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WAGE BARGAINING WITH DISCOUNT RATES VARYING IN TIME UNDER DIFFERENT STRIKE DECISIONS *

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Abstract. We present a non-cooperative union-firm wage bargaining model in which the union must choose between strike and holdout if a proposed wage contract is rejected. The innovative element that our model brings to the existing literature on wage bargaining concerns the parties' preferences which are not expressed by constant discount rates, but by sequences of discount factors varying in time. First, we determine subgame perfect equilibria if the strike decision of the union is exogenous. We analyze the case when the union is committed to strike in each disagreement period, the case when the union is committed to strike only when its own offer is rejected, and the case of the never strike exogenous decision. A comparison of the results is provided, among the cases of the exogenous strike decisions. Next, we consider the general model with no assumption on the commitment to strike. We find subgame perfect equilibria in which the strategies supporting the equilibria in the exogenous cases are combined with the minimum-wage strategies, provided that the firm is not less patient than the union. If the firm is more impatient than the union, then the firm is better off by playing the no-concession strategy. We find a subgame perfect equilibrium for this case.

Keywords. Union – firm bargaining, alternating offers, varying discount rates, subgame perfect equilibrium.

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1. Introduction

Collective wage bargaining between firms and unions (workers' representatives) is one of the most central issues in labor economics. Both cooperative and non-cooperative approaches to collective wage bargaining are applied in the literature; for a short survey, see [13], for broader surveys of bargaining models see, *e.g.*, Osborne and Rubinstein [11, 12]. Some authors apply a dynamic (strategic) approach to wage bargaining and focus on the concept of *subgame perfect equilibrium* (that will be denoted here by *SPE*). Several modified versions of Rubinstein's game [7, 14] to firm-union negotiations are proposed. Haller and Holden [8] extend Rubinstein's model to incorporate the choice of calling a strike in union-firm negotiations. It is assumed that in each period until an agreement is reached the union must decide whether or not it will strike in that period. Both parties have the same discount factor δ. Fernandez and Glazer [6] consider essentially the same wage-contract sequential bargaining, but with the union and firm using different discount factors δ_u, δ_f. We will refer to their model as the *F-G model*. Holden [9] assumes a weaker type of commitment in the F-G model. Also Bolt [3] studies the F-G model. Houba and Wen [10] apply the method of [19] to derive the exact bounds of equilibrium payoffs in the F-G model and characterize the equilibrium strategy profiles that support these extreme equilibrium payoffs for all discount factors.

Although numerous versions of wage bargaining between unions and firms are presented in the literature, a common assumption is the stationarity of parties' preferences that are described by constant discount factors. In real bargaining, however, due to time preferences, discount factors of parties may vary in time. Cramton and Tracy [5] emphasize that stationary bargaining models are very rare in real-life situations. In the framework of the original Rubinstein model, several other authors discuss non-stationarity of parties' preferences. Binmore [1], for instance, analyses preferences that do not necessarily satisfy the stationarity assumption and shows through an example that for any (positive) time interval between two consecutive offers, there may exist a continuum of SPE (see also [2], pp. 187−188). Coles and Muthoo [4] study an alternating offers bargaining model in which the set of utilities evolves through time in a non-stationary way, but additionally assume that this set evolves smoothly through time. They show that in the limit as the time interval between two consecutive offers becomes arbitrarily small, there exists a unique SPE. In [4] a short survey of works that consider bargaining situations with players having time-varying payoffs is also presented. Rusinowska [15–18] generalizes the original model of Rubinstein to bargaining models with preferences described by sequences of discount rates or/and bargaining costs varying in time.

In the present paper, we investigate the *union-firm wage bargaining with discount rates varying in time* which generalizes the F-G wage bargaining with constant discount rates. While several generalizations of the original Rubinstein model with non-stationary preferences have been presented in the literature, to the best of our knowledge no such generalized F-G model has been analyzed before. First, we consider three games in this generalized setup, where the union strike decision is taken as *exogenous*: the case when the union is committed to strike in each period in which there is a disagreement, the case when the union is committed to go on strike only when its own offer is rejected, and the case of "never strike" decision. We determine SPE for these games and compare the results among the three cases of the exogenous strike decisions. As mentioned in Section 3 and shown by Fact 3.2, while the F-G model coincides with Rubinstein's model under the "always-strike decision", the generalized wage bargaining model and the generalization of Rubinstein's model do not coincide.

The study of the exogenous strike decisions is aligning with some real-life observations. In some countries and in some sectors, workers do not have legal rights to make official strikes, and consequently, in some environments strikes never take place. On the contrary, if the strikes are formally allowed, sometimes unions call for the non-stop strikes. Our comparison of the exogenous cases shows that, in fact, it would be more profitable for unions to use a "mixed" strike strategy: striking if the union's offer is rejected, but holding out if the union rejects an offer. We show that what the union would get under equilibrium in such a case of the mixed strike decision is higher than what it would get under the equilibria of the extreme strike decisions (always striking or always holding out). Our results for the cases with the exogenous strike decisions (Thms. 3.3 and 4.3, and Fact 5.1) generalize some previous results for constant discount rates: Lemma 1 in [6], formulas (3) and (4) in [8], and Lemma 2 in [6].

After considering the exogenous strike decisions, we investigate a general model with no assumption on the commitment to strike. The analysis of the three exogenous cases helps us to investigate SPE for the general case. Our Fact 6.1 shows that Lemma 2 of [6] on the minimum wage contract obtained in equilibrium remains valid for the general model. We find SPE in which the strategies supporting the equilibria in the exogenous cases (always strike, and strike only after rejection of own proposals) are combined with the minimum-wage strategies, provided that the union is sufficiently patient. The corresponding results (Props. 6.2 and 6.3) generalize Lemmas 3 and 4 of [6], and Proposition 1(i) of [3]. The latter SPE is restricted to the situations when the firm is at least as patient as the union. If the firm is more impatient than the union, then the firm is better off by playing the no-concession strategy (reject all offers and always make an unacceptable offer). This result is presented in Proposition 6.4. We find a SPE for this case (Thm. 6.5) which generalizes Proposition 1(ii) by Bolt [3].

The approach used in the paper and in our follow-up research on the F-G model that we intend to conduct is based on generalizing the analytical method used in the works on the F-G model [3,6,8–10]. Such an approach to wage bargaining is different from the approach to Rubinstein's bargaining game applied by Binmore [1]. He defines a model which is very similar to Rubinstein's model, except that in [1] it is not required that a player makes an offer in every period when there is his turn to do so. Then Binmore [1] proposes an alternative method which provides a geometric characterization of SPE for the introduced model. Such a "geometric technique" allows to refine the Rubinstein's results, in particular, by considering the case where the "cake" to be divided does not shrink steadily over time. We believe that in order to find SPE for the wage bargaining model with strike decisions and discount factors varying in time, it is more straightforward to use the "traditional" approach and to determine analytically SPE in the model.

The remainder of the paper is as follows. In Section 2 we present the generalized wage bargaining model with discount rates varying in time. Section 3 concerns the exogenous strike decision when the union is supposed to go on strike in each period in which there is a disagreement. In Section 4 we analyze the exogenous strike decision when the union goes on strike only after rejection of its own proposals. In Section 5 we briefly discuss the exogenous no-strike decision case when the union is supposed to go never on strike. Section 6 is devoted to SPE in the general model. We conclude in Section 7 with mentioning some possible applications of the model and our future research agenda.

2. Wage bargaining with discount factors varying in time

The bargaining procedure between the union and firm is the following [6, 8]. There is an existing wage contract that specifies the wage that a worker is entitled to per day of work, which has come up for renegotiation. Two parties (union and firm) bargain sequentially over discrete time and a potentially infinite horizon. They alternate in making offers of wage contracts that the other party is free either to accept or to reject. Upon either party's rejection of a proposed wage contract, the union must decide whether or not to strike in that period. Under the previous contract $w_0 \in (0,1]$, the union receives w_0 and the firm receives $1-w_0$. By the new contract $W \in [0,1]$, the union and firm will get W and $1-W$, respectively. Figure 1 presents the first three periods of this wage bargaining.

In period 0 the union proposes W^0. If the firm accepts the new wage contract, then the agreement is reached and the payoffs are $(W^0, 1-W^0)$. If the firm rejects it, then the union can either go on strike, and then both parties get $(0,0)$ in the current period, or go on with the previous contract with payoffs $(w_0, 1-w_0)$. After the union goes on strike or holds out, it is the firm's turn to make a new offer Z^1 in period 1, which assigns Z^1 to the union and $(1-Z^1)$ to the firm. If the union accepts this offer, then the agreement is reached, otherwise the union either goes on strike or holds out, and then makes its offer W^2 in period 2. This procedure goes on until an agreement is reached, and upon either party's rejection of a proposed contract the union decides whether or not to strike in that period. W^{2t} denotes the offer of the union made in an even-numbered period $2t$, and Z^{2t+1} denotes the offer of the firm made in an odd-numbered period $2t+1$.

