C. R. Acad. Sci. Paris, Ser. I 357 (2019) 389–394

Contents lists available at ScienceDirect

C. R. Acad. Sci. Paris, Ser. I

www.sciencedirect.com

Functional analysis/Complex analysis

Construction of some classes of commutative Banach and $C^\star$-algebras of Toeplitz operators ☆

Une construction de quelques classes d'algèbre commutatives de Banach et de $C^\star$ via les opérateurs de Toeplitz

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ARTICLE INFO

Article history:
Received 21 October 2018
Accepted after revision 8 April 2019
Available online 18 April 2019

Presented by the Editorial Board

ABSTRACT

In the present paper, we construct commutative algebras generated by Toeplitz operators on the Segal–Bargmann space $H^2_s(\mathbb{C}^n)$ and on the true-k-Fock spaces. Analogous to the commutative Banach algebras constructed by N. Vasilevski for the case of the unit ball, we obtain a commutative algebra on $H^2_s(\mathbb{C}^n)$ formed only by Toeplitz operators and a composition formula is obtained. Employing a natural extension for the notion of Toeplitz operators, we introduce "true-k-Toeplitz operators" acting on the true-k-Fock spaces. We provide a commutative $C^\star$-algebra generated by such operators, whose symbol depends on the real and imaginary parts of the complex variable in a certain sense.

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RÉSUMÉ

Dans cet article, nous construisons des algèbres commutatives générées par les opérateurs de Toeplitz sur l'espace de Segal–Bargmann $H^2_s(\mathbb{C}^n)$ et sur les espaces true-k-Fock. En utilisant une méthode analogue à celle mise en œuvre sur les algèbres de Banach commutatives construites par N. Vasilevski, nous obtenons une algèbre commutative sur $H^2_s(\mathbb{C}^n)$ formée uniquement par les opérateurs de Toeplitz, et une formule de composition est aussi obtenue. En utilisant une extension naturelle de la notion d'opérateur de Toeplitz, nous introduisons les opérateurs « true-k-Toeplitz » agissant sur les espaces true-k-Fock. Nous fournissons une $C^\star$-algèbre commutative générée par ces opérateurs, dont les symboles dépendent des parties réelle et imaginaire de la variable complexe dans un certain sens.

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☆ The results presented in the article were taken from the author's Ph.D. thesis while at Georg-August University of Göttingen (Germany), back in 2012. The project was supported by an "Emmy-Noether scholarship" of DFG (*Deutsche Forschungsgemeinschaft*).

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https://doi.org/10.1016/j.crma.2019.04.003
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1. Introduction

The problem of describing symbol classes that generate commutative algebras of Toeplitz operators on Bergman spaces is still open. In the case of the unit disc, the commutative property for $C^\star$-algebras is characterized by a geometrical property [4]. More precisely, these algebras arise from symbol classes that are radial with respect to geodesics. In the case of the Segal–Bargmann space, as well as of Bergman spaces over general bounded symmetric domains [5], a geometric characterization for obtaining the algebraic property of commutativity is not clear. Motivated by the work done in [14], for the case of the unit ball in higher dimensions, one has to include Banach algebras of commuting Toeplitz operators.

In the present paper, we provide two types of commutative Banach and $C^\star$-algebras of Toeplitz operators on the Segal–Bargmann space. Some results are obtained for the "true-k-Toeplitz operators" acting on the true k-Fock spaces as introduced in [12]. Our contribution is based on explicit computational results and so it would be a natural step for finding a geometrical classification for the classes of symbols that generate commutative Toeplitz algebras on the Segal–Bargmann and the true-k-Fock spaces.

Our first result is motivated by the original work of N. Vasilevski in [14]. The author constructed classes of k-quasi-homogeneous symbols for which the Banach algebras generated by the Toeplitz operators are commutative on each standard weighted Bergman space over the unit ball of $\mathbb{C}^n$. The result remains true on weakly pseudo-convex domains as it was shown in [8] (cf. [7] for the case of the projective space). Also, it was exhibited in [13] that parabolic quasi-radial quasi-homogeneous symbols generate commutative Toeplitz algebras. Subordinated to the quasi-parabolic and quasi-hyperbolic groups of the automorphism of the unit ball, commutative Banach algebras were presented in [1,2]. A natural extension of the work done in [14] was given in [15] for the class of quasi-radial and pseudo-homogeneous symbols.

