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A new Carleman inequality for parabolic systems with a single observation and applications

Une nouvelle inégalité de Carleman pour des systèmes paraboliques avec une seule observation et applications

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ABSTRACT

In this Note, we present Carleman estimates for linear reaction–diffusion–convection systems of two equations and linear reaction–diffusion systems of three equations. These estimates are the key for proving controllability results for semilinear reaction–diffusion–convection systems of order 2 and reaction–diffusion systems of order 3. They allow us to derive results for identification of n coefficients by $(n-2)$ observations.

RÉSUMÉ

On établit des inégalités d'observabilité pour des systèmes linéaires de réaction–diffusion–convection d'ordre 2 et des systèmes linéaires de réaction–diffusion d'ordre 3, basées sur des estimations de Carleman. Elles permettent de démontrer des résultats de contrôlabilité aux trajectoires par une seule force localisée en espace ainsi que des résultats d'identification de n coefficients par $(n-2)$ observations.

Version française abrégée

La contrôlabilité de systèmes paraboliques est un sujet relativement nouveau. Les principaux résultats ont été obtenus dans [10,2,1,8,3]. La question reste largement ouverte. Ce travail généralise les résultats de [9] dans le cas de deux équations de réaction–diffusion–convection. Il démontre, à notre connaissance, le premier résultat de contrôlabilité de trois équations de réaction–diffusion (par une seule force localisée). Il est basé sur l'obtention d'une nouvelle inégalité de Carleman (8). On obtient les résultats suivants :

Observabilité du système (3). *Sous les hypothèses du Théorème* 1.1, *il existe une fonction* β, *une constante* $K>0$ (*voir* (2)) *et des constantes positives* s_0, C, *telles que pour tout* $u_0, v_0, f, g \in L^2(\Omega)$ *et* $|\tau_1-\tau_2|<1$ *l'estimation de Carleman* (4) *ait lieu pour tout* $s \geqslant s_0$ *et tout* (u, v) *solutions de* (3).

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Observabilité du système (5). *Sous les hypothèses du Théorème* 1.2*, il existe une fonction positive* $\beta \in C^2(\overline{\Omega})$*, une constante* K *et deux constantes positives* s_0, C*, telles que pour tout* $u_0, v_0, w_0, f, g, h \in L^2(\Omega)$ *et* $\tau \in \mathbb{R}$ *l'estimation de Carleman* (6) *ait lieu pour tout* $s \geqslant s_0$ *et toutes les solutions* (u, v, w) *de* (5)*.*

1. Introduction and main results

The subject of controllability of parabolic systems is a relatively new subject. Some of the main results have been obtained in [10,2,1,8,3]. Many issues yet remain open. In this article we generalize the results of [4,9] which address the case of two reaction–diffusion–convection equations. We derive global Carleman estimates for a reaction–diffusion–convection system of two equations and for reaction–diffusion systems of three equations with the observation of one of the three unknown functions. These estimates are the key in the proof of controllability results for two and three coupled parabolic equations as well as a result on the identification of coefficients for three coupled parabolic equations (see [5]). To be more precise, let $\Omega \subset \mathbb{R}^n$, $n \geqslant 1$ be a bounded connected open set of class C^2. Let $T > 0$ and let ω be a non-empty open subset of Ω. We define $\Omega_T = \Omega \times (0, T)$ and $\Sigma_T = \partial\Omega \times (0, T)$. Let us consider second-order elliptic-selfadjoint operators given by $\operatorname{div}(H_l \nabla) = \sum_{i,j=1}^n \partial_i (h_{ij}^l(x)\partial_j)$ for $l = 1, 2$ and a positive constant h_0, with

$$h_{ij}^l \in W^{1,\infty}(\Omega), \quad h_{ij}^l(x) = h_{ji}^l(x) \quad \text{a.e. in } \Omega, \quad \text{and} \quad \sum_{i,j=1}^n h_{ij}^l(x)\xi_i\xi_j \geqslant h_0|\xi|^2, \quad \forall \xi \in \mathbb{R}^n. \tag{1}$$