The key difference between the F-G model and our wage bargaining lies in preferences of both parties and, as a consequence, in their utility functions. While Fernandez and Glazer [6] assume stationary preferences described by constant discount rates δ_u and δ_f, we consider a model with preferences of the union and

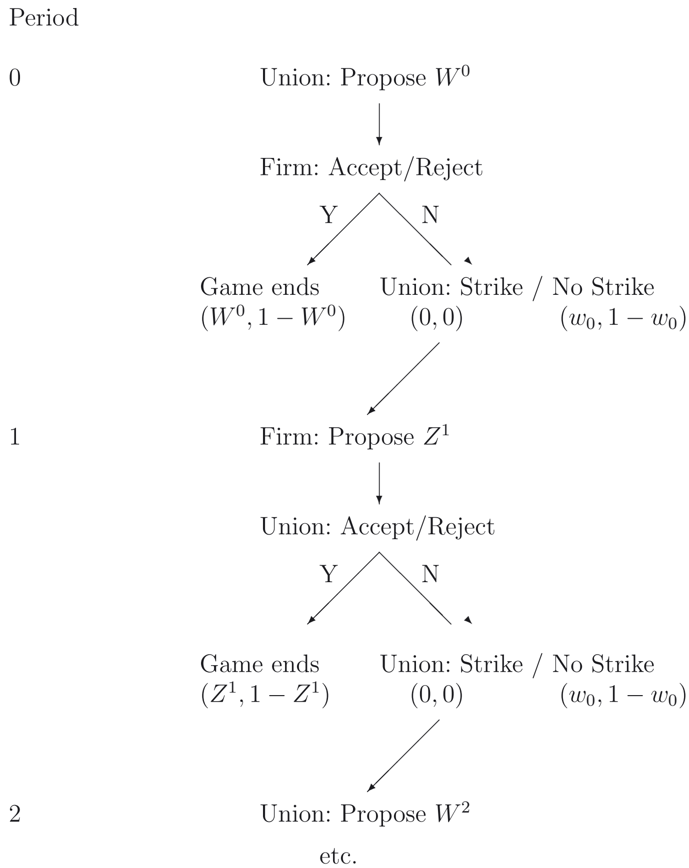

FIGURE 1. Non-cooperative bargaining game between the union and the firm.

the firm described by *sequences of discount factors varying in time*, $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$, respectively, where:

$\delta_{u,t}$ = discount factor of the union in period $t \in \mathbb{N}$, $\delta_{u,0} = 1$, $0 < \delta_{u,t} < 1$ for $t \geq 1$;

$\delta_{f,t}$ = discount factor of the firm in period $t \in \mathbb{N}$, $\delta_{f,0} = 1$, $0 < \delta_{f,t} < 1$ for $t \geq 1$.

The *result* of the wage bargaining is either a pair (W, T), where W is the wage contract agreed upon and $T \in \mathbb{N}$ is the number of proposals rejected in the bargaining, or a *disagreement* $(0, \infty)$, *i.e.*, the situation in which the parties never reach an agreement. The following notation for each $t \in \mathbb{N}$ is introduced:

$$\delta_u(t) := \prod_{k=0}^{t} \delta_{u,k}, \quad \delta_f(t) := \prod_{k=0}^{t} \delta_{f,k} \quad \text{and} \tag{2.1}$$

$$\text{for } 0 < t' \leq t,\ \delta_u(t',t) := \frac{\delta_u(t)}{\delta_u(t'-1)} = \prod_{k=t'}^{t} \delta_{u,k},\ \delta_f(t',t) := \frac{\delta_f(t)}{\delta_f(t'-1)} = \prod_{k=t'}^{t} \delta_{f,k}. \tag{2.2}$$

The utility of the result (W,T) for the union is equal to the discounted sum of wage earnings

$$U(W,T) = \sum_{t=0}^{\infty} \delta_u(t) u_t \tag{2.3}$$

where $u_t = W$ for each $t \geq T$ and, if $T > 0$ then for each $0 \leq t < T$:

$u_t = 0$ if there is a strike in period $t \in \mathbb{N}$
$u_t = w_0$ if there is no strike in period t.

The utility of the result (W,T) for the firm is equal to the discounted sum of profits

$$V(W,T) = \sum_{t=0}^{\infty} \delta_f(t) v_t \tag{2.4}$$

where $v_t = 1 - W$ for each $t \geq T$ and, if $T > 0$ then for each $0 \leq t < T$:

$v_t = 0$ if there is a strike in period t;
$v_t = 1 - w_0$ if there is no strike in period t.

We set $U(0,\infty) = V(0,\infty) = 0$. We analyze $(\delta_{u,t})_{t\in\mathbb{N}}$ and $(\delta_{f,t})_{t\in\mathbb{N}}$ that are bounded by a certain number smaller than 1, *i.e.*, we assume that

there exist $a < 1$ *and* $b < 1$ *such that* $\delta_{u,t} \leq a$ *and* $\delta_{f,t} \leq b$ *for each* $t \in \mathbb{N}$. (2.5)

The conditions given in (2.5) are sufficient for the convergence of the series that define $U(W,T)$ and $V(W,T)$ in (2.3) and (2.4). The convergence follows immediately from the comparison test applied to the geometric series.

We also introduce a kind of generalized discount factors which take into account the sequences of discount rates varying in time and the fact that the utilities are defined by the discounted streams of payoffs. We have for every $t \in \mathbb{N}_+$

$$\Delta_u(t) := \frac{\sum_{k=t}^{\infty} \delta_u(t,k)}{1 + \sum_{k=t}^{\infty} \delta_u(t,k)}, \quad \Delta_f(t) := \frac{\sum_{k=t}^{\infty} \delta_f(t,k)}{1 + \sum_{k=t}^{\infty} \delta_f(t,k)} \tag{2.6}$$

and consequently, for every $t \in \mathbb{N}_+$

$$1 - \Delta_u(t) = \frac{1}{1 + \sum_{k=t}^{\infty} \delta_u(t,k)}, \quad 1 - \Delta_f(t) = \frac{1}{1 + \sum_{k=t}^{\infty} \delta_f(t,k)}. \tag{2.7}$$

Note that for every $t \in \mathbb{N}_+$

$$\sum_{k=t}^{\infty} \delta_f(t,k) \geq \sum_{k=t}^{\infty} \delta_u(t,k) \quad \text{if and only if} \quad \Delta_f(t) \geq \Delta_u(t)$$

Obviously, for the special case of constant discount rates, *i.e.*, if $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for every $t \in \mathbb{N}_+$, we have $\Delta_u(t) = \delta_u$ and $\Delta_f(t) = \delta_f$.

In what follows, $\Delta_u(t)$ and $\Delta_f(t)$ will be called the *generalized discount factors* of the union and the firm in period t, respectively.

Furthermore, we introduce the additional definition and notation. Let (s_u, s_f) be the following family of strategies:

- *strategy of the union* s_u*: in period* $2t$ $(t \in \mathbb{N})$ *propose* $\overline{W}^{2t}$*; in period* $2t+1$ *accept an offer* y *if and only if* $y \geq \overline{Z}^{2t+1}$;
- *strategy of the firm* s_f*: in period* $2t+1$ *propose* $\overline{Z}^{2t+1}$*; in period* $2t$ *accept an offer* x *if and only if* $x \leq \overline{W}^{2t}$.

A strategy of the union specifies also its strike decision.

In the first part of our analysis, presented in Sections 3, 4 and 5, we assume that the union commits to a specific strike decision and consider the family (s_u, s_f) of the parties' strategies. This assumption will be then relaxed in Section 6, where SPE for the general model are presented.

3. Going always on strike under a disagreement

We analyze the case when the strike decision of the union is exogenous and the union is supposed to go on strike in each period in which there is a disagreement. Fernandez and Glazer [6] show that in such a case, if preferences are defined by constant discount factors, then there is a unique SPE of the wage bargaining game. It coincides with the SPE in Rubinstein's model and leads to an agreement $\overline{W} = \frac{1-\delta_f}{1-\delta_u\delta_f}$ reached in period 0. In this paper we generalize the equilibrium result obtained in [6] to the model with discount factors varying in time.

First of all, we deliver necessary and sufficient conditions for (s_u, s_f) to be a SPE. According to these conditions, in every even (odd, respectively) period the firm (the union, respectively) is indifferent between accepting the equilibrium offer of the union (of the firm, respectively) and rejecting that offer. This is formalized in the following proposition.

