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A new time domain estimation of k-factors GARMA processes

Estimation du minimum de distance de Hellinger dans les processus GARMA à k facteurs

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ABSTRACT

We address the problem of parameter estimation of long memory time series. We consider k-factors Gegenbauer Autoregressive Moving Average (k-GARMA) processes and we estimate their parameters by the minimum Hellinger distance estimator. We establish the consistency of the estimator and the asymptotic normality for some bandwidth choice.

RÉSUMÉ

Nous étudions le problème d'estimation dans les séries temporelles fortement dépendantes. Nous considérons les processus Gegenbaeur autorégressifs à moyenne mobile (GARMA) à k facteurs pour les modéliser et nous estimons leurs paramètres par la méthode du minimum de distance de Hellinger. Nous établissons la consistance de l'estimateur et la normalité asymptotique pour un certain choix de fenêtre de lissage.

1. Introduction

We consider a more general class of long memory models for time series, the k-factor GARMA model. First introduced by Gray et al. [3], these processes have the ability in providing a good characterization of both cyclical and long memory behavior of many time series.

Definition 1. The process $(X_t)_{t\in Z}$ is a k-factor GARMA process if it can be written as:

$$\phi(B)\prod_{i=1}^{k}\left(I-2\nu_i B+B^2\right)^{d_i}(X_t-\mu)=\theta(B)\varepsilon_t \tag{1}$$

where k is an integer, ε_t is a white noise sequence with variance σ^2 such that $E(\varepsilon_t^4)\leqslant\infty$, μ is the mean of the process, $|\nu_i|\leqslant 1$ for $i=1,\dots,k$, d_i is a long-memory parameter and $\phi(B)$ and $\theta(B)$ are autoregressive and moving average polynomials of degrees p and q respectively.

For $i=1,\dots,k$, $\lambda_i=\cos^{-1}(\nu_i)$ are the Gegenbauer frequencies or G-frequencies. For the sake of simplicity and without loss of generality, in the remainder of the paper we assume that $\mu=0$.

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An estimation procedure for the parameters is proposed in this paper. For the parametric estimation approach, we refer to Kouamé and Hili [6], and the references therein. The estimator introduced here belongs to the Minimum Hellinger Distance (MHD) class. This type of estimator was first introduced by Beran [1] for independent series and was studied by some authors such as Hili [4] for dependent series under mixing conditions.

Relative to other time domain approaches, such as maximum likelihood estimation, it presents the additional advantage that it is not necessary to specify a particular distribution for the innovation process. Also, it is robust (see Beran [1], Hili [4,5]).

The invertibility and stationarity conditions are established and proved by Woodward et al. [8] and Giraitis and Leipus [2].

Let $\psi = (d, \nu, \phi, \theta) \in \Psi \subset R^{2k+p+q}$ be the vector of parameters of interest where Ψ is a compact set and ψ_0 the vector of the true values. Let now consider a stationary and invertible k-factor GARMA process X_t and f_ψ be the probability density of that process. Let $\hat{f}_n$ be a nonparametric density estimate computed from $X_1, \ldots, X_n$. It is obtained by using the kernel density estimator

$$\hat{f}_n(u) = \frac{1}{nh} \sum_{t=1}^{n} K\left(\frac{u - X_t}{h}\right) \tag{2}$$

where $h = h_n$ is a sequence of bandwidths ($h \to 0$, $nh \to \infty$) and K a kernel function.

Then we define $\hat{\psi}_n$ as the value of $\psi \in \Psi$ which minimizes the Hellinger distance between $\hat{f}_n$ and f_ψ; that is,

$$\hat{\psi}_n = \arg\min_{\psi \in \Psi} H_2(\hat{f}_n, f_\psi) \tag{3}$$

where

$$H_2(\hat{f}_n, f_\psi) = \left(\int_{\Re} \left|\hat{f}_n^{\frac{1}{2}}(u) - f_\psi^{\frac{1}{2}}(u)\right|^2 \mathrm{d}u\right)^{\frac{1}{2}}$$

2. Hypothesis and asymptotic properties

To establish the asymptotic properties of $\hat{\psi}_n$, we need some assumptions:

(C1) $\psi_1 \neq \psi_2$ implies $f_{\psi_1} \neq f_{\psi_2}$ on a set of positive Lebesgue measure.