Following [7], given $\omega \subset \Omega$, $K > 0$ to be defined below, let us introduce $\beta \in C^2(\overline{\Omega})$ such that $\beta > 0$ in $\bar{\Omega}$, $|\nabla\beta| > 0$ in $\Omega \setminus \omega$,

$$\begin{aligned}
&\eta(x,t) := \frac{e^{2\lambda K} - e^{\lambda\beta(x)}}{t(T-t)}, \quad \forall (x,t) \in \Omega_T, \qquad \rho(t) := \frac{e^{\lambda\beta(x)}}{t(T-t)}, \quad \forall (x,t) \in \Omega_T,\\
&\eta^* = \max_{\overline{\Omega}} \eta, \qquad \eta_- = \min_{\overline{\Omega}} \eta, \qquad \alpha = 4\eta_- - 3\eta^*, \qquad \rho^* = \max_{\overline{\Omega}} \rho,\\
&I(\tau,\varphi) = \int_{\Omega_T} (s\rho)^{\tau-1} e^{-2s\eta} \Bigg(|\varphi_t|^2 + \sum_{1 \leqslant i \leqslant j \leqslant n} \big|\partial^2_{x_i x_j}\varphi\big|^2 + (s\lambda\rho)^2|\nabla\varphi|^2 + (s\lambda\rho)^4|\varphi|^2 \Bigg)\, dx\, dt.
\end{aligned} \tag{2}$$

• Let $a, b, c, d \in L^\infty(\Omega_T)$ and $A, C, D \in L^\infty(\Omega_T)^n$, $B \in L^\infty(\Omega)^n$. Let $u_0, v_0 \in L^2(\Omega)$, $f, g \in L^2(\Omega_T)$. We consider the following reaction–diffusion–convection system:

$$\begin{cases}
\partial_t u = \operatorname{div}(H_1 \nabla u) + a\,u + b\,v + A \cdot \nabla u + B \cdot \nabla v + f & \text{in } \Omega_T,\\
\partial_t v = \operatorname{div}(H_2 \nabla v) + cu + dv + C \cdot \nabla u + D \cdot \nabla v + g & \text{in } \Omega_T,\\
u(\cdot,t) = v(\cdot,t) = 0 & \text{on } \Sigma_T,\\
u(\cdot,0) = u_0(\cdot), \qquad v(\cdot,0) = v_0(\cdot) & \text{in } \Omega.
\end{cases} \tag{3}$$

Theorem 1.1. *Let us assume that* ω *is of class* C^2, $\omega \subset \Omega$ *is such that for some* $\gamma \subset \partial\Omega$, $|\gamma| \neq 0$ *with* $\gamma \subset \partial\omega \cap \partial\Omega$ *we have that* $|B(x) \cdot \nu(x)| \neq 0$, $x \in \gamma$, $B \in W^{2,\infty}(\omega)^n$, $A \in W^{1,\infty}(\omega_T)^n$ *and* $b \in W^{2,\infty}(\omega_T)$*. Then, there exist a positive function* $\beta \in C^2(\overline{\Omega})$*, a constant* $K > 0$ *(see* (2)*) and two positive constants* s_0, C*, such that for every* $(u_0, v_0) \in L^2(\Omega)^2$ *and* $|\tau_1 - \tau_2| < 1$*, the following Carleman estimate holds*:

$$\begin{aligned}
I(\tau_1, u) + I(\tau_2, v) \leqslant C\Bigg(&\int_{\omega_T} e^{-2s\alpha} \big(s\rho^*\big)^{\tau^*} |u|^2\, dx\, dt\\
&+ \int_{\omega_T} e^{-2s\eta}(s\rho)^{3+\tau_2} |Qf|^2\, dx\, dt + \int_{\Omega_T} e^{-2s\eta}\big((s\rho)^{\tau_1}|f|^2 + (s\rho)^{\tau_2}|g|^2\big)\, dx\, dt \Bigg)
\end{aligned} \tag{4}$$

for all $s \geqslant s_0$ *and for all* (u, v) *solution of* (3)*.* Q *is a bounded operator in* $L^2(\omega)$ *defined in* (10) *and* $\tau^* = 4\tau_2 - 3\tau_1 + 15$*.*

• Let $(a_{ij})_{1 \leqslant i,j \leqslant 3} \in L^\infty(\Omega_T)$. Let $H_l = (h_{ij}^l)_{1 \leqslant i,j \leqslant 3}$, $1 \leqslant l \leqslant 3$, be defined in (1). We consider the following 3×3 reaction–diffusion system

$$\begin{cases}
\partial_t u = \operatorname{div}(H_1 \nabla u) + a_{11}u + a_{12}v + a_{13}w + f & \text{in } \Omega_T,\\
\partial_t v = \operatorname{div}(H_2 \nabla v) + a_{21}u + a_{22}v + a_{23}w + g & \text{in } \Omega_T,\\
\partial_t w = \operatorname{div}(H_3 \nabla w) + a_{31}u + a_{32}v + a_{33}w + h & \text{in } \Omega,\\
u = v = w = 0 & \text{on } \Sigma_T,\\
u(\cdot,0) = u_0, \qquad v(\cdot,0) = v_0, \qquad w(\cdot,0) = w_0 & \text{in } \Omega.
\end{cases} \tag{5}$$