Proposition 3.1. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$*, where* $\delta_{i,0} = 1$*,* $0 < \delta_{i,t} < 1$ *for* $t \geq 1$*,* $i = u, f$*. Assume that the strike decision is given exogenously and the union is committed to strike in every period in which there is a disagreement. Then* (s_u, s_f) *is a SPE of this game if and only if the offers satisfy the following infinite system of equations: for each* $t \in \mathbb{N}$

$$1 - \overline{W}^{2t} = \left(1 - \overline{Z}^{2t+1}\right) \Delta_f(2t+1) \quad \textit{and} \quad \overline{Z}^{2t+1} = \overline{W}^{2t+2} \Delta_u(2t+2). \tag{3.1}$$

Proof. $(\Leftarrow)$ Let (s_u, s_f) be defined by (3.1) which can be equivalently written as

$$1 - \overline{W}^{2t} + \left(1 - \overline{W}^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = \left(1 - \overline{Z}^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) \tag{3.2}$$

and

$$\overline{Z}^{2t+1} + \overline{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k) = \overline{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k). \tag{3.3}$$

Consider an arbitrary subgame starting in period $2t$ with the union making an offer. Under (s_u, s_f) the union gets $\overline{W}^{2t} + \overline{W}^{2t} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$ and the firm gets $(1-\overline{W}^{2t}) + (1-\overline{W}^{2t}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$. Suppose that the union deviates from s_u. If it proposes a certain $x > \overline{W}^{2t}$, then it gets $\overline{Z}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$. From (3.2), $0 \leq 1 - W^{2t} = (\overline{W}^{2t} - \overline{Z}^{2t+1}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$, and therefore $\overline{W}^{2t} \geq \overline{Z}^{2t+1}$. Consequently, $\overline{W}^{2t} + \overline{W}^{2t} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k) \geq \overline{Z}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$, and hence the union would not be better off by this deviation. If the union proposes a certain $x < \overline{W}^{2t}$, then it gets $x + x\sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$, but then it is worse off, since $x + x\sum_{k=2t+1}^{\infty} \delta_u(2t+1,k) < \overline{W}^{2t} + \overline{W}^{2t} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$. If the firm rejects $\overline{W}^{2t}$, then it gets at most $(1-\overline{Z}^{2t+1}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$, which by virtue of equation (3.2) is equal to $(1-\overline{W}^{2t}) + (1-\overline{W}^{2t}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$, so the firm would not be better off.

The analysis of a subgame starting in $2t+1$ with the firm proposing is analogous to the study of a subgame starting in $2t$, except that we use (3.3) instead of (3.2).

Consider a subgame starting in period $2t$ with the firm replying to an offer x. Let $x \leq \overline{W}^{2t}$. Under (s_u, s_f) the firm accepts it and gets $(1-x) + (1-x)\sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$. Suppose that the firm rejects such x. We already know that it is optimal for the firm to propose $\widetilde{Z}^{2t+1}$ in $(2t+1)$, so the firm would get $(1-\widetilde{Z}^{2t+1}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$, but from (3.2), $(1-x)+(1-x)\sum_{k=2t+1}^{\infty} \delta_f(2t+1,k) \geq (1-\overline{W}^{2t}) + (1-\overline{W}^{2t}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k) = (1-\overline{Z}^{2t+1}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$.

Hence, the firm would not be better off by this deviation. Let $x > \overline{W}^{2t}$. Under (s_u, s_f) the firm rejects it and proposes $\overline{Z}^{2t+1}$ which is accepted. The union gets then $\overline{Z}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$ and the firm $(1-\widetilde{Z}^{2t+1}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$. If the firm accepts such x, then it gets $(1-x) + (1-x)\sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$. But from (3.2), $(1-x) + (1-x)\sum_{k=2t+1}^{\infty} \delta_f(2t+1,k) < (1-\overline{W}^{2t}) + (1-\overline{W}^{2t}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k) = (1-\overline{Z}^{2t+1}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$, so the firm would be worse off by this deviation.

The analysis of subgames starting in period $2t+1$ by the union replying is analogous to the analysis of the corresponding subgames starting in period $2t$ by the firm replying.

=> Let (s_u, s_f) be a SPE. Consider a subgame starting in period $2t$ with the union making an offer. Using (s_u, s_f) gives $(1-\overline{W}^{2t}) + (1-\overline{W}^{2t}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$ to the firm. By rejecting $\overline{W}^{2t}$ the firm would get $(1-\overline{Z}^{2t+1}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)$. Since (s_u, s_f) is a SPE,

$$\left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)\geq\left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k).$$

Suppose that

$$\left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)>\left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k).$$

Then there exists $\tilde{x}>\overline{W}^{2t}$ such that

$$\left(1-\overline{W}^{2t}\right)+\left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f\left(2t+1,k\right)>(1-\tilde{x})+(1-\tilde{x})$$
$$\times\sum_{k=2t+1}^{\infty}\delta_f\left(2t+1,k\right)>\left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f\left(2t+1,k\right).$$

Since $\tilde{x}>\overline{W}^{2t}$, the firm rejects it and gets $\left(1-\overline{Z}^{2t+1}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k)$, but it would be better off by accepting this offer. Hence, we get a contradiction and prove (3.2). Proving (3.3) is analogous by considering a subgame starting in period $2t+1$ with the firm proposing. □

Rusinowska [15, 16] determines SPE for the generalized Rubinstein model with preferences described by sequences of discount rates varying in time. More precisely, she considers an alternating offers bargaining model [14] in which preferences of player $i=1,2$ are expressed not by a constant discount rate $0<\delta_i<1$ as in the original Rubinstein model, but by a sequence of discount rates $(\delta_{i,t})_{t\in N}$ varying in time, where $\delta_{i,0}=1$, $0<\delta_{i,t}<1$ for $t\geq 1$. In her model, the utility $\widetilde{U}_i$ to player $i=1,2$ of the result (W,T), where $W\in[0,1]$ is the agreement and $T\in\mathbb{N}$ is the number of periods rejected in the bargaining, is equal to

$$\widetilde{U}_i(W,T)=W_i\prod_{k=0}^{T}\delta_{i,k},\ \text{where } W_1=W \text{ and } W_2=1-W \tag{3.4}$$

and the utility of the disagreement $(0,\infty)$ is equal to $\widetilde{U}_i(0,\infty)=0$. Note that this generalized bargaining model differs from the generalized wage bargaining proposed in the present paper, in particular, because in the latter the utility of the union is defined as the discounted *sum* of wage earnings (see formula (2.3)) and the utility of the firm is defined by the discounted *sum* of profits (see formula (2.4)). While the F-G model coincides with Rubinstein's model under the "always-strike decision", the generalized wage bargaining model and the generalization of Rubinstein's model mentioned above do not coincide. Consequently, as shown in Fact 3.2, the result on SPE in the generalized Rubinstein model by Rusinowska [15, 16] cannot be applied to the generalized wage bargaining model introduced in the present paper.

Fact 3.2. *The generalized wage bargaining model in which the strike decision is given exogenously and the union is committed to strike in every disagreement period does not coincide with the generalized Rubinstein model with discount rates varying in time, and in general the SPE of the two models are different.*

Proof. In order to find the SPE offers in the generalized Rubinstein model with players 1 and 2 being the union and the firm, respectively, we need to solve the following infinite system of equations for each $t \in \mathbb{N}$ ([15, 16])

$$1 - \overline{W}^{2t} = \left(1 - \overline{Z}^{2t+1}\right) \delta_{f,2t+1} \quad \text{and} \quad \overline{Z}^{2t+1} = \overline{W}^{2t+2} \delta_{u,2t+2}. \tag{3.5}$$

In order to find the SPE offers in the generalized wage bargaining model with the exogenous "always strike" decision we need to solve (3.1) for each $t \in \mathbb{N}$. For the model with constant discount rates δ_u and δ_f these two infinite systems (3.1) and (3.5) are equivalent. For each $t \in \mathbb{N}$, $\Delta_f(2t+1) = \delta_f$ and $\Delta_u(2t+2) = \delta_u$, so inserting this into (3.1) gives equivalently (3.5), since $\delta_{f,2t+1} = \delta_f$, $\delta_{u,2t+2} = \delta_u$. However, these two infinite systems are NOT equivalent if we consider the generalized wage bargaining model, because

$$\Delta_f(2t+1) = \frac{\delta_{f,2t+1}(1 + \sum_{k=2t+2}^{\infty} \delta_f(2t+2,k))}{1 + \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k)}$$

$$\Delta_u(2t+2) = \frac{\delta_{u,2t+2}(1 + \sum_{k=2t+3}^{\infty} \delta_u(2t+3,k))}{1 + \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k)}$$

and for any $t \neq t'$ usually

$$\sum_{k=t}^{\infty} \delta_f(t,k) \neq \sum_{k=t'}^{\infty} \delta_f(t',k), \quad \sum_{k=t}^{\infty} \delta_u(t,k) \neq \sum_{k=t'}^{\infty} \delta_u(t',k)$$

and therefore usually

$$\Delta_f(2t+1) \neq \delta_{f,2t+1}, \quad \Delta_u(2t+2) \neq \delta_{u,2t+2}.$$