(C2) (i) $f_\psi(.)$ is uniformly continuous and 2 times continuously differentiable ($r \geqslant 2$);
(ii) $\sup_{x \in \mathbb{R}} |\frac{\partial}{\partial x} f_\psi(x)| < +\infty$.

(C3) (i) $K(.)$ is a bounded positive density function;
(ii) $\exists N_1 > 0\ \sup_u |K(u+v) - K(u)| \leqslant N_1 |v|$ for all $v \in \mathbb{R}$.

(C4) $\int_{\mathbb{R}} K(u)\,\mathrm{d}u = 1$, $\int_{\mathbb{R}} uK(u)\,\mathrm{d}u = 0$ and $\int_{\mathbb{R}} |u|^2 |K(u)|\,\mathrm{d}u < +\infty$.

(C5) $E(|\varepsilon_t|^4) < +\infty$.

Proposition 1. *Under assumptions* (C2)–(C5), *if the distribution function of ε_t is differentiable with bounded, continuous and integrable derivatives then, $\hat{f}_n(u)$ almost surely converges to $f_\psi(u)$ for all $u \in \mathbb{R}$ as $n \to \infty$.*

Sketch of proof. Let $d = \max(d_i)$ for $i = 1, \ldots, k$ and $h = n^{-\delta}$, $0 < \delta < (1 - 2d) \wedge 1/2$.

Then, for $\lambda < \delta \wedge [((1 - 2d) \wedge 1/2) - \delta]$, we show that

$$n^{\lambda} \sup_{u \in \mathbb{R}} \left|\hat{f}_n(u) - f_\psi(u)\right| \to 0 \quad \text{a.s.,} \quad n \to \infty \tag{4}$$

Theorem 1 *(Consistency). Let ψ_0 the true value of the parameter ψ be an interior point of the compact set Ψ. Then, under assumptions of Proposition* 1, *$\hat{\psi}_n$ converges a.s. to ψ_0 as $n \to \infty$.*

Sketch of proof. Let denote $\mathcal{F}$ the set of all densities with respect to the Lebesgue measure on $\mathbb{R}$. Define the functional $T : \mathcal{F} \to \Psi$ in the following:

Let $g \in \mathcal{F}$. Denote

$$A(g) = \left\{\psi \in \Psi:\ H_2(g, f_\psi) = \min_{\theta \in \Psi} H_2(g, f_\theta)\right\} \tag{5}$$

where H_2 is the Hellinger distance. If $A(g)$ is reduced to a unique element, then define $T(g)$ as the value of this element. Elsewhere we choose an arbitrary but unique element of these minimums and call it $T(g)$.

Table 1
Results of Monte Carlo simulations for performance of MDH estimator.

d_0	$B(\hat{d})$	$RMSE(\hat{d})$
$T = 300$		
−0.2	−0.005	0.0104
0.4	0.0188	0.0334

From Proposition 1 $H_2(\hat{f}_n, f_\psi) = (\int_{\Re} |\hat{f}_n^{\frac{1}{2}}(u) - f_\psi^{\frac{1}{2}}(u)|^2 \, du)^{\frac{1}{2}} \to 0$ a.s. when $n \to \infty$. And from the continuity of the functional T, we deduce that $\hat{\psi}_n = T(\hat{f}_n) \to T(f_{\psi_0}) = \psi_0$ a.s., $n \to \infty$.

In the following, we discuss the asymptotic distribution of the kernel estimate.

Proposition 2. *Assume that f_1 the probability density function of ε_t is Lipschitz and three times differentiable with bounded, continuous and integrable derivatives. Under conditions* (C2), (C4) *and* (C5); *let assume that $h = Cn^\alpha$ where $C > 0$ is a constant and $-1 < \alpha < 0$. Then if $\alpha < 2d$,*

$$(nh)^{\frac{1}{2}}\big[\hat{f}_n(x) - f_\psi(x)\big] \quad \Rightarrow \quad N\bigg[0, f_\psi(x)\int_R K^2(s)\,ds\bigg] \tag{6}$$

Sketch of proof. We derive this proposition from Theorem 2 and Corollary 1 of Wu and Mielniczuk [9].