Theorem 1.2. *Suppose that there exists $j \in \{2,3\}$ such that $|a_{1j}(x,t)| \geqslant C > 0$ for all $(x,t) \in \omega_T$ and that $H_2 = H_3$. Let $k_j = \frac{6}{j}$, $B_{k_j} = -2H_2(\nabla a_{1k_j} - \frac{a_{1k_j}}{a_{1j}} \nabla a_{1j})$ and*

$$b_j = 2H_2 \nabla a_{1j} \left(\frac{\nabla a_{1k_j} a_{1j} - \nabla a_{1j} a_{1k_j}}{a_{1j}^2} \right) + \frac{a_{1k_j} \operatorname{div}(H_2 \nabla a_{1j})}{a_{1j}} - \operatorname{div}(H_2 \nabla a_{1k_j}) - \frac{a_{1k_j}^2 a_{k_j j} + a_{1j} a_{1k_j} a_{jj} - a_{1j} a_{1k_j} a_{k_j k_j} - a_{1j}^2 a_{j k_j}}{a_{1j}}.$$

Furthermore, we assume that either $B_{k_j}(x,t) = 0$ and $b_j(x,t) \neq 0$ on ω_T, or a_{12} and a_{13} are time independent, $\partial\omega \cap \partial\Omega = \gamma$, $|\gamma| \neq 0$, $a_{12}, a_{13} \in W^{4,\infty}(\omega)$ and $B_{k_j} \cdot \nu(x) \neq 0$, on γ. Then, there exist a positive function $\beta \in C^2(\overline{\Omega})$, positive constants $K > 0$, s_0, C, such that for every $u_0, v_0, w_0 \in L^2(\Omega)$, $f, g, h \in L^2(\Omega_T)$, the following Carleman estimate holds for $s \geqslant s_0$, all (u,v,w) solution of (5) *and Q defined by* (10):

$$I(\tau,u) + I(\tau,v) + I(\tau,w) \leqslant C\bigg(\int_{\omega_T} s^{(\tau+33)} (\rho^*)^{\tau+31} e^{(-4s\alpha+2s\eta)} (|u|^2 + |f|^2)\, dx\, dt + \int_{\omega_T} e^{-2s\eta} (s\rho)^{3+\tau} (|Qg|^2 + |Qh|^2)\, dx\, dt + \int_{\Omega_T} e^{-2s\eta} (s\rho)^{\tau} (|f|^2 + |g|^2 + |h|^2)\, dx\, dt \bigg). \tag{6}$$

Remark. Note that if all the coefficients are constants then $B_{k_j} = 0$ and the assumptions of Theorem 1.2 are reduced to $b_j \neq 0$ which corresponds to the Kalman type rank condition of [3].

2. The main lemma

The proofs of the previous theorems will be a consequence of this crucial lemma:

Lemma 2.1. *Let the assumptions of Theorem 1.1 be satisfied. Suppose that $u \in C([0,T]; H^2(\Omega) \cap H_0^1(\Omega)) \cap C^1([0,T]; L^2(\Omega))$. Let H_1 defined in* (1), *$a \in L^\infty(\Omega_T)$ and $A \in L^\infty(\Omega_T)^n$. Then, for all $\varepsilon > 0$, there exists $C_\varepsilon > 0$ such that any solution v of*

$$bv + B \cdot \nabla v = \partial_t u - \operatorname{div}(H_1 \nabla u) - au - A \cdot \nabla u - f \quad \text{in } \omega_T, \qquad v(\cdot,t) = 0 \quad \text{on } \gamma_T, \tag{7}$$

satisfies:

$$\int_{\widetilde{\omega}_T} (s\rho)^{\tau_2+3} e^{-2s\eta} |v|^2\, dx\, dt \leqslant C_\varepsilon \int_{\omega_T} e^{-2s\alpha} (s\rho^*)^{\tau^*} |u|^2\, dx\, dt + \int_{\omega_T} e^{-2s\eta} (s\rho)^{3+\tau_2} |Qf|^2\, dx\, dt + \varepsilon I(\tau_2, v) \tag{8}$$

with $\widetilde{\omega} \subset \omega$, Q defined in (10).