As an illustrative example, consider $\delta_{f,1} = \delta_{u,1} = \frac{1}{2}$, $\delta_{f,t} = \delta_{u,t} = \frac{1}{3}$ for each $t \geq 2$. Then

$$\sum_{k=1}^{\infty} \delta_f(1,k) = \frac{3}{4}, \quad \sum_{k=2t+1}^{\infty} \delta_f(2t+1,k) = \frac{1}{2} \text{ for each } t \geq 1.$$

Solving the system (3.5) gives $\overline{W}^0 = \frac{5}{8}$, $\overline{W}^{2t} = \frac{3}{4}$ for each $t \geq 1$, $\overline{Z}^{2t+1} = \frac{1}{4}$ for each $t \in N$, but this solution does not satisfy the first equation of (3.1), *i.e.*, $1 - \overline{W}^0 \neq \left(1 - \overline{Z}^1\right) \Delta_f(1)$. □

By solving the infinite system (3.1), we can determine the SPE offers made by the union and the firm, as presented in Theorem 3.3. Since we will compare the SPE offers under different exogenous strike decisions, in the statement of the corresponding results (but not in their proofs), we will use additional notations. For the 'always strike' decision case, the SPE offers will be denoted by $\overline{W}^{2t}_{AS}$ and $\overline{Z}^{2t+1}_{AS}$ for every $t \in \mathbb{N}$.

Theorem 3.3. *Consider the generalized wage bargaining model with preferences described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. Assume that the strike decision is given exogenously and the union is committed to strike in every disagreement period. Then there is the unique SPE of the form (s_u, s_f), in which the offers of the parties, for each $t \in \mathbb{N}$, are given by*

$$\overline{W}^{2t}_{AS} = 1 - \Delta_f(2t+1) + \sum_{m=t}^{\infty}(1 - \Delta_f(2m+3)) \prod_{j=t}^{m} \Delta_u(2j+2)\Delta_f(2j+1) \quad (3.6)$$

$$\overline{Z}^{2t+1}_{AS} = \overline{W}^{2t+2}_{AS} \Delta_u(2t+2). \quad (3.7)$$

Proof. We solve the system (3.1) which is equivalent, for each $t \in \mathbb{N}$, to

$$\overline{W}^{2t} - \overline{Z}^{2t+1}\Delta_f(2t+1) = 1 - \Delta_f(2t+1)$$
$$\text{and} \quad \overline{Z}^{2t+1} - \overline{W}^{2t+2}\Delta_u(2t+2) = 0 \quad (3.8)$$

and gives immediately (3.7). Note that (3.8) is a regular triangular system $AX = Y$, with $A = [a_{ij}]_{i,j\in\mathbb{N}^+}$, $X = [(x_i)_{i\in\mathbb{N}^+}]^T$, $Y = [(y_i)_{i\in\mathbb{N}^+}]^T$, where for each $t, j \geq 1$

$$a_{t,t} = 1,\ a_{t,j} = 0 \text{ for } j < t \text{ or } j > t+1 \quad (3.9)$$

and for each $t \in \mathbb{N}$

$$a_{2t+1,2t+2} = -\Delta_f(2t+1), \quad a_{2t+2,2t+3} = -\Delta_u(2t+2) \quad (3.10)$$

$$x_{2t+1} = \overline{W}^{2t}, \quad x_{2t+2} = \overline{Z}^{2t+1}, \quad y_{2t+1} = 1 - \Delta_f(2t+1), \quad y_{2t+2} = 0. \quad (3.11)$$

Any regular triangular matrix A possesses the (unique) inverse matrix B, *i.e.*, there exists B such that $BA = I$, where I is the infinite identity matrix. The matrix $B = [b_{ij}]_{i,j\in\mathbb{N}^+}$ is also regular triangular, and its elements are the following:

$$b_{t,t} = 1,\ b_{t,j} = 0 \text{ for each } t, j \geq 1 \text{ such that } j < t \quad (3.12)$$

$$b_{2t+1,2t+2} = \Delta_f(2t+1),\ b_{2t+2,2t+3} = \Delta_u(2t+2) \text{ for each } t \in \mathbb{N} \quad (3.13)$$

and for each $t, m \in \mathbb{N}$ and $m > t$

$$b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3),$$

$$b_{2t+2,2m+3} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+3)\Delta_u(2m+2) \quad (3.14)$$

$$b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+1),$$

$$b_{2t+1,2m+2} = \prod_{j=t}^{m-1} \Delta_u(2j+2)\Delta_f(2j+1)\Delta_f(2m+1). \tag{3.15}$$

Next, by applying $X = BY$ we get $\overline{W}^{2t}$ as given by (3.6). Obviously $\overline{W}^{2t} \geq 0$. Let us consider the sequence of partial sums for $k > t$

$$S_k = 1 - \Delta_f(2t+1) + \sum_{m=t}^{k-1}(1 - \Delta_f(2m+3)) \prod_{j=t}^{m} \Delta_u(2j+2)\Delta_f(2j+1).$$

The sequence is increasing and also $S_k \leq 1$ for each $k > t$, and therefore $\overline{W}^{2t} = \lim_{k\to+\infty} S_k \leq 1$. Since $0 \leq \overline{W}^{2t+2} \leq 1$, we have $0 \leq \overline{Z}^{2t+1} < 1$. □

Formula (3.6) presents the SPE offer made by the union in an even period. It is determined by the generalized discount factors of the union in all even periods following the given period and by the generalized discount factors of the firm in all odd periods following that period. Shaked and Sutton [19] provide a nice interpretation of the solution in the wage bargaining à la Rubinstein for constant discount rates: the payoff of the firm (which is the first mover in their model) coincides with the sum of the shrinkages of the cake which occur during the time periods when the offers made in even periods are rejected. For the common discount rate δ, we have $\frac{1}{1+\delta} = (1-\delta)(1+\delta^2+\delta^4+\cdots)$ which explains this interpretation, because the cake shrinks from δ^{2t} to δ^{2t+1}, *i.e.*, by $(1-\delta)\delta^{2t}$ if it is rejected in period $2t$. As Shaked and Sutton [19] mention, this also holds for the (constant) discount rates which are not equal. In our case, we notice a similar (but generalized) pattern, with the generalized discount factors.

According to (3.7), the SPE offer made by the firm in an odd period is equal to the SPE offer made by the union in the subsequent period, discounted by the generalized discount factor of the union. In other words, what the union can earn by accepting the SPE offer made by the firm in an odd period is equal to what the union could earn by rejecting that offer and submitting its SPE offer in the subsequent even period (that would be accepted by the firm).

Note that the more patient the union is in the subsequent periods, the more is proposed to the union in a given period under the SPE, both by the union and by the firm.

Example 3.4. When we apply our result to the wage bargaining studied by Fernandez and J. Glazer [6], we get obviously their result (see Lem. 1 in [6]). Let us calculate the share $\overline{W}^0$ that the union proposes for itself at the beginning of the game. We have $\delta_{f,2t+1} = \delta_f$ and $\delta_{u,2t+2} = \delta_u$ for each $t \in \mathbb{N}$. Hence, for each $t \in \mathbb{N}$

$$\overline{W}^{2t}_{AS} = (1-\delta_f) + (1-\delta_f)\left[\delta_f\delta_u + (\delta_f\delta_u)^2 + \cdots\right] = \frac{1-\delta_f}{1-\delta_f\delta_u}\cdot$$

Example 3.5. Let us analyze a model in which the union and the firm have the following sequences of discount factors varying in time: for each $t \in \mathbb{N}$

$$\delta_{f,2t+1} = \delta_{u,2t+1} = \frac{1}{2}, \quad \delta_{f,2t+2} = \delta_{u,2t+2} = \frac{1}{3}$$

Hence, for each $j \in \mathbb{N}$

$$\sum_{k=2j+1}^{\infty} \delta_f(2j+1,k) = \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2} + \cdots$$
$$= \frac{1}{2}\left(1 + \frac{1}{6} + \frac{1}{6^2} + \cdots\right) + \frac{1}{6}\left(1 + \frac{1}{6} + \frac{1}{6^2} + \cdots\right) = \frac{4}{5}, \varDelta_f(2j+1) = \frac{4}{9}$$
$$\sum_{k=2j+2}^{\infty} \delta_u(2j+2,k) = \frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{3} + \cdots$$
$$= \frac{1}{3}\left(1 + \frac{1}{6} + \frac{1}{6^2} + \cdots\right) + \frac{1}{6}\left(1 + \frac{1}{6} + \frac{1}{6^2} + \cdots\right) = \frac{3}{5}, \ \varDelta_u(2j+2) = \frac{3}{8}.$$

Hence, by virtue of (3.6) the offer of the union in period 0 in the SPE is equal to

$$\overline{W}^0_{AS} = \frac{5}{9} + \frac{4}{9}\cdot\frac{3}{8}\cdot\frac{5}{9} + \left(\frac{4}{9}\cdot\frac{3}{8}\right)^2\cdot\frac{5}{9} + \cdots = \frac{5}{9}\left(1 + \frac{1}{6} + \frac{1}{6^2} + \cdots\right) = \frac{2}{3}.$$

Note again that if we would apply the generalization of the original Rubinstein model to this example, then we would get $\overline{W}^0 = \frac{3}{5}$.

