $i \in \mathcal{P}$, $2 > C > 1$, $T/C \geq R$, $\gamma\ |1 - (2D_i + U + L)/R| \in \mathbb{N}^+$ for all $i \in \mathcal{R}$.

We have that any optimal sequence begins for the drawpoint with the minimum value $I_i - D_i$ and then, our goal is to determine a sequence for the remaining drawpoints in $\mathcal{R}$.

Consider the remaining drawpoints in $\mathcal{R}$ such that:

(1) The first 3 drawpoints have $(2D_i + U + L)/R = C = \beta/\gamma$. These drawpoints are tight, meaning that their position in the sequence is just after of workload for RHB is equal to C.
(2) The other $3a$ drawpoints have $\frac{C-1}{2} < 1 - (2D_i + U + L)/R = \alpha_i/\gamma < \frac{C-1}{4}$, $\sum_{i=1}^{3a} \alpha_i/\gamma 3(C - 1)$

Note that this construction can be completed in time polynomial in a, and it gives feasible solution of 3-PARTITION instance, where the drawpoints in $\mathcal{R}$ are sequenced such that the tight drawpoints divide the time into 3 intervals of length C. Also each of the $3a$ other drawpoints is scheduled in exactly one of these intervals.

This feasible solution of 3-PARTITION instance represents the optimal solution for sequence for the drawpoints in $\mathcal{R}$, where all waiting times of the sequence are zero. □

4. Exact formulation: MIP model

We consider the above statement of the coordination problem and propose an MIP model. Without loss of generality, we adopt $B_i = 1$ for all drawpoint i, since for the case of a drawpoint k with $B_k = m > 1$ bucketfuls, it can simulated by m drawpoints with $B_l = 1, 1 \leq l \leq m$.

For convenience, we define $P_i = (2\ D_i + U + L)/R$ and $S_i = I_i - D_i$. The first value is the ore amount that RBH can process within the time window while LHD visit to drawpoint i and, second one is the difference in working time between to start the workload of drawpoint i from initial point and the dumping site/turning point. Note that $P_i R$ corresponds to the working time of a LHD work cycle come-load-go-download for an ore bucketful from/to the dumping site/turning point. Formally, the MIP is defined as follows:

4.1. Sets, parameters and variables

Sets	
$\mathcal{P}$	: Set of all drawpoints to work on during the working shift.
$\mathcal{L}$	: Subset of drawpoints on the left side of drift.
$\mathcal{R}$	: Subset of drawpoints on the right side of drift.

Variables:	
$x_{ij} \in \{0,1\}$	: Indicates if one bucketful of drawpoint $i \in \mathcal{P}$ is extracted in the $j \in \mathcal{P}$ position of the sequence.
$y_j \in \{0,1\}$	: Indicates if the LHD vehicle turns side once the work in the $j \in \mathcal{P}$ position of the sequence is realized.
$z_j \in \{0,1\}$	: Auxiliary variable that indicates if the ore-pass is full in the ending of $j \in \mathcal{P}$ position in the sequence.
$p_j \in \mathbb{R}^+$	: Amount of ore in the ore-pass to be processed by the RBH in the ending of $j \in \mathcal{P}$ position in the sequence.
$w_j \in \mathbb{R}^+$	: Waiting time of the LHD vehicle when it arrives to drawpoint located in the $j \in \mathcal{P}$ position of the sequence.

Parameters:	
P_i	: Ore amount processed by RBH when the drawpoint $i \in \mathcal{P}$ is visited.
S_i	: Difference in working time between to start the workload of drawpoint $i \in \mathcal{P}$ from initial point and the dumping site/turning point.
R	: Rate at which RBH reduces ore bucketfuls
C	: Maximum ore capacity of the ore-pass
T	: Turning time required to pass to the opposite side of the drift.
F	: Time required to go from dumping site to initial point and finish the working shift
M	: Auxiliary parameter that is very big

4.2. Objective function

$$\text{minimize} \quad \sum_{i\in\mathcal{P}} x_{i1} S_i + \sum_{j=1}^{|\mathcal{P}|} w_j + T \sum_{j=1}^{|\mathcal{P}|} y_j + F + R \sum_{i\in\mathcal{P}} P_i \tag{4.1}$$