In order to make the proof clearer to the reader, we are going to consider the simplest case where

$$\Omega = (0,1) \times \Omega', \quad \omega = (0,\epsilon) \times \omega', \quad \gamma = \{0\} \times \omega', \quad B(x) = (1,0,\ldots,0), \quad x = (x_1, x'),$$

with $\Omega' \subset \mathbb{R}^{n-1}$ open and smooth enough, and $\omega' \subset \Omega'$ open and non-empty. The general case follows from to this simpler one by suitable change of variables. With this setting, (7) has the form $bv + \partial_{x_1} v = \partial_t u - \operatorname{div}(H_1 \cdot \nabla u) - au - A \cdot \nabla u - f$. The proof will be done in 3 steps.

• **Step 1: An equation for v.** Let $L := \partial_{x_1} + b$, with $D(L) = \{v \in H^1(\omega);\ v(0,x') = 0, \text{ on } \omega'\}$.

$$L^{-1}(w)(x,t) = e^{\int_0^{x_1} b(y_1,x',t)\, dy_1} \int_0^{x_1} e^{-\int_0^{y_1} b(x_1,x',t)\, dx_1} w(y_1,x',t)\, dy_1, \quad \forall w \in L^2(\omega_T).$$

For, $p, q \in L^\infty(\omega_T)$, let us define $K(p,q)w(x,t) = p(x,t) \int_0^{x_1} q(y_1,x',t) w(y_1,x',t)\, dy_1$. Note that $K(p,q) \in \mathcal{L}(L^2(\omega_T))$ and that $L^{-1} = K(p,q)$ with $p(x,t) = e^{\int_0^{x_1} b(y_1,x',t)\, dy_1}$, $q(x,t) = e^{-\int_0^{x_1} b(y_1,x',t)\, dy_1}$. Applying L^{-1} to (7), we obtain the following result:

Let H_1, A, b, B satisfying assumptions of Theorem 1.1 *and p, q previously defined. There exist $(p_i, q_i)_{2 \leqslant i \leqslant n} \in W^{2,\infty}(\omega_T)^{2(n-1)}$, $(\tilde{p}_i, \tilde{q}_i)_{1 \leqslant i \leqslant n} \in W^{1,\infty}(\omega_T)^{2n}$, $k \in L^\infty(\omega_T)$ such that, for any $u \in C([0,T]; H^2(\Omega) \cap H_0^1(\Omega)) \cap C^1([0,T]; L^2(\Omega))$, the solution v of* (7) *satisfies*:

$$v(x,t) = \partial_t K(\tilde{p_1},\tilde{q_1})u(x,t) + \sum_{i=2}^{n} \partial^2_{x_i} K(p_i,q_i)u(x,t) + \sum_{i=2}^{n} \partial_{x_i} K(\tilde{p_i},\tilde{q_i})u(x,t) + K(p,aq)u(x,t) \\ + k(x,t)u(x,t) + K(p,q)f(x,t) + pq(x,t)h_1\big(0,x',t\big)\partial_{x_1}u\big(0,x',t\big), \quad \text{a.e. in } \omega_T. \tag{9}$$

• **Step 2: An observability inequality for v with two observations u on ω_T and $\partial_\nu u$ on $\gamma \times (0,T)$.** We multiply (9) by $(s\rho)^{\tau_2+3}\xi e^{-2s\eta}v$ and integrate on Ω_T (where ξ is a cut-off function supported in ω). We then obtain

$$\lambda^4 \int_{\widetilde{\omega}_T} (s\rho)^{\tau_2+3} e^{-2s\eta}|v|^2\,dx\,dt \leqslant C_\varepsilon\bigg(\lambda^8 s^{\tau_2+7}\int_{\omega_T} M\big(x',t\big)|u|^2\,dx\,dt + \lambda^4\int_{\omega_T} e^{-2s\eta}(s\rho)^{3+\tau_2}\big|K(p,q)f\big|^2\,dx\,dt \\ + \lambda^4\int_{\omega'_T}(s\rho)^{\tau_2+3}e^{-2s\eta}\big|\partial_{x_1}u\big(0,x',t\big)\big|^2\,dx'\,dt\bigg) + \varepsilon I(\tau_2,v),$$

with $M(x',t) = \int_0^1 \rho^{\tau_2+7}e^{-2s\eta}dx_1$. It remains to estimate the boundary term.