4. Going on strike only after rejection of own proposals

Haller and Holden [8] consider also another game with the exogenous strike decision, in which the union goes on strike only after its own proposal is rejected and it holds out if a proposal of the firm is rejected. They analyze the model with the same discount factor δ and show that in such a game there is the unique SPE with the union's offer equal to $\overline{W} = \frac{1+\delta w_0}{1+\delta}$. We generalize this game to discount rates varying in time.

Similarly as Proposition 3.1 for the case of always strike decision, Proposition 4.1 presents necessary and sufficient conditions for (s_u, s_f) to be a SPE for the case of "going on strike only after rejection of own proposals", if the firm is at least as patient as the union, *i.e.*, more precisely, if the generalized discount factor of the firm in every even period is at least as high as the generalized discount factor of the union in this even period. According to these conditions, each party is indifferent between accepting and rejecting the equilibrium offer in every period in which it is the turn of that party to reply to the offer.

Proposition 4.1. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$*, where* $\delta_{i,0} = 1$*,* $0 < \delta_{i,t} < 1$ *for* $t \geq 1$*,* $i = u, f$*, and*

$$\Delta_f(2t+2) \geq \Delta_u(2t+2) \text{ for each } t \in \mathbb{N} \tag{4.1}$$

Assume that the strike decision is given exogenously and the union is committed to strike only after rejection of its own proposals. Then (s_u, s_f) *is a SPE of this game if and only if the offers satisfy the following infinite system of equations: for each* $t \in \mathbb{N}$

$$\begin{aligned} 1 - \overline{W}^{2t} &= \left(1 - \overline{Z}^{2t+1}\right)\Delta_f(2t+1) \quad \text{and} \\ \overline{Z}^{2t+1} &= w_0\left(1 - \Delta_u(2t+2)\right) + \overline{W}^{2t+2}\Delta_u(2t+2) \end{aligned} \tag{4.2}$$

Proof. (⇐) The analysis of subgames that start with replies to an offer as well as of a subgame starting in period $2t$ with the union making an offer is analogous to the analysis of the corresponding subgames of the going always on strike case.

Consider a subgame starting in period $2t+1$ with the firm making an offer. Under (s_u, s_f), the union gets $\overline{Z}^{2t+1} + \overline{Z}^{2t+1}\sum_{k=2t+2}^{\infty}\delta_u(2t+2,k)$ and the firm $(1-\overline{Z}^{2t+1}) + (1-\overline{Z}^{2t+1})\sum_{k=2t+2}^{\infty}\delta_f(2t+2,k)$. Suppose that the firm deviates from s_f and proposes a certain $y < \overline{Z}^{2t+1}$. Then the firm gets $(1-w_0) + (1-\overline{W}^{2t+2})\sum_{k=2t+2}^{\infty}\delta_f(2t+2,k)$. Note that $\overline{Z}^{2t+1} \geq w_0$, otherwise the union would prefer to reject $\overline{Z}^{2t+1}$ and to get w_0 in period $2t+1$. From (4.2), $0 \leq \overline{Z}^{2t+1} - w_0 = (\overline{W}^{2t+2} - \overline{Z}^{2t+1})\sum_{k=2t+2}^{\infty}\delta_u(2t+2,k)$, and therefore $\overline{W}^{2t+2} \geq \overline{Z}^{2t+1}$. By virtue of (4.1), $(\overline{W}^{2t+2} - \overline{Z}^{2t+1})\sum_{k=2t+2}^{\infty}\delta_f(2t+2,k) = (\overline{Z}^{2t+1} - w_0)\frac{\sum_{k=2t+2}^{\infty}\delta_f(2t+2,k)}{\sum_{k=2t+2}^{\infty}\delta_u(2t+2,k)} \geq (\overline{Z}^{2t+1} - w_0)$. Hence, we have $(1-\overline{Z}^{2t+1}) + (1-\overline{Z}^{2t+1})\sum_{k=2t+2}^{\infty}\delta_f(2t+2,k) \geq (1-w_0) + (1-\overline{W}^{2t+2})\sum_{k=2t+2}^{\infty}\delta_f(2t+2,k)$, so this deviation would not be profitable to the firm. The proofs that other deviations are not profitable to the deviating party are similar to the going always on strike case.

(⇒) The proof is analogous to the proof of Proposition 3.1. □

Remark 4.2. From the proof of Proposition 4.1 we can note that if $\Delta_f(2t+2) < \Delta_u(2t+2)$ for some $t \in \mathbb{N}$, then in the corresponding subgame starting in period $2t+1$ with the firm making an offer, (s_u, s_f) as defined by (4.2) would not be a Nash equilibrium, and consequently would not be a SPE of the game.

By solving the infinite system (4.2), we determine the SPE offers made by the union and the firm, as presented in Theorem 4.3. For the 'strike only after rejection' case, the SPE offers will be denoted by $\overline{W}_{SAR}^{2t}$ and $\overline{Z}_{SAR}^{2t+1}$ for every $t \in \mathbb{N}$.

Theorem 4.3. *Consider the generalized wage bargaining model with preferences described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$*, where* $\delta_{i,0} = 1$*,* $0 < \delta_{i,t} < 1$ *for* $t \geq 1$*,* $i = u, f$ *and condition (4.1) is satisfied, i.e.,*

$$\Delta_f(2t+2) \geq \Delta_u(2t+2) \text{ for each } t \in \mathbb{N}.$$

Assume that the strike decision is given exogenously and the union is committed to strike only after rejection of its own proposals. Then there is the unique SPE of the form (s_u, s_f), in which the offers of the parties for each $t \in \mathbb{N}$ are given by

$$\begin{aligned}\overline{W}^{2t}_{SAR} &= 1 - \Delta_f(2t+1) + w_0\Delta_f(2t+1)(1-\Delta_u(2t+2)) \\ &\quad + \sum_{m=t}^{\infty}(1-\Delta_f(2m+3) + w_0\Delta_f(2m+3)(1-\Delta_u(2m+4))) \\ &\quad \times \prod_{j=t}^{m}\Delta_u(2j+2)\Delta_f(2j+1) \end{aligned} \tag{4.3}$$

$$\overline{Z}^{2t+1}_{SAR} = w_0\left(1-\Delta_u(2t+2)\right) + \overline{W}^{2t+2}_{SAR}\Delta_u(2t+2). \tag{4.4}$$

Proof. We need to solve (4.2) for each $t \in \mathbb{N}$, which is equivalent for each $t \in \mathbb{N}$ to

$$\overline{W}^{2t} - \overline{Z}^{2t+1}\Delta_f(2t+1) = 1 - \Delta_f(2t+1) \quad \text{and} \tag{4.5}$$

$$\overline{Z}^{2t+1} - \overline{W}^{2t+2}\Delta_u(2t+2) = w_0\left(1-\Delta_u(2t+2)\right). \tag{4.6}$$

From (4.6) we get (4.4). (4.5) and (4.6) is a regular triangular system $AX = Y$ with $A = [a_{ij}]_{i,j\in\mathbb{N}^+}$, $X = [(x_i)_{i\in\mathbb{N}^+}]^T$, $Y = [(y_i)_{i\in\mathbb{N}^+}]^T$, where A is the same as for Theorem 3.3 and is described by (3.9) for $t, j \geq 1$ and (3.10) for $t \in \mathbb{N}$.

$$x_{2t+1} = \overline{W}^{2t}, \quad x_{2t+2} = \overline{Z}^{2t+1},$$
$$y_{2t+1} = 1 - \Delta_f(2t+1), \quad y_{2t+2} = w_0\left(1-\Delta_u(2t+2)\right).$$

Since we have the same A as in the always-strike decision, its (unique) inverse matrix B is the same. By applying $X = BY$ we get $\overline{W}^{2t}$ as in (4.3). From (4.4) $0 \leq \overline{Z}_{2t+1} \leq 1$. Also $\overline{W}^{2t} \geq 0$. The proof that $\overline{W}^{2t} \leq 1$ goes analogously as in Theorem 3.3. □

Remark 4.4. Note that $\overline{W}^{2t}_{SAR}$ given in (4.3) can be written equivalently as

$$\begin{aligned}\overline{W}^{2t}_{SAR} = \overline{W}^{2t}_{AS} + w_0\Bigg(&\Delta_f(2t+1)(1-\Delta_u(2t+2)) \\ &+ \sum_{m=t}^{\infty}\Delta_f(2m+3)(1-\Delta_u(2m+4))\prod_{j=t}^{m}\Delta_u(2j+2)\Delta_f(2j+1)\Bigg)\end{aligned} \tag{4.7}$$

and hence, $\overline{W}^{2t}_{SAR} > \overline{W}^{2t}_{AS}$. This has an intuitive interpretation. Going on strike only after rejection of own proposals (*i.e.*, in even periods) gives a greater wage contract than going on strike in every disagreement period, because the first strategy creates an asymmetry in costs of rejecting. Under the first strategy, it is more costly for the firm to reject the union's offer (which leads to the strike) than it is for the union to reject the firm's offer (which leads to the holdout).