4.3. Constraints

$$\sum_{i\in\mathcal{P}} x_{ij} = 1 \qquad j = 1, \ldots, |\mathcal{P}| \tag{4.2}$$

$$p_j \geq p_{j-1} + \left(1 - \sum_{i\in\mathcal{P}} P_i x_{ij}\right) - \frac{w_j}{R} - T\frac{y_{j-1}}{R} \qquad j = 2, \ldots, |\mathcal{P}| \tag{4.3}$$

$$p_j + M(z_j - 1) \leq p_{j-1} + \left(1 - \sum_{i\in\mathcal{P}} P_i x_{ij}\right) - \frac{w_j}{R} - T\frac{y_{j-1}}{R} \quad j = 2, \dots, |\mathcal{P}| \quad (4.4)$$

$$p_j \leq z_j C + y_{j-1} \quad j = 2, \dots, |\mathcal{P}| \quad (4.5)$$

$$y_{j-1} \leq p_j \quad j = 2, \dots, |\mathcal{P}| \quad (4.6)$$

$$\sum_{i\in\mathcal{L}} x_{ij} - \sum_{i\in\mathcal{L}} x_{i(j-1)} \leq y_j \quad j = 2, \dots, |\mathcal{P}| \quad (4.7)$$

$$\sum_{i\in\mathcal{L}} x_{ij} - \sum_{i\in\mathcal{L}} x_{i(j-1)} \geq -y_j \quad j = 2, \dots, |\mathcal{P}| \quad (4.8)$$

The objective function represented by expression (4.1) minimizes the sum over the time used during the whole process of carrying the material from each drawpoint to the RBH, the waiting times, the turning times and the last trip of LHD from dumping site/turning point to the initial point. In practice, the function objective considers the working time of a LHD work cycle come-load-go-download for an ore bucketful from/to the dumping site/turning point in every drawpoint of the drift $\left(R\sum_{i\in\mathcal{P}} P_i\right)$, excluding the first one in the sequence where the LHD starts from the initial point (note that $S_i + RP_i = D_i + I_i + U + L$).

Expression (4.2) states that in each step the LHD goes to a single drawpoint.

Expressions (4.3)–(4.6) are a set of constraints that defines the accumulated ore that will be in each step in the RBH, which state the waiting time for each period subject to that the accumulate ore to be processed by RHB: (i) has to be nonnegative, (ii) cannot exceed the maximum capacity of the ore-pass, and (iii) will be equal than one in the ending of the first position later that the LHD turned. The latter fact follows from the cases assumptions: the bucketful of LHD will be empty before to turn and $T/C \geq R$.

Expressions (4.7) and (4.8) indicate the relation between turning to the opposite side and going to drawpoint of the other subset.

Note that the initial conditions $p_1 = 1, w_1 = 0$ are due to the RHB begins to work when the first bucketful is unloaded at the dumping point by case assumption. Also, we remark that for an arbitrary instance where LHD turns sides in position $k-1, k = 2, \dots, \mathcal{P}$, the variables $p_k = 1$ and $w_k = 0$ hold by $T/C \geq R$ and constraints (4.5) and (4.6), respectively.

Figure 3 shows a feasible solution for a problem instance with 5 drawpoints in each side of the drift, where the constraints and initial conditions are illustrated.

Clearly, the above constraints imply that in any optimal solution the LHD only turns to the opposite side of the drift only once and therefore the objective function (4.1) can be restated as follows.

$$\sum_{i\in\mathcal{P}} x_{i1} S_i + \sum_{j=1}^{|\mathcal{P}|} w_j + T + F + R\sum_{i\in\mathcal{P}} P_i, \quad (4.9)$$

where T, F and $R\sum_{i\in\mathcal{P}} P_i$ are constants.

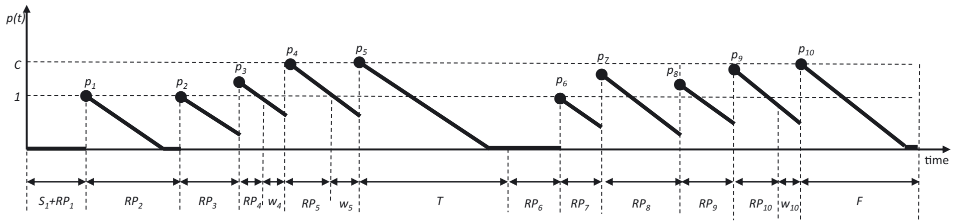


FIGURE 3. Example of feasible solution for a plan-driven strategy with 5 drawpoints in each side of the drift, where $p(t)$ is the amount of ore in the ore-pass to be processed by RHB in the time t.