Inspired by the aforementioned results, we provide a construction of Toeplitz Banach algebras that are commutative on each Segal–Bargmann space $H^2_s(\mathbb{C}^n)$ endowed with the standard Gaussian measures, $\mu_s(z) = (\frac{s}{\pi})^n \mathrm{e}^{-s|z|^2}$, and parametrized by $s > 0$. More precisely, and analogously to the case of the unit ball, we provide symbol classes of k-quasi-homogeneous functions for which each class generates commutative Toeplitz algebras on $H^2_s(\mathbb{C}^n)$.

Starting by a fixed tuple $k = (k_1, \cdots, k_m)$ of positive integers with $|k| = n$ and $k_1 \leqslant k_2 \leqslant \cdots \leqslant k_m$, we consider any other tuple $h = (h_1, \cdots, h_m) \in \mathbb{N}_0^m$ with $h \leq k - 1$ and satisfying

$$h_j \geq \begin{cases} 1 & \text{if } k_j \neq 1, \\ h_l & \text{if } j > l \text{ and } k_j = k_l. \end{cases} \tag{1}$$

Let $p = (p_{(1)}, \cdots, p_{(m)})$, $q = (q_{(1)}, \cdots, q_{(m)}) \in \mathbb{N}^{k_1} \times \cdots \times \mathbb{N}^{k_m}$, where

$$p_{(j)} = (p_{j,1}, \cdots, p_{j,h_j}, 0 \cdots, 0) \text{ and } q_{(j)} = (0, \cdots, 0, q_{j,h_j+1}, \cdots, q_{j,k_j}) \tag{2}$$

satisfies

$$|p_{(j)}| = |q_{(j)}|. \tag{3}$$

Denote by $\mathcal{R}_k$ the space of all measurable k-quasi-radial functions, i.e. those depending only on their radial components. More precisely, a measurable function $\varphi \in \mathcal{R}_k$ if $\varphi(z) = \varphi(r_1, \cdots, r_m)$, where

$$z_{(j)} = r_j \xi_{(j)} \in \mathbb{C}^{k_j}, r_j = |z_{(j)}| \text{ and } \xi_{(j)} \in \mathbb{S}^{2k_j - 1}.$$

We are now able to state the first result of this note.

Theorem 1.1. *Let $k = (k_1, \cdots, k_m)$ be a tuple of positive integers with $|k| = n$ and $k_1 \leqslant k_2 \leqslant \cdots \leqslant k_m$. Fix another tuple $h \in \mathbb{N}_0^m$ satisfying (1). Consider the following class of k-quasi-homogeneous symbols*

$$\mathcal{R}_k(h) = \left\{ \varphi \xi^p \overline{\xi}^q \mid \varphi \in \mathcal{R}_k \cap L^\infty; \, p, q \text{ satisfies (2) and (3)} \right\}.$$

Then the Toeplitz operators with symbols in $\mathcal{R}_k(h)$ generate a commutative Banach algebra of Toeplitz operators in $\mathcal{L}(H^2_s(\mathbb{C}^n))$. Moreover, for $k \neq (1, \cdots, 1)$ these algebras are not $C^\star$-algebras.

The second result concerns with the construction of commutative $C^\star$-algebras of the so-called "true-k-Toeplitz operators", where k is an n-tuple of positive integers. These operators act on the true-k-Fock spaces $F^2_{(k)}$ over $\mathbb{C}^n$ (cf. [12] for the properties of these spaces) and they are Toeplitz operators when $|k| = n$. For a bounded measurable function h on $\mathbb{C}^n$, we introduce the true-k-Toeplitz operator $T_h^{(k)}$ to be defined on $F^2_{(k)}$ as follows

$$T_h^{(k)} := P_{(k)} M_h, \tag{4}$$

where $P_{(k)}$ is the orthogonal projection from $L^2(\mathbb{C}^n, \mathrm{d}\mu_1)$ onto $F^2_{(k)}$ and M_h is the multiplication operator by the symbol h.

Concerning the operators defined by (4), we have the following commutative result.

Theorem 1.2. *Let $k=(k_1,k_2,\cdots,k_n)\in\mathbb{N}^n$ and $\mathcal{H}$ be a linear subspace of $\mathbb{R}^n$. Then the $C^\star$-algebra generated by the set*

$$\left\{T^{(k)}_{a(A(x))\mathrm{e}^{iu\cdot y}} \mid a\in L^\infty(\mathbb{R}^n),\ A\in\mathcal{L}(\mathbb{R}^n) \text{ with } \ker A=\mathcal{H} \text{ and } u\in\mathcal{H}\right\} \tag{5}$$

is commutative. In particular, when $|k|=n$, we obtain a $C^\star$-algebra generated by Toeplitz operators and being commutative on each Segal–Bargmann space $H^2_s(\mathbb{C}^n)$.