Since $\overline{W}^{2t+2}_{SAR} > \overline{W}^{2t+2}_{AS}$, we have also $\overline{Z}^{2t+1}_{SAR} = w_0\,(1-\varDelta_u(2t+2)) + \overline{W}^{2t+2}_{SAR}\varDelta_u(2t+2) > \overline{W}^{2t+2}_{AS}\varDelta_u(2t+2) = \overline{Z}^{2t+1}_{AS}$, and therefore $\overline{Z}^{2t+1}_{SAR} > \overline{Z}^{2t+1}_{AS}$.

Example 4.5. Let us apply this result to the wage bargaining studied by [6], *i.e.*, we have $\delta_{f,t} = \delta_f$ and $\delta_{u,t} = \delta_u$ for each $t \in \mathbb{N}$. Hence, for each $t \in \mathbb{N}$

$$\begin{aligned}\overline{W}^{2t}_{SAR} &= (1-\delta_f + w_0\delta_f(1-\delta_u))\left[1+\delta_f\delta_u + (\delta_f\delta_u)^2 + \cdots\right]\\ &= \frac{1-\delta_f + w_0\delta_f(1-\delta_u)}{1-\delta_f\delta_u} = w_0 + \frac{(1-\delta_f)(1-w_0)}{1-\delta_f\delta_u}\cdot\end{aligned}$$

If additionally we assume that $\delta_f = \delta_u = \delta$, then $\overline{W}^{2t}_{SAR} = \frac{1+\delta w_0}{1+\delta}$, which coincides with the result by Haller and Holden [8].

Example 4.6. We analyze the model presented in Example 3.5. By virtue of (4.3) the offer of the union in period 0 in the SPE is equal to

$$\begin{aligned}\overline{W}^{0}_{SAR} &= \left(\frac{5}{9} + \frac{4}{9}\cdot\frac{5}{8}\cdot w_0\right)\left[1 + \frac{4}{9}\cdot\frac{3}{8} + \left(\frac{4}{9}\cdot\frac{3}{8}\right)^2 + \cdots\right]\\ &= \frac{2+w_0}{3} > \frac{2}{3} = \overline{W}^{0}_{AS}.\end{aligned}$$

5. Going never on strike

In case of the exogenous 'never-strike' decision of the union, the unique SPE leads to the minimum wage contract w_0. The SPE offers for this case are denoted by $\overline{W}^{2t}_{NS}$ and $\overline{Z}^{2t+1}_{NS}$. We have the following:

Fact 5.1. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t\in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. Assume that the no-strike decision is given exogenously and the union never goes on strike. Then there is the unique SPE of the form (s_u, s_f), where $\overline{W}^{2t}_{NS} = \overline{Z}^{2t+1}_{NS} = w_0$ for each $t \in \mathbb{N}$.*

Proof. Suppose that the union never goes on strike. Similar as in the proof of Proposition 3.1 one can show that if (s_u, s_f) is a SPE, then it must hold for each $t \in \mathbb{N}$

$$\begin{aligned}\left(1-\overline{W}^{2t}\right) + \left(1-\overline{W}^{2t}\right)\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) = (1-w_0) + \left(1-\overline{Z}^{2t+1}\right)\\ \times\sum_{k=2t+1}^{\infty}\delta_f(2t+1,k) \quad (5.1)\end{aligned}$$

and

$$\overline{Z}^{2t+1} + \overline{Z}^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k) = w_0 + \overline{W}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k). \quad (5.2)$$

Obviously, $\overline{W}^{2t} = \overline{Z}^{2t+1} = w_0$ for each $t \in \mathbb{N}$ is a solution of this system of equations, and we know from the infinite matrices theory that this system has the only one solution. One can easily show that (s_u, s_f) with $\overline{W}^{2t} = \overline{Z}^{2t+1} = w_0$ for $t \in \mathbb{N}$ is a SPE. □

Remark 5.2. Note that $\overline{W}^{2t}_{SAR}$ given in (4.3) can also be written equivalently as

$$\overline{W}^{2t}_{SAR} = w_0 + (1-w_0)\Bigg(1 - \varDelta_f(2t+1) + \sum_{m=t}^{\infty}(1-\varDelta_f(2m+3)) \prod_{j=t}^{m} \varDelta_u(2j+2)\varDelta_f(2j+1)\Bigg) \quad (5.3)$$

and therefore $\overline{W}^{2t}_{SAR} > w_0 = \overline{W}^{2t}_{NS}$ if $w_0 < 1$. This means that striking only after rejection of own proposals gives to the union the minimum wage contract plus the solution of the case "going always on strike" with the size of the "cake" equal to $1 - w_0$ instead of 1.

Moreover, $1 - \overline{W}^{2t}_{SAR} = (1-w_0)(1-\overline{W}^{2t}_{AS})$, which means that in this case the firm gets what it would have under the "going always on strike" equilibrium with the size of the cake equal to $1 - w_0$.

Since $\overline{W}^{2t+2}_{SAR} > w_0$, we have also $\overline{Z}^{2t+1}_{SAR} = w_0\,(1 - \varDelta_u(2t+2)) + \overline{W}^{2t+2}_{SAR}\varDelta_u(2t+2) = w_0 + \varDelta_u(2t+2)(\overline{W}^{2t+2}_{SAR} - w_0) > w_0 = \overline{Z}^{2t+1}_{NS}$.

6. Subgame perfect equilibria in the general model

After finding the unique SPE for each of the three cases with the exogenous strike decisions, now we will show that the strategies forming these SPE also appear in the SPE for the general model, *i.e.*, for the model with no assumption on the commitment to strike.

First of all, we consider the pair of strategies analyzed in Section 5. It appears that Lemma 2 of [6] remains valid for the general wage bargaining model with discount factors varying in time. We have the following:

Fact 6.1. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$*, where* $\delta_{i,0} = 1$*,* $0 < \delta_{i,t} < 1$ *for* $t \geq 1$*,* $i = u, f$*. There is a SPE in which an agreement of* w_0 *is reached immediately in period 0. This SPE is the following 'minimum-wage equilibrium':*

- *the union plays* s_u *with* $\overline{W}^{2t} = w_0$ *for each* $t \in \mathbb{N}$ *and never goes on strike;*

- *the firm plays* s_f *with* $\overline{Z}^{2t+1} = w_0$ *for each* $t \in \mathbb{N}$.

Proof. It is easy to show that the 'minimum-wage' strategies form a SPE for the general wage bargaining game. If one party changes its strategy, with the strategy of the another party being fixed, then the deviating party cannot be better off: neither if at some point it makes an offer different from w_0, nor when it accepts (rejects) an offer which gives the party less (more) than the considered profile of strategies (w_0 for the union and $1 - w_0$ for the firm). The union will not be better off when it decides to change its 'never strike' decision and goes on strike when there is a disagreement. □

Next, we consider the pair of strategies presented for the always strike case in Theorem 3.3 of Section 3. If we combine this pair of strategies with the 'minimum-wage' strategies, then we find a SPE for the general wage bargaining, provided that the union is sufficiently patient (*i.e.*, the generalized discount factors of the union in all odd periods are sufficiently high). The following proposition generalizes Lemma 3 of [6].