5. APPROXIMATION ALGORITHM AND DMP

Although, MIP model proposed can be solved by using specialized software (*e.g.*, LINGO, CPLEX, Gurobi, among others), in practice, the use of such tools may be difficult given that skills and knowledge would be needed for the decision makers (*e.g.* LHD operator). On the other hand, a decision support system with an easy-to-use interface surely can help in this task, but an investment would be necessary (*e.g.*, optimization engine, customized software, training, among others).

Therefore, the study of simple-to-execute DMP based on a resolution algorithm is a very interesting subject for OR approaching to real implementation. Ideally, this should be simple to execute for the workers and the resolution algorithm should lead to optimal solution in polynomial time. Two good examples are Johnson's rule [6] for minimizing the makespan in flow shops with n jobs and two machines, and Smith's rule [9] for minimizing the total weighted completion time of n jobs in a single machine. In both cases, the optimal result is found in a polynomial time.

Unfortunately, no polynomial time algorithm to find an optimal solution by considering the NP-hardness of our problem (see Thm. 3.1) and then, a polynomial algorithm that computes a feasible solution whose value is within a range of δ close to the optimum for any instances can be a good approach.

5.1. OUR APPROXIMATION ALGORITHM

We develop a simply algorithm, so called ALGO A, which runs in polynomial time and yields a solution at most two times the minimum makespan of drift workload.

To define our algorithm, we introduce some definitions and notations. We say that a drawpoint i belongs to the increasing type $\mathcal{P}^+$, if the workload in the RHB increases in the time interval where it is carried out, and no-increasing type $\mathcal{P}^-$ an otherwise. Formally, we define $\mathcal{P}^+ = \{i \in \mathcal{P}, 1 > P_i\}$ and $\mathcal{P}^- = \{i \in \mathcal{P}, 1 \leq P_i\}$, and the similar way $\mathcal{L}^+$, $\mathcal{L}^-$, $\mathcal{R}^+$ and $\mathcal{R}^-$. For convenience, we assume that $\mathcal{L}^+$ and $\mathcal{R}^+$ are in increasing order by P_i, whereas $\mathcal{L}^-$ and $\mathcal{R}^-$ are in decreasing order.

Let $\mathcal{Z} \in \{\mathcal{L}, \mathcal{R}\}$ be the side of drift where the first drawpoint in the sequence σ is located. We denote $\pi(\mathcal{Z}, j)^+$ the drawpoint in the position j in the set $\mathcal{Z}^+$ and the similar way $\pi(\mathcal{Z}, j)^-$, $\pi(\mathcal{P} \setminus \mathcal{Z}, j)^+$ and $\pi(\mathcal{P} \setminus \mathcal{Z}, j)^-$

We now distinguish two instances instances of our problem:

(1) A set of instances $\daleth$ features for $\mathcal{L}^+, \mathcal{R}^+ = \{\emptyset\}$; $\mathcal{L}^-, \mathcal{R}^- = \{\emptyset\}$; $\mathcal{L}^+, \mathcal{R}^- = \{\emptyset\}$ or $\mathcal{L}^-, \mathcal{R}^+ = \{\emptyset\}$.
(2) A set of instances $\beth$, which are not in $\daleth$.

We show that any instance in $\daleth$ only depends of first drawpoint in each side of the drift and so, the optimal solution admits an algorithm in polynomial time.