It should be mentioned that whenever $\mathcal{H}=0$, we obtain the horizontal symbols as introduced in [3]. The authors proved that Toeplitz operators on the Segal–Bargmann space with such type of symbols is unitary equivalent to a multiplication operator on $L^2(\mathbb{R}^n)$. The result was then generalized to poly-Fock and true-poly-Fock spaces in [10], where the Toeplitz operators are unitary equivalent to a matrix multiplication operator. Moreover, the authors provided an explicit description for the $C^\star$-algebra generated by such symbols. For the case of bounded vertical symbols over poly-Bergman and true-poly Bergman spaces in the upper half plane, the $C^\star$-algebra was also characterized in [9].

In our case, we show that the operators in (5) are unitary equivalent to a composition of a shift and a multiplication operator on $L^2(\mathbb{R}^n)$.

2. Sketch of the proof of Theorem 1.1

We present a commutativity result between Toeplitz operators with k-quasi-homogeneous symbols having some exponential growth at infinity and then provide a proof for Theorem 1.1. Following a usual extension argument for densely bounded operators, we are interested in Toeplitz operators whose domain includes the space of all holomorphic polynomials $\mathbb{P}[z]$ on $\mathbb{C}^n$. For a fixed $s>0$, we thus introduce the following symbol class

$$\mathcal{E}_s := \left\{g:\mathbb{C}^n\longrightarrow\mathbb{C} \mid \text{ there are } c,d>0 \text{ with } c<\frac{s}{2} \text{ and } |g(z)|\leqslant d\,\mathrm{e}^{c|z|^2}\right\}.$$

Given a measurable function $g\in\mathcal{E}_s$, it is easy to check that T^s_g is well defined on $\mathbb{P}[z]$. In particular, we obtain a densely defined operator T^s_g on $H^2_s(\mathbb{C}^n)$.

Using Prop. 1.4.9 in [11], we apply Toeplitz operators with k-quasi-homogeneous symbols to the monomials and we get:

Lemma 2.1. *Let $\varphi\in\mathcal{R}_k\cap\mathcal{E}_s$ then for each $\alpha\in\mathbb{N}_0^n$ the Toeplitz operator $T^s_{\varphi\xi^p\bar{\xi}^q}$ fulfills:*

$$T^s_{\varphi\xi^p\bar{\xi}^q}z^\alpha = \begin{cases}\delta_{\varphi,k,p,q,s}(\alpha)z^{\alpha+p-q} & \alpha+p-q\in\mathbb{N}_0^n\\ 0 & \textit{else},\end{cases}$$

where

$$\delta_{\varphi,k,p,q,s}(\alpha)=\frac{2^m s^{n+|\alpha+p-q|}(\alpha+p)!}{(\alpha+p-q)!\prod_{j=1}^m(k_j-1+|\alpha_{(j)}+p_{(j)}|)!}\int_{\mathbb{R}^m}\varphi(r_1,\cdots,r_m)\prod_{j=1}^m r_j^{|2\alpha_{(j)}+p_{(j)}-q_{(j)}|+2k_j-1}\mathrm{e}^{-sr_j^2}\,\mathrm{d}r_j.$$

The following commutative result follows by an application of the above lemma.

Proposition 2.1. *Let $p,q,u,v\in\mathbb{N}_0^n$ be such that $p\cdot q=0$, $u\cdot v=0$ and*

$$|p_{(j)}|=|q_{(j)}|,\quad |u_{(j)}|=|v_{(j)}| \quad \textit{for all } j=1,\cdots,m.$$

Suppose that, for each $l=1,\cdots,n$ one of the following conditions holds:

- $p_l=q_l=0$,
- $u_l=v_l=0$,
- $p_l=u_l=0$,
- $q_l=v_l=0$.

Then given $\varphi,\psi\in\mathcal{R}_k\cap\mathcal{E}_s$, the Toeplitz operators $T^s_{\varphi\xi^p\bar{\xi}^q}$ and $T^s_{\psi\xi^u\bar{\xi}^v}$ commute on $\mathbb{P}[z]$.

