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Functional Analysis

Corrigendum to the Note “A Striktpositivstellensatz for measurable functions” [C. R. Acad. Sci. Paris, Ser. I 347 (7–8) (2009) 381–384]

Mihai Putinar

Mathematics Department, University of California, Santa Barbara, CA 93106, USA

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The main result and its corollary need an additional hypothesis, as follows:

Theorem 1. *Let Q be a countably generated Archimedean quadratic module contained in the algebra $\mathcal{A} = \mathbb{R}[x_1, \dots, x_d, h_1, \dots, h_m]$ spanned by the coordinate functions and by Borel measurable functions $h_1, \dots, h_m$ on $\mathbb{R}^d$. Assume that Q has the moment property, that is, every linear functional on $\mathcal{A}$ which is non-negative on Q is representable by a positive measure. If a function $f \in \mathcal{A}$ is positive on $P(Q)$, then $f \in Q$.*

Similarly,

Corollary 2. *Let $q_1, \dots, q_n$ be elements of the algebra $\mathcal{A} = \mathbb{R}[x_1, \dots, x_d, h_1, \dots, h_m]$ generated by the coordinate functions and by Borel measurable functions $h_1, \dots, h_m$ on $\mathbb{R}^d$. Let $\Sigma\mathcal{A}^2$ denote the convex cone of sums of squares, and consider the Borel measurable set*

$$P(q_0, q_1, \dots, q_n) = \{x \in \mathbb{R}^d;\ q_i(x) \geqslant 0,\ 0 \leqslant i \leqslant n\},$$

where $q_0(x) = 1 - (x_1^2 + \dots + x_d^2 + h_1^2 + \dots + h_m^2)$.

If a function $f \in \mathcal{A}$ is positive on $P(q_0, q_1, \dots, q_n)$, then $f \in Q$, provided that the quadratic module $Q = \Sigma\mathcal{A}^2 + q_0\Sigma\mathcal{A}^2 + \dots + q_n\Sigma\mathcal{A}^2$ possesses the moment property.

In its turn, the moment property of the cone Q can be restated as a density of Q in the set of all elements of $\mathcal{A}$ which are non-negative on $P(Q)$, in the strongest locally convex topology carried by $\mathcal{A}$.

For details see: Jean-Bernard Lasserre, Mihai Putinar, Positivity and optimization for semi-algebraic functions, arXiv: 0910.5250.

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E-mail address: mputinar@math.ucsb.edu.
URL: http://www.math.ucsb.edu/~mputinar.

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