Lemma 5.1. *An optimal sequence σ^* for an arbitrary instance in $\daleth$ only depends of first drawpoint in each side of the drift. In particular, the first drawpoint $\sigma^*(1)$ in an optimal sequence σ^* is defined by the* $\arg\min$ *between*

$$\min_{k \in \mathcal{L}} \left\{ S_{\sigma(k)} + R \max\left\{0, P_L + P_{\sigma(k)}\right\}\right\} + R \max\left\{0, P_R + \min_{i \in \mathcal{R}} P_{\sigma(i)}\right\}$$

and

$$\min_{k \in \mathcal{R}} \left\{ S_{\sigma(k)} + R \max\left\{0, P_R + P_{\sigma(k)}\right\}\right\} + R \max\left\{0, P_L + \min_{i \in \mathcal{L}} P_{\sigma(i)}\right\} \tag{5.1}$$

where $P_L = |\mathcal{L}| - \sum_{i \in \mathcal{L}} P_i - C$ *and* $P_R = |\mathcal{R}| - \sum_{i \in \mathcal{R}} P_i - C$

Proof. Consider an arbitrary instance in $\daleth$. Fix the first position of sequence for both sides of the drift. In each side, we have the total waiting time is independent of the sequence after the first drift. Thus, we have that for any instance in $\daleth$ the optimal value of problem is whose minimizing of the expression (4.9), which is equivalent to minimize

$$S_{\sigma(1)} + R \max\left\{0, 1 + \sum_{i \in \mathcal{Z}} (1 - P_i) + (P_{\sigma(1)} - 1) - C\right\}$$
$$+ R \max\left\{0, 1 + \sum_{i \in \mathcal{P} \setminus \mathcal{Z}} (1 - P_i) + \left(\min_{i \in \mathcal{P} \setminus \mathcal{Z}} P_i - 1\right) - C\right\}$$

Since, the above problem only depends of first drawpoint in each side of the drift. Clearly, the first drawpoint $\sigma^*(1)$ in an optimal sequence σ^* is defined by equation (5.1). $\square$

We describe our algorithm ALGO A, such as follows:

Steps 1−3 runs in time $O(n \log n)$ and so, ALGO A runs in time $O(n \log n)$. Now, we show that ALGO A guarantees at most two times the minimum makespan of drift workload.

Algorithm 1: ALGO A

Data: Data of an instance $\mathcal{I}$: C, R, T, F and P_i, S_i for all $i \in \mathcal{P}$
Result: A sequence σ for the LHD working path
Step 1 Assign $\sigma(1)$ by equation (5.1) and fix the drift side $c := \mathcal{Z}$ to start.
Step 2: Compute the sets $\mathcal{R}^-, \mathcal{R}^+, \mathcal{L}^-$ and $\mathcal{L}^+$, excluding the initial drawpoint $\sigma(1)$
Assign $j := 1$, $p_1 := 1$, $w_1 = 0$, $a := 2$, $b := 1$;$\sigma(2) := \pi(\mathcal{Z}, 1)^+$
Step 3: While $j + 2 \leq |\mathcal{Z}|$ **do**
$p_{j+1} = \max\{0, p_j + 1 - P_{\sigma(j+1)}\}$
  if $p_{j+1} \leq C$ **then**
    $w_{j+1} = 0$
    $\sigma(j+2) = \pi(c, a)^+$
    $a = a + 1$
  else $p_{j+1} = C$
    $w_{j+1} = (p_j + 1 - P_{\sigma(j+1)} - C)R$
    $\sigma(j+2) = \pi(c, b)^-$
    $b = b + 1$
  end if
end while
Step 4: if $j + 2 < |\mathcal{P}|$ **then**
$j := |\mathcal{Z}| + 1$, $p_{|\mathcal{Z}|+1} := 1$, $w_{|\mathcal{Z}|+1} := 0$, $a := 3$, $b := 1$, $c := \mathcal{P} \backslash \mathcal{Z}$, $\sigma(j) := \pi(c, 1)^+$,
$\sigma(j+1) := \pi(c, 2)^+$
Go to Step 3.
end if

Theorem 5.2. *The algorithm ALGO A produce a sequence σ such that the value objective is less than 2 the minimum makespan of drift workload.*

Proof. We focus on the set of instances $\beth$, since ALGO A yields the optimal sequence for any an arbitrary instance in $\daleth$ by Lemma 5.1. We denote ALGO A($\mathcal{I}$) and OPT($\mathcal{I}$) the algorithm value and the optimal value for an arbitrary instance $\mathcal{I} \in \beth$, respectively. Also, we denote LB and UB the lower bound for the OPT($\mathcal{I}$) value and upper bound for the ALGO A($\mathcal{I}$) value.