Theorem 1.1 then follows by noting that symbols in $\mathcal{R}_k(h)$ satisfy the conditions in the above proposition.

3. Sketch of the proof of Theorem 1.2

Following the notation in [12], we introduce the operator $\tilde{R}_{(k)} := R^{\star}_{(k)} U$ on $L^2(\mathbb{C}^n, \mathrm{d}\mu_1)$. By slightly modifying the arguments in [12] one can prove that

Proposition 3.1. *The restriction*

$$\tilde{R}_{(k)} | F^2_{(k)}(\mathbb{C}^n) : F^2_{(k)}(\mathbb{C}^n) \longrightarrow L^2(\mathbb{R}^n, \mathrm{d}x)$$

and the adjoint operator

$$\tilde{R}^{\star}_{(k)} = U^{\star} R_{(k)} : L^2(\mathbb{R}^n, \mathrm{d}x) \longrightarrow F^2_{(k)}(\mathbb{C}^n)$$

are isometric isomorphisms. Furthermore, it holds

$$\tilde{R}_{(k)} \tilde{R}^{\star}_{(k)} = I : L^2(\mathbb{R}^n, \mathrm{d}x) \longrightarrow L^2(\mathbb{R}^n, \mathrm{d}x),$$

and

$$\tilde{R}^{\star}_{(k)} \tilde{R}_{(k)} = P_{(k)} : L^2(\mathbb{C}^n, \mathrm{d}\mu) \longrightarrow F^2_{(k)}(\mathbb{C}^n).$$

Applying the above isometric isomorphism, it is easy to check that the true-k-Toeplitz operator $T^{(k)}_h$ on $F^2_{(k)}(\mathbb{C}^n)$ is unitary equivalent to the following operator on $L^2(\mathbb{R}^n, \mathrm{d}x)$

$$S_h = R^{\star}_{(k)} U M_h U^{\star} R_{(k)}.$$

In the below, we note that, for special types of symbols, the above operator is a composition of a shift and of a multiplication operator on $L^2(\mathbb{R}^n, \mathrm{d}x)$:

Proposition 3.2. *Let $k = (k_1, k_2, \cdots, k_n)$ be a tuple of positive integers and consider a bounded measurable function*

$$\theta(z) = \theta(x, y) = a(A(x))\, \mathrm{e}^{\mathrm{i}u \cdot y},$$

where $u \in \mathbb{R}^n$ and A is an endomorphism of $\mathbb{R}^n$. Then, for any $\psi \in L^2(\mathbb{R}^n, \mathrm{d}x)$, we have

$$(S_\theta \psi)(x) = \psi(x - \frac{u}{\sqrt{2}}) \int_{\mathbb{R}^n} a\big(A(\frac{x+y}{\sqrt{2}})\big) \tilde{h}_{k-\mathbf{1}}(y + \frac{u}{\sqrt{2}}) \tilde{h}_{k-\mathbf{1}}(y)\, \mathrm{d}y,$$

where $\tilde{h}_{k-\mathbf{1}}$ is a product of Hermite polynomials as denoted in [12].

Applying the above proposition, we obtain commutative true-k-Toeplitz operators.

Proposition 3.3. *Consider two bounded functions on $\mathbb{C}^n$*

$$\theta(z) = \theta(x, y) = a\big(A(x)\big)\, \mathrm{e}^{\mathrm{i}u \cdot y} \quad \text{and} \quad \gamma(z) = \gamma(x, y) = b\big(B(x)\big)\, \mathrm{e}^{\mathrm{i}t \cdot y},$$

where A and B are endomorphisms of $\mathbb{R}^n$, $t \in \ker A$ and $u \in \ker B$. Then $T^{(k)}_\theta$ and $T^{(k)}_\gamma$ commute on $F^2_{(k)}(\mathbb{C}^n)$.

