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Corrigendum to "On a question of Serre" [C. R. Acad. Sci. Paris, Ser. I 350 (2012) 741–744]

Corrigendum à « Sur une question de Serre » [C. R. Acad. Sci. Paris, Ser. I 350 (2012) 741–744]

Alexander D. Rahm

University of Luxembourg, Mathematics Research Unit, Maison du Nombre (MNO), 6, avenue de la Fonte, L-4364 Esch-sur-Alzette, Luxembourg

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As the note "On a question of Serre" [7] by the author had been published prematurely, the proof of the key lemma remaining sketchy, the author did contact an expert for the Borel–Serre compactification, Lizhen Ji, for the project of establishing a detailed version of that key proof. Lizhen Ji then found out that there are cases where that proof does not apply. So it is necessary to correct the scope of the lemma, which is possible using additional symmetries discovered by Luigi Bianchi (cf. [1], [2]).

The Bianchi groups, $\mathrm{SL}_2(\mathcal{O})$ over the ring $\mathcal{O}$ of integers in an imaginary quadratic field $\mathbf{Q}(\sqrt{-m})$, act naturally on hyperbolic 3-space $\mathcal{H}$ (cf. the monographs [4], [5], [6]). *Throughout this corrigendum, we will exclude the ring $\mathcal{O}$ from being the Gaussian integers, in $\mathbf{Q}(\sqrt{-1})$, or the Eisensteinian integers, in $\mathbf{Q}(\sqrt{-3})$.* Those two special cases can easily be treated separately. For a subgroup Γ of finite index in $\mathrm{SL}_2(\mathcal{O})$, consider the Borel–Serre compactification ${}_\Gamma\backslash\widehat{\mathcal{H}}$ of the orbit space ${}_\Gamma\backslash\mathcal{H}$, constructed in the appendix of [9], which joins a 2-torus $\mathbf{T}_\sigma$ to ${}_\Gamma\backslash\mathcal{H}$ at every cusp σ.

Question 1 *(Serre [9]). Consider the map α induced on homology when attaching the boundary into the Borel–Serre compactification ${}_\Gamma\backslash\widehat{\mathcal{H}}$. How can one determine the kernel of α (in degree 1)?*

The following lemma was the key for the approach to Question 1 pursued in the note under correction. In the remainder of this corrigendum, we consider an imaginary quadratic field $\mathbf{Q}(\sqrt{-D})$ with D a square-free natural integer, and we set $\Gamma = \mathrm{SL}_2(\mathcal{O}_{\mathbf{Q}(\sqrt{-D})})$. We decompose the 2-torus $\mathbf{T}_\sigma$ in the classical way into a 2-cell, two edges and a vertex.

Lemma 2. *Let n be the number of prime divisors of D. Let $N = 2^{n-1}$ for $D \equiv 2$ or $3 \mod 4$, $N = 2^n$ for $D \equiv 1 \mod 4$. Then ${}_\Gamma\backslash\mathcal{H}$ admits at least N cusps σ such that the inclusion of $\mathbf{T}_\sigma$ into ${}_\Gamma\backslash\widehat{\mathcal{H}}$ makes exactly one of the edges of $\mathbf{T}_\sigma$ become the boundary of a 2-chain.*

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E-mail address: Alexander.Rahm@uni.lu.

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The hypothesis on σ was missing in the note under correction, and a completed proof is given in [8], making use of this hypothesis.

Corollary 3. *If the class group of $\mathcal{O}$ is isomorphic to $\mathbf{Z}/2^m\mathbf{Z}$ for some $m \in \mathbf{N}$, then for all cusps σ of ${}_\Gamma\backslash\mathcal{H}$, the inclusion of $\mathbf{T}_\sigma$ into ${}_\Gamma\backslash\widehat{\mathcal{H}}$ makes exactly one of the edges of $\mathbf{T}_\sigma$ become the boundary of a 2–chain.*

Proof. By [3, thm. 3.22], our hypothesis on the class group of $\mathcal{O}$ is equivalent to $m = \begin{cases} n-1, & D \equiv 2 \text{ or } 3 \mod 4, \\ n, & D \equiv 1 \mod 4. \end{cases}$

It is well known that each ideal class of $\mathcal{O}$ corresponds to one cusp of ${}_\Gamma\backslash\mathcal{H}$, so there are N cusps, and Lemma 2 yields the claim. □

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