To obtain the LB, we evaluate the sequence σ obtained from ALGO A in the simple expression defined by (5.1). For the case of UB, we consider the makespan of new sequence σ' obtained from σ such that, the first drawpoints in both side of drift are the same that are in σ and the contribution over total waiting time of no-increasing drawpoints type $\mathcal{P}^-$ is removed. Clearly, this new sequence has a makespan value greater than the makespan value of sequence σ given by ALGO A. The value makespan of new sequence σ' is:

$$UB = R\sum_{i\in\mathcal{P}} P_i + S_{\sigma(1)} + R\max\left\{0, 1 + \sum_{i\in\mathcal{Z}^+}(1 - P_i) + (P_{\sigma(1)} - 1) - C\right\}$$
$$+R\max\left\{0, 1 + \sum_{i\in\{\mathcal{P}\backslash\mathcal{Z}\}^+}(1 - P_i) + \left(\min_{i\in\{\mathcal{P}\backslash\mathcal{Z}\}^+} P_i - 1\right) - C\right\} + T + F$$

$$\le R\sum_{i\in\mathcal{P}} P_i + S_{\sigma(1)} + R\max\left\{0, 1+\sum_{i\in\mathcal{Z}}(1-P_i) + (P_{\sigma(1)}-1) - C\right\} + T + F$$
$$+R\max\left\{0, 1+\sum_{i\in\mathcal{P}\setminus\mathcal{Z}}(1-P_i) + \left(\min_{i\in\mathcal{P}\setminus\mathcal{Z}} P_i - 1\right) - C\right\} - R\sum_{i\in\mathcal{P}^-}(1-P_i)$$
$$= LB + R\sum_{i\in\mathcal{P}^-}(P_i - 1)$$

Finally, we show:

$$\begin{aligned}\frac{ALGOA(\mathcal{I})}{OPT(\mathcal{I})} < \frac{UB}{LB} = \frac{LB + R\sum_{i\in\mathcal{P}^-}(P_i-1)}{LB} &= \left(1 + \frac{R\sum_{i\in\mathcal{P}^-}(P_i-1)}{LB}\right) \\ &< \left(1 + \frac{R\sum_{i\in\mathcal{P}^-}(P_i-1)}{R\sum_{i\in\mathcal{P}} P_i}\right) \\ &< 2,\end{aligned}$$

which concludes the proof. □

Therefore, our algorithm ALGO A runs in polynomial time and computes a feasible solution whose value is within a factor at most 2 of the optimum.

5.2. Our DMP

In this section, we define a set of seven decision rules from ALGO A, which work in conjunction to develop a DMP for the LHD operators. The seven decision rules above are integrated into a DMP in a sequential way as shown in Figure 4.

R1 to R6 are obtained from ALGO A. They show the different steps to consider in the generation of the work sequence for LHD vehicle. R7 gives a feasibility condition for the fulfillment of drift workload within the working shift by considering ALGO A. In practice, R7 uses the fact that the computed workload time is at most two times the optimal value and therefore, if the computed workload time is greater or equal than the available time of two working shift TWS, then the optimal value is greater than TWS and the infeasibility holds.

In order to compute a in practice value for the approximation of makespan by using the DMP, a numerical analysis is performed. For this, a total of 90 cases (plan-driven strategies) are considered, which have different topologies for the drifts; and the production goals for each drawpoint are between 29 and 280 ore tons. The data set is base on data of *El Teniente* Chilean copper underground mine available from [3]. The optimal solution is determined by using MIP proposed in Section 4.

The study of numerical cases shows that DMP leads to good results, since they help to meet the production goal as imposed by the plan-driven strategy in an integrated way. In practice, 80 among 90 of the cases analyzed were feasible and 7 of 10 infeasible cases were detected by using the approximation value (R7 rule). In the