Proof. The operator $T^{(k)}_\theta T^{(k)}_\gamma$ is unitary equivalent to $S_\theta S_\gamma$ and is given by

$$\begin{aligned}(S_\theta S_\gamma \varphi)(x) &= (S_\gamma \varphi)(x - \frac{u}{\sqrt{2}}) \int_{\mathbb{R}^n} a(A(\frac{x+y}{\sqrt{2}})) \tilde{h}_{k-\mathbf{1}}(y + \frac{u}{\sqrt{2}}) \tilde{h}_{k-\mathbf{1}}(y)\, \mathrm{d}y \\ &= \varphi(x - \frac{u}{\sqrt{2}} - \frac{t}{\sqrt{2}}) \int_{\mathbb{R}^n} b(B(\frac{x - \frac{u}{\sqrt{2}} + v}{\sqrt{2}})) \tilde{h}_{k-\mathbf{1}}(v + \frac{t}{\sqrt{2}}) \tilde{h}_{k-\mathbf{1}}(v)\, \mathrm{d}v \int_{\mathbb{R}^n} a(A(\frac{x+y}{\sqrt{2}})) \tilde{h}_{k-\mathbf{1}}(y + \frac{u}{\sqrt{2}}) \tilde{h}_{k-\mathbf{1}}(y)\, \mathrm{d}y. \quad (6)\end{aligned}$$

Similarly, the operator $T^{(k)}_\gamma T^{(k)}_\theta$ is unitary equivalent to the operator $S_\gamma S_\theta$ on $L^2(\mathbb{R}^n, \mathrm{d}x)$:

$$(S_\gamma S_\theta \varphi)(x) = (S_\theta \varphi)(x - \frac{t}{\sqrt{2}}) \int_{\mathbb{R}^n} b(B(\frac{x+v}{\sqrt{2}}))\tilde{h}_{k-\mathbf{1}}(v + \frac{t}{\sqrt{2}})\tilde{h}_{k-\mathbf{1}}(v)\,\mathrm{d}v$$

$$= \varphi(x - \frac{t}{\sqrt{2}} - \frac{u}{\sqrt{2}}) \int_{\mathbb{R}^n} a(A(\frac{x - \frac{t}{\sqrt{2}} + y}{\sqrt{2}}))\tilde{h}_{k-\mathbf{1}}(y + \frac{u}{\sqrt{2}})\tilde{h}_{k-\mathbf{1}}(y)\,\mathrm{d}y \int_{\mathbb{R}^n} b(B(\frac{x+v}{\sqrt{2}}))\tilde{h}_{k-\mathbf{1}}(v + \frac{t}{\sqrt{2}})\tilde{h}_{k-\mathbf{1}}(v)\,\mathrm{d}v. \quad (7)$$

It follows that Eqs. (6) and (7) are equal whenever $t \in \ker A$ and $u \in \ker B$. □

The proof of Theorem 1.1 follows by applying the above proposition to the symbols considered in (5).

4. Final remarks

In this section, we provide further results that are interesting in the analysis of Toeplitz operators.

Addressed to the problem of composition of Toeplitz operators, we note that the product of two such operators is rarely a Toeplitz operator. Concerning the commutative Banach algebra obtained in Theorem 1.1, it turns out that the algebra is formed only by Toeplitz operators and a composition formula for the symbols is obtained.

Corollary 4.1. *Let $k = (k_1, \cdots, k_m)$ be a tuple of positive integers with $|k| = n$ and $k_1 \leqslant k_2 \leqslant \cdots \leqslant k_m$. Given $p, q \in \mathbb{N}_0^n$ orthogonal multi-indices with $|p_{(j)}| = |q_{(j)}|$ for all $j = 1, \cdots, m$. Then, for any $\varphi \in \mathcal{R}_k \cap \mathcal{E}_s$, the following equality holds true on $\mathbb{P}[z]$:*

$$T^s_\varphi T^s_{\xi^p \overline{\xi}^q} = T^s_{\varphi \xi^p \overline{\xi}^q}.$$

In particular, by fixing another tuple h satisfying (1) we obtain the following composition formula

$$T^s_{\varphi \xi^p \overline{\xi}^q} T^s_{\psi \xi^u \overline{\xi}^v} = T^s_{\varphi \psi \xi^{(p+u)} \overline{\xi}^{(q+v)}},$$

for any $\varphi \xi^p \overline{\xi}^q, \psi \xi^u \overline{\xi}^v \in \mathcal{R}_k(h)$.

Concerning the commutative $C^\star$-algebra obtained in Theorem 1.2, one can choose $k = (1, \cdots, 1)$ to obtain a commutative $C^\star$-algebra generated by Toeplitz operators on $H^2_1(\mathbb{C}^n)$. Using a suitable change of variable, the commutativity result holds true on every Segal Bergman space $H^2_s(\mathbb{C}^n)$:

Corollary 4.2. *Let $\mathcal{H}$ be a linear subspace of $\mathbb{R}^n$; then the $C^\star$-algebra generated by the Toeplitz operators*

$$\{T^s_{a(A(x))\,\mathrm{e}^{\mathrm{i}u\cdot y}} \mid a \in L^\infty(\mathbb{R}^n),\ A \in \mathcal{L}(\mathbb{R}^n) \text{ with } \ker A = \mathcal{H} \text{ and } u \in \mathcal{H}\}$$

is commutative.