First we prove that $\frac{nh}{\sqrt{nh}}[\hat{f}_n(x) - E\hat{f}_n(x)] = \frac{M_n}{\sqrt{nh}} + o_p(1)$ where for $h \to 0$ and $nh \to +\infty$, $(nh)^{-\frac{1}{2}} M_n \Rightarrow N(0, \sigma^2(x))$ with $\sigma^2(x) = f_\psi(x) \int K^2(s)\,ds$.

Then we show that $\frac{nh}{\sqrt{nh}}[E\hat{f}_n(x) - f_\psi(x)] \to 0$, $n \to \infty$; which yield Proposition 2.

Theorem 2 *(Asymptotic distribution). Assuming that conditions of Propositions* 1 *and* 2 *are satisfied, then the limiting distribution of $n^{1/2}(\hat{\psi}_n - \psi)$ is $N(0, \Sigma^2)$ where*

$$\Sigma^2 = \frac{1}{4}\bigg[\int \dot{g}_\psi(x)\dot{g}'_\psi(x)\bigg]^{-1} \int K^2(u)\,du \tag{7}$$

where $g_\psi = f_\psi^{1/2}$; $\dot{g}_\psi = \frac{\partial g_\psi}{\partial \psi}$ and $(')$ the transpose.

Sketch of proof. From Theorem 2 in Beran [1], the limit law of $(\hat{\psi}_n - \psi)$, is that of $\int V_\psi(u)[\hat{f}_n^{\frac{1}{2}}(u) - f_\psi^{\frac{1}{2}}(u)]\,du$; where $V_\psi(u) = [\int \dot{g}_\psi(u)\dot{g}'_\psi(u)\,du]^{-1}\dot{g}_\psi(u)$.

By using the following algebraic identity from $f_\psi > 0$: $\hat{f}_n^{1/2} - f_\psi^{1/2} = \frac{\hat{f}_n - f_\psi}{2f_\psi^{1/2}} - \frac{(\hat{f}_n - f_\psi)^2}{2f_\psi^{1/2}(\hat{f}_n^{1/2} + f_\psi^{1/2})^2}$ and Proposition 1, we have now to determine the limit law of: $n^{1/2}\int \frac{V_\psi(u)}{2f_\psi^{1/2}(u)}[\hat{f}_n(u) - f_\psi(u)]\,du$.

From Proposition 2, this limit law is $N(0, \Sigma^2)$ where: $\Sigma^2 = \frac{1}{4}\int V_\psi(u)(V_\psi(u))'\,du \int K^2(s)\,ds$. Which proves Theorem 2.

3. Numerical simulations

In this section we did some numerical simulations for minimum Hellinger distance estimator (MDH) to show its performance. We generate two long memory processes, GARMA$(0, d, \nu, 0)$, stationary and invertible for $\nu = 0.8$ and $d \in \{-0.2, 0.4\}$, the process $(\varepsilon_t \sim N(0, \sigma_\varepsilon))$ where $\sigma_\varepsilon^2 = 1$.

We choose sample length $T = 300$. For each model we make 100 independent replications. Now, notice that the model density f_ψ analytical expression is intractable. So following the method of Takada [7], we will replaced the density f_ψ by the nonparametric estimator $\tilde{f}^*_{\psi,n}$ defined as follows:

Let $(\tilde{X}_1^s(\psi), \ldots, \tilde{X}_n^s(\psi))$ be the s-th replication of the simulated sequence from the model GARMA, $s = 1, \ldots, S$. That is the simulated sequence has length $S \times T$ (choose here $S = 100$)

$$\tilde{f}^*_{\psi,n}(u) = \frac{1}{S}\sum_{s=1}^{S}\bigg[\frac{1}{nh}\sum_{t=1}^{n} K\bigg(\frac{u - \tilde{X}_t^s(\psi)}{h}\bigg)\bigg]$$

We estimate only parameter d assuming $\nu = 0.8$ known. We calculate the bias and the root mean square error (RMSE). Results are in Table 1.

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