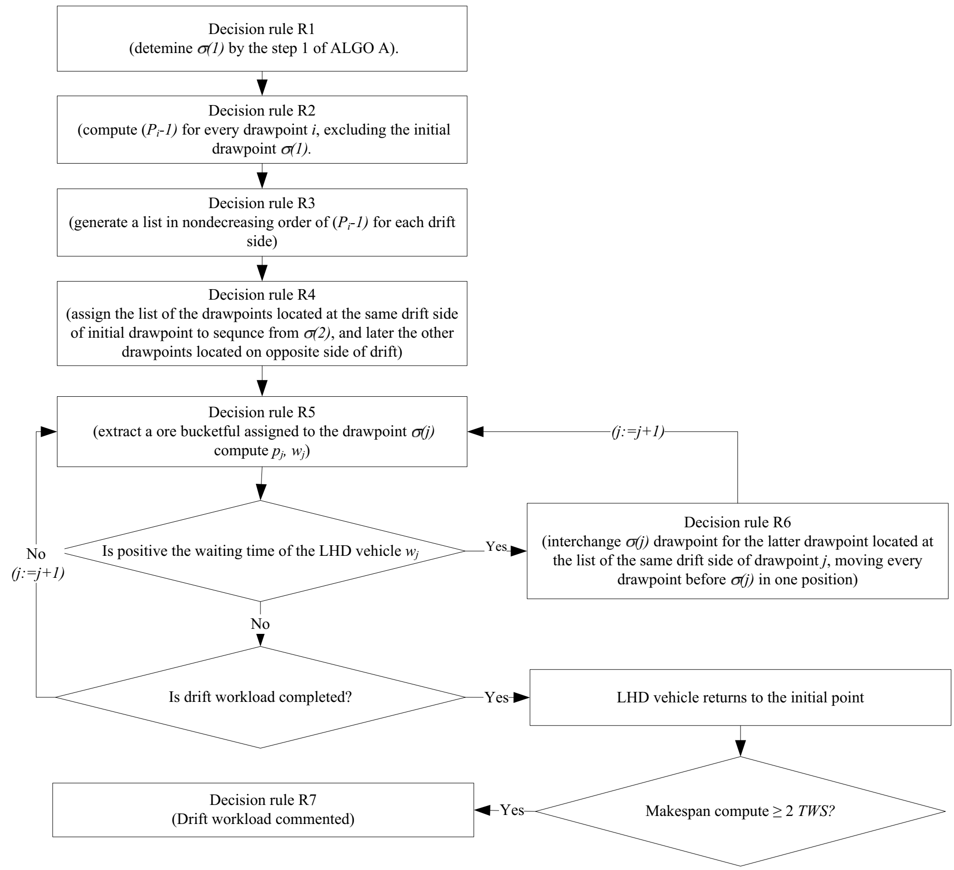
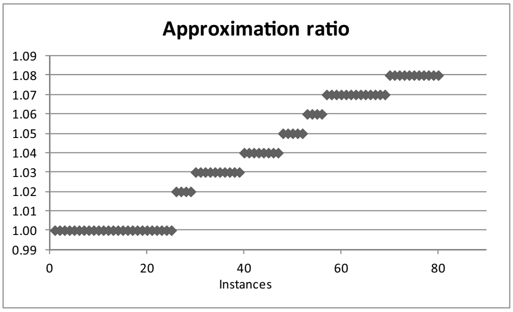


FIGURE 4. Flow diagram of the DMP proposed for the LHD operation problem.

feasible cases, 31.25% were matched the optimal solution; whereas the remaining cases presented an approximation value of 1.08 (the worst case by considering the maximum value). The obtained results are shown in the Figure 5.

6. Final remark

Generally, the plan-driven strategies elaborated by higher-level management are estimated based on historical data and considered a bounded area for the drawpoints selection in order to guarantee the execution of workload demanded. Therefore, the goal presented by the plan-driven strategy can be seem as a consequence of DPM historically used by the LHD operators in practice. From above perspective, we can explain the very good approximation ratio in practice and conclude that the proposed DMP can be an interesting and fast approach to improve the currently performance of operations levels in real implementation context.



Infeasible cases	10
Infeasible cases detected	7
Feasible cases	80
Hits	25
Average	1.04
Maximum	1.08
Minimum	1.00
Stand. Desviation	0.03
Median	1.04

FIGURE 5. Analysis of approximation ratio based on *El Teniente* Chilean copper underground mine data.

In a methodological setting, we recommend the steps developed in this work for similar studies, *i.e.* statement of the problem, analysis of the NP-hardness, study of a resolution exact, generation of a polynomial algorithm with certain performance guaranteed and a simple-to-execute DMP based on the polynomial algorithm generated.

Finally, further research is proposed regarding two lines researches: on one hand, the improvement of approximation ratio of algorithm proposed; and on other hand, online algorithms for LHD vehicle operations problem, since information of the drawpoints to be worked on is only available in a given point in time into the working shift.

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