Finally, we note that one can combine the results of Theorem 1.1 and Corollary 4.2 to form a more general commutative Banach algebra generated by Toeplitz operators.

Acknowledgements

Most of the above-mentioned results were obtained during my doctoral studies in Georg-August University of Göttingen back in 2012. A detailed proof of the results are presented in the dissertation (cf. Ch. 3 in [6]). I take this opportunity to express my heartfelt thanks to my supervisor Prof. Wolfram Bauer, who has been most generous with his time and ideas during those years. The paper is dedicated to Ibn Al-Hassan Al-Moaammal and to my family.

References

[1] W. Bauer, N. Vasilevski, Banach algebras of commuting Toeplitz operators on the unit ball via the quasi-hyperbolic group, Oper. Theory, Adv. Appl. 218 (2012) 155–175.
[2] W. Bauer, N. Vasilevski, Commutative Toeplitz Banach algebras on the ball and quasi-nilpotent group action, Integral Equ. Oper. Theory 72 (2) (2012) 223–240.
[3] K. Esmeral, N. Vasilevski, $C^\star$-algebra generated by horizontal Toeplitz operators on the Fock space, Bol. Soc. Mat. Mexicana 22 (2016) 567–582.
[4] S. Grudsky, R. Quiroga-Barranco, N. Vasilevski, Commutative C^*-algebras of Toeplitz operators and quantization on the unit disc, J. Funct. Anal. 234 (2006) 1–44.
[5] H. Issa, Compact Toeplitz operators for weighted Bergman spaces on bounded symmetric domains, Integral Equ. Oper. Theory 70 (2011) 569–582.
[6] H. Issa, The Analysis of Toeplitz Operators, Commutative Toeplitz Algebras and Applications to Heat Kernel Constructions, Ph.D. thesis, Georg-August Universität, Göttingen, Germany, 2012, Handle: 11858/00-1735-0000-000D-F066-5.
[7] R. Quiroga-Barranco, A. Sánchez-Nungaray, Toeplitz operators with quasi-radial quasi-homogeneous symbols and bundles of Lagrangian frames, J. Oper. Theory 71 (1) (2014) 199–222.

[8] R. Quiroga-Barranco, A. Sánchez-Nungaray, Toeplitz operators with quasi-homogeneous quasi-radial symbols on some weakly pseudoconvex domains, Complex Anal. Oper. Theory 9 (2015) 1111–1134.
[9] J. Ramírez-Ortega, A. Sánchez-Nungaray, Toeplitz operators with vertical symbols acting on the poly-Bergman spaces of the upper half-plane, Complex Anal. Oper. Theory 9 (8) (2015) 1801–1817.
[10] A. Sánchez-Nungaray, C. González-Flores, R.R. López-Martínez, J.L. Arroyo-Neri, Toeplitz operators with horizontal symbols acting on the poly-Fock spaces, J. Funct. Spaces 2018 (2018) 8031259.
[11] W. Rudin, Function Theory in the Unit Ball of $\mathbb{C}^n$, A Series of Comprehensive Studies in Mathematics, Springer, 1991.
[12] N. Vasilevski, Poly-Fock spaces, Oper. Theory, Adv. Appl. 117 (2000) 371–386.
[13] N. Vasilevski, Parabolic quasi-radial quasi-homogeneous symbols and commutative algebras of Toeplitz operators, Oper. Theory, Adv. Appl. 202 (2010) 553–568.
[14] N. Vasilevski, Quasi-radial quasi-homogeneous symbols and commutative Banach algebras of Toeplitz operators, Integral Equ. Oper. Theory 66 (2010) 141–152.
[15] N. Vasilevski, On Toeplitz operators with quasi-radial and pseudo-homogeneous symbols, in: M. Pereyra, S. Marcantognini, A. Stokolos, W. Urbina (Eds.), Harmonic Analysis, Partial Differential Equations, Banach Spaces, and Operator Theory, vol. 2, in: Association for Women in Mathematics Series, vol. 5, Springer, Cham, Switzerland, 2017.