Proposition 6.2. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$, *where* $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ *for* $t \geq 1$, $i = u, f$. *If*

$$w_0 \leq \overline{Z}_{AS}^{2t+1} \Delta_u(2t+1) \text{ for every } t \in \mathbb{N} \tag{6.1}$$

then there exists a SPE in which the agreement of $\overline{W}_{AS}^{0}$ *is reached in period 0, where* $\overline{W}_{AS}^{0}$ *is given in Theorem 3.3. This SPE is formed by the following profile of strategies:*

- *The union plays* s_u *with* $\overline{W}^{2t} = \overline{W}_{AS}^{2t}$ *for each* $t \in \mathbb{N}$ *and always goes on strike if there is a disagreement, where* $\overline{W}_{AS}^{2t}$ *is given in (3.6),*
- *The firm plays* s_f *with* $\overline{Z}^{2t+1} = \overline{Z}_{AS}^{2t+1}$ *for each* $t \in \mathbb{N}$, *where* $\overline{Z}_{AS}^{2t+1}$ *is given in (3.7),*
- *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the strategies given in the 'minimum-wage equilibrium'.*

Proof. Note that from assumption (6.1) it follows that $\overline{W}_{AS}^{2t} \geq w_0$ and $\overline{Z}_{AS}^{2t+1} \geq w_0$ for every $t \in \mathbb{N}$, because we have $\overline{Z}_{AS}^{2t+1} \geq \overline{Z}_{AS}^{2t+1} \Delta_u(2t+1) \geq w_0$, and from (3.1), $1 - \overline{W}_{AS}^{2t} = \left(1 - \overline{Z}_{AS}^{2t+1}\right) \Delta_f(2t+1) \leq (1 - w_0)\, \Delta_f(2t+1)$. Hence, $\overline{W}_{AS}^{2t} \geq 1 - (1 - w_0)\, \Delta_f(2t+1) = w_0 + (1 - \Delta_f(2t+1))(1 - w_0) \geq w_0$.

In order for the union not to deviate from its strike decision in any $2t$ period when no agreement is reached, it must hold $w_0 + w_0 \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k) \leq \overline{Z}_{AS}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k)$, which is equivalent to (6.1). Hence, the required condition holds.

In order for the union not to deviate from its strike decision in any $2t+1$ period when no agreement is reached, it must hold $w_0 \leq \overline{W}_{AS}^{2t+2} \Delta_u(2t+2)$, but this is satisfied, since from (6.1), $w_0 \leq \overline{Z}_{AS}^{2t+1} \Delta_u(2t+1) \leq \overline{Z}_{AS}^{2t+1} = \overline{W}_{AS}^{2t+2} \Delta_u(2t+2)$.

Consider a (proper) subgame such that the union has already deviated in an earlier period. Then, if the parties play the considered profile of strategies, then they use the minimum-wage equilibrium strategies. Hence, from Fact 6.1, this profile is a Nash equilibrium in every subgame starting after the subgame with the deviation.

Consider a subgame such that the union has not deviated before. If the union deviates now in period $2t$ and proposes $x \neq \overline{W}_{AS}^{2t} \geq w_0$, then the firm switches to the minimum-wage strategy and the union cannot be better off by this deviation. Also the firm cannot be better off by deviating in $2t+1$ and proposing $y \neq \overline{Z}_{AS}^{2t+1}$. Finally, it is easy to show that no party can be better off by a deviation when replying to an offer of the other party. □

Consider now the pair of strategies presented for the "going on strike only after rejection of own proposals" case in Theorem 4.3 of Section 4. If we combine this pair of strategies with the 'minimum-wage' strategies, then we find a SPE for the general wage bargaining, provided that the firm is at least as patient as the union in every even period and that the union is sufficiently patient in every odd period (where the parties' patience is represented by the generalized discount factors in a given period). The following proposition generalizes Lemma 4 of [6] and Proposition 1(i) of [3].

Proposition 6.3. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$*, where* $\delta_{i,0} = 1$*,* $0 < \delta_{i,t} < 1$ *for* $t \geq 1$*,* $i = u, f$*. If*

$$w_0 \leq \overline{Z}_{SAR}^{2t+1} \Delta_u(2t+1) \text{ for every } t \in \mathbb{N} \tag{6.2}$$

and condition (4.1) is satisfied, i.e.,

$$\Delta_f(2t+2) \geq \Delta_u(2t+2) \text{ for each } t \in \mathbb{N}$$

then there exists a SPE in which the agreement of $\overline{W}_{SAR}^{0}$ *is reached in period 0, where* $\overline{W}_{SAR}^{0}$ *is given in Theorem 4.3. This SPE is supported by the following "generalized alternating strike strategies":*

- *The union plays* s_u *with* $\overline{W}^{2t} = \overline{W}_{SAR}^{2t}$ *for each* $t \in \mathbb{N}$*, goes on strike after rejection of its own proposals and holds out after rejecting firm's offers, where* $\overline{W}_{SAR}^{2t}$ *is given in (4.3).*
- *The firm plays* s_f *with* $\overline{Z}^{2t+1} = \overline{Z}_{SAR}^{2t+1}$ *for each* $t \in \mathbb{N}$*, where* $\overline{Z}_{SAR}^{2t+1}$ *is given in (4.4),*
- *If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the strategies given in the 'minimum-wage equilibrium'.*

Proof. From (5.3), if $w_0 < 1$ then we have $\overline{W}_{SAR}^{2t} > w_0$ and $\overline{Z}_{SAR}^{2t+1} > w_0$ for every $t \in \mathbb{N}$. If in period $2t$, when no agreement is reached, the union deviates from its strike decision, then it is not better of by virtue of condition (6.2). If in period $2t{+}1$, when no agreement is reached, the union deviates from its 'hold out' decision, then it is worse off, since $w_0 \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k) < w_0 + \overline{W}_{SAR}^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2,k)$. The remaining parts of the proof goes similarly to the proof of Proposition 6.2. □

Next, we will find a SPE for a particular case of the wage bargaining when condition (4.1) is not satisfied, *i.e.*, for the game with $\Delta_u(2t+2) > \Delta_f(2t+2)$ for each $t \in \mathbb{N}$. In such a case, given the generalized alternating strike strategy of the union, the firm is better off by playing the so called no-concession strategy instead of the generalized alternating strike strategy. The *no-concession strategy of the firm* is defined as follows:

– *Reject all offers of the union in every even period* $2t$*, and make an unacceptable offer (e.g.,* $Z_{NC}^{2t+1} = 0$*) in every odd period* $2t+1$*.*

We can prove the following result.

Proposition 6.4. *If there exists* $T \in \mathbb{N}$ *such that* $\Delta_u(2t+2) > \Delta_f(2t+2)$ *for each* $t \geq T$*, then the pair of the generalized alternating strike strategies is not a SPE. In particular, for* $T = 0$*, this pair is not a Nash equilibrium.*

Proof. Assume that there exists $T \in \mathbb{N}$ such that $\Delta_u(2t+2) > \Delta_f(2t+2)$ for each $t \geq T$. Then we have the following:

$$\begin{aligned}\sum_{m=T}^{\infty} (1-\Delta_f(2m+3)) \prod_{j=T}^{m} \Delta_f(2j+1)\Delta_u(2j+2) \\ &= \sum_{m=T}^{\infty} \frac{\prod_{j=T}^{m} \Delta_f(2j+1)\Delta_u(2j+2)}{1+\sum_{k=2m+3}^{\infty} \delta_f(2m+3,k)} \\ &> \sum_{m=T}^{\infty} \frac{\prod_{j=T}^{m} \delta_{f,2j+1}\delta_{f,2j+2}}{1+\sum_{k=2T+1}^{\infty} \delta_f(2T+1,k)} \\ &= \frac{\sum_{m=T}^{\infty} \delta_f(2T+1,2m+2)}{1+\sum_{k=2T+1}^{\infty} \delta_f(2T+1,k)}\cdot\end{aligned}$$

Hence, we have

$$\sum_{m=T}^{\infty} (1-\Delta_f(2m+3)) \prod_{j=T}^{m} \Delta_f(2j+1)\Delta_u(2j+2) > \frac{\sum_{m=T}^{\infty} \delta_f(2T+1,2m+2)}{1+\sum_{k=2T+1}^{\infty} \delta_f(2T+1,k)}\cdot \qquad (6.3)$$

Consider a subgame starting in period $2T$ in which the union proposes $\overline{W}_{SAR}^{2T}$ and no deviation of the union has taken place before. Then, the generalized alternating strike strategies lead to the agreement $\overline{W}_{SAR}^{2T}$ reached in period $2T$. If the firm switches to the no-concession strategy, then it gets the (normalized) payoff $(1 - Y_{NC}^{2T})$ equal to

$$1 - Y_{NC}^{2T} = (1 - w_0)\frac{\sum_{m=T}^{\infty} \delta_f(2T+1, 2m+1)}{1 + \sum_{m=2T+1}^{\infty} \delta_f(2T+1, m)}$$
$$= (1 - w_0)\left[\Delta_f(2T+1) - \frac{\sum_{m=T}^{\infty} \delta_f(2T+1, 2m+2)}{1 + \sum_{m=2T+1}^{\infty} \delta_f(2T+1, m)}\right].$$

Note that

$$1 - \overline{W}_{SAR}^{2T} = (1 - w_0)\left(1 - \overline{W}_{AS}^{2T}\right) = (1 - w_0)\Bigg(\Delta_f(2T+1)$$
$$- \sum_{m=T}^{\infty}(1 - \Delta_f(2m+3)) \prod_{j=T}^{m} \Delta_f(2j+1)\Delta_u(2j+2)\Bigg).$$

Hence, $1 - Y_{NC}^{2T} > 1 - \overline{W}_{SAR}^{2T}$, as it is equivalent to (6.3), which shows that the firm is better off by switching to the no-concession strategy. □

The intuition behind this result is the following. Since the firm is more impatient than the union and its disagreement payoff in even periods is very low, the firm is willing to disagree forever, *i.e.*, to make unacceptable offers and alternate between strikes and paying the old contract w_0, rather than paying the contract $\overline{W}_{SAR}^{0}$. For this case, the SPE is modified as presented in the following theorem.

Theorem 6.5. *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors* $(\delta_{i,t})_{t\in N}$*, where* $\delta_{i,0} = 1$*,* $0 < \delta_{i,t} < 1$ *for* $t \geq 1$*,* $i = u, f$*, where*

$$\Delta_u(2t+2) > \Delta_f(2t+2) \text{ for each } t \in \mathbb{N} \tag{6.4}$$

and for each $t \in \mathbb{N}$

$$w_0 \leq \Delta_u(2t+1)\left((1 - \Delta_u(2t+2))w_0 + \Delta_u(2t+2)\widetilde{W}^{2t+2}\right) \tag{6.5}$$

where

$$\widetilde{W}^{2t} = \frac{1 + \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2) + w_0 \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+1)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1, m)}. \tag{6.6}$$

Then there exists a SPE in which an agreement is reached only in even periods. This SPE is supported by the following 'modified generalized alternating strike strategies':

(A) Union:

- *In every period* $2t$ *propose* $\widetilde{W}^{2t}$ *given by (6.6).*

- *In every period $2t+1$ accept an offer y if and only if $y \geq (1-\Delta_u(2t+2))w_0 + \Delta_u(2t+2)\widetilde{W}^{2t+2}$.*
- *Strike in even periods and hold out in odd periods if no agreement is reached.*
- *If the union deviates, then play the minimum-wage strategy.*

(B) Firm:

- *in every period $2t+1$ propose $\widetilde{Z}^{2t+1} = 0$;*
- *in every period $2t$ accept an offer x if and only if $x \leq \widetilde{W}^{2t}$;*
- *if the union deviates, then play the minimum-wage strategy.*

Proof. Note that for $\widetilde{W}^{2t}$ given by (6.6), if $w_0 < 1$, then we have $\widetilde{W}^{2t} > w_0$ for every $t \in \mathbb{N}$. If in period $2t$, when no agreement is reached, the union deviates from its strike decision, then it is not better off by virtue of condition (6.5). Moreover, as $\widetilde{W}^{2t} > w_0$, the union would be worse off by deviating from the hold out decision in period $2t+1$.

In any (proper) subgame, where the union has already deviated before, no party would be better off by deviating on its own from the required minimum-wage strategy.

Suppose that there was no deviation by the union before. In any even period $2t$, the union prefers to offer $\widetilde{W}^{2t}$: by proposing less than $\widetilde{W}^{2t}$ it would be worse off, and by proposing more than $\widetilde{W}^{2t}$, it would get at most $w_0 \sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)$ which is less than $\widetilde{W}^{2t}\left(1+\sum_{k=2t+1}^{\infty} \delta_u(2t+1,k)\right)$. Consider any odd period $2t+1$. The firm's no-concession payoff from that period onward will be

$$\frac{(1-w_0)\left(1+\sum_{m=t}^{\infty} \delta_f(2t+2, 2m+3)\right)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

given the strategy of the union. Hence, the firm will not offer more to the union than

$$1 - \frac{(1-w_0)\left(1+\sum_{m=t}^{\infty} \delta_f(2t+2, 2m+3)\right)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

$$= \frac{w_0 + \sum_{m=t}^{\infty} \delta_f(2t+2,2m+2) + w_0\sum_{m=t}^{\infty} \delta_f(2t+2, 2m+3)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

$$Z^{2t+1} \leq \frac{w_0 + \sum_{m=t}^{\infty} \delta_f(2t+2,2m+2) + w_0\sum_{m=t}^{\infty} \delta_f(2t+2, 2m+3)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}.$$

In period $2t+1$, the union will reject any offer and hold out, because

$$\frac{w_0 + \sum_{m=t}^{\infty} \delta_f(2t+2,2m+2) + w_0\sum_{m=t}^{\infty} \delta_f(2t+2, 2m+3)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

$$= w_0(1-\Delta_f(2t+2)) + \frac{\widetilde{W}^{2t+2}\left(\delta_{f,2t+2} + \delta_{f,2t+2}\sum_{m=2t+3}^{\infty} \delta_f(2t+3,m)\right)}{1+\sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

$$< w_0(1 - \varDelta_f(2t+2)) + \frac{\widetilde{W}^{2t+2} \sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}{1 + \sum_{m=2t+2}^{\infty} \delta_f(2t+2,m)}$$

$$= w_0(1 - \varDelta_f(2t+2)) + \widetilde{W}^{2t+2} \varDelta_f(2t+2) = \varDelta_f(2t+2)(\widetilde{W}^{2t+2} - w_0) + w_0$$

$$< \varDelta_u(2t+2)(\widetilde{W}^{2t+2} - w_0) + w_0 = w_0(1 - \varDelta_u(2t+2)) + \widetilde{W}^{2t+2} \varDelta_u(2t+2).$$

The last inequality comes from (6.4) and from the fact that $\widetilde{W}^{2t+2} > w_0$. □

Theorem 6.5 generalizes Proposition 1(ii) of [3]. Under this SPE, the union offers $\widetilde{W}^{2t}$ in every period $2t$, and accepts an offer in period $2t+1$ only if it gives to the union at least as much as what the union would get by rejecting, holding out and getting its offer $\widetilde{W}^{2t+2}$ in $2t+2$. Note that the union's offer $\widetilde{W}^{2t}$ in period $2t$ is equal to its (normalized) payoff Y_{NC}^{2t} which it would get when the firm uses the no-concession strategy from period $2t$, *i.e.*,

$$\widetilde{W}^{2t} = Y_{NC}^{2t} = 1 - (1 - w_0) \frac{\sum_{m=t}^{\infty} \delta_f(2t+1, 2m+1)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1,m)}$$
$$= \frac{1 + \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+2) + w_0 \sum_{m=t}^{\infty} \delta_f(2t+1, 2m+1)}{1 + \sum_{m=2t+1}^{\infty} \delta_f(2t+1,m)}.$$

Moreover, under this SPE, the firm always makes unacceptable offers, but accepts an offer in period $2t$ if it gives to him at least its no-concession payoff $1 - Y_{NC}^{2t}$. Both parties switch to the minimum-wage strategies if the union deviates.

7. Conclusion

There are several issues in our agenda for future research on the generalized wage bargaining model. As mentioned in the Introduction, Houba and Wen [10] applied the method of Shaked and Sutton [19] to derive the exact bounds of equilibrium payoffs in the original F-G model. In order to find the infimum and supremum SPE payoffs under nonstationary discounting, the issues identified in [10] and the references therein should be taken into account. In our follow-up research, we intend to apply their method to our generalized wage bargaining model. Since we assume that the sequence of discount rates of a party can be arbitrary, with the only restriction that the infinite series that determines the utility for the given party must be convergent, first we will describe the conditions in a general case for the supremum of the union's SPE payoffs in any even period and for the infimum of the firm's SPE payoffs in any odd period. Then, we will solve the conditions for particular cases of the sequences of discount rates.

Several authors analyze the issues of bargaining power, both in the standard bargaining models and in the wage bargaining with constant discount rates. Since discount rates are usually crucial in determining bargaining power of parties, it is of importance to study these issues in our framework with discount rates varying in time.

Furthermore, we would like to provide a detailed analysis of some applications of the generalized wage bargaining model to real-life situations. The generalized wage bargaining version in which utilities of bargainers are of the type (2.3) and (2.4) are more suitable to model reality than the original bargaining with constant discount rates. Patience of parties may obviously be changing over time, due to many circumstances, *e.g.*, economic, financial, political, social, environmental, health or climatic issues. Moreover, in many situations, the utility of an agreement is counted not only in one step (the given period when the agreement is achieved), but it is a long-term utility. If we negotiate wages for workers or a price of a pharmaceutical product, the agreement is valid for a longer time. Even if the time of implementing the given agreement is finite, its expiration time might be unknown. Consequently, it is more appropriate to define the utilities by the type (2.3) and (2.4).

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