• **Step 3: Estimates of the boundary term.** Observe that for any f and h in $H^2(\omega)$, $\int_\omega \partial_{x_1} f(hf)\,dx = -\frac{1}{2}\int_\omega |f|^2\partial_{x_1}(h)\,dx + \frac{1}{2}\int_{\omega'}(|f|^2h)(\varepsilon)\,dx' - (|f|^2h)(0)$. We apply this formula for $f = \partial_{x_1}u$, $f = u$ and h such that $h(\varepsilon) = 0$. After some computations, we obtain

$$\int_{\omega'_T}(s\rho)^{\tau_2+3}e^{-2s\eta}\big|\partial_{x_1}u\big(0,x',t\big)\big|^2dx'\,dt \leqslant \varepsilon\int_{\Omega_T}(s\rho)^{\tau_1-1}e^{-2s\eta^*}\big|\partial^2_{x_1}u\big|^2dx\,dt \\ + C_\varepsilon s^{n^*}\int_{\omega_T}(\rho)^{-\tau_1+1}e^{2s\eta^*}N^2\big(x',t\big)|\partial_{x_1}u|^2\,dx\,dt,$$

with $N(x',t) = \int_0^1(\rho)^{\tau_2+3}e^{-2s\eta}\,dx_1$, $\eta^* = \max_{\overline{\Omega}}\eta$. Therefore $\varepsilon\int_{\Omega_T}(s\rho)^{\tau_1-1}e^{-2s\eta^*}|\partial^2_{x_1}u|^2\,dx\,dt \leqslant \varepsilon I(\tau_1,u)$. Besides,

$$\lambda^4\int_{\omega_T}(\rho)^{-\tau_1+1}s^{n^*}e^{2s\eta^*}N^2\big(x',t\big)|\partial_{x_1}u|^2\,dx\,dt \leqslant \varepsilon I(\tau_1,u) + C_\varepsilon\int_{\omega_T}\big(s\rho^*\big)^{4\tau_2-3\tau_1+15}e^{-8s\eta_-+6s\eta^*}|u|^2\,dx\,dt.$$

If $-4\eta_- + 3\eta^* < 0$, the last integral is bounded. To obtain this, we choose $K \geqslant \max\{\frac{2ln2}{\|\beta\|_\infty}, \|\beta\|_\infty\}$. The proof for the case where $B = (1,0,\dots,0)$ is now complete. The general case (see [5]) is obtained by introducing new coordinates such that $B\cdot\nabla$ becomes ∂_{x_1}. This can be done by choosing ω sufficiently small. We denote by Λ the corresponding diffeomorphism defined in ω. Thus, with $\widetilde{L}(v\circ\Lambda) = \partial_{x_1}(v\circ\Lambda) + (b\circ\Lambda)(v\circ\Lambda)$, we denote

$$Qf = \big(\widetilde{L}^{-1}(f\circ\Lambda)\big)\circ\Lambda^{-1}. \tag{10}$$

3. Sketch of the proof of Theorems 1.1 and 1.2

Proof of Theorem 1.1. Consider $\widetilde{\omega} \subset\subset \omega$ a non-empty open subset If $|\tau_1 - \tau_2| < 1$, a direct application of Carleman estimates for scalar parabolic equations leads to

$$I(\tau_1,u) + I(\tau_2,v) \leqslant C\bigg(\lambda^4\int_{\tilde{\omega}_T}(s\rho)^{\tau_1+3}e^{-2s\eta}|u|^2\,dx\,dt + \lambda^4\int_{\tilde{\omega}_T}(s\rho)^{\tau_2+3}e^{-2s\eta}|v|^2\,dx\,dt\bigg) \\ + \int_{\Omega_T}(s\rho)^{\tau_1}e^{-2s\eta}|f|^2\,dx\,dt + \int_{\Omega_T}(s\rho)^{\tau_2}e^{-2s\eta}|g|^2\,dx\,dt.$$

The main question is to remove the term $\int_{\widetilde{\omega}_T}(s\rho)^{\tau_2+3}e^{-2s\eta}|v|^2\,dx\,dt$. The main difficulty here is the presence of *first order terms* in v. Roughly speaking, the idea is to *locally transform* (*in* $\omega\times(0,T)$) the first equation of (3) as $v = Lu$ where L is a *partial differential operator* (*first order in time and second order in space*). To achieve this, we need the condition $B\cdot\nu \neq 0$ on $\gamma\times(0,T)$ where γ is a part of the boundary of $\Omega\cap\omega$ and that ω is a neighborhood of γ. One can apply Lemma 2.1 and deduce the result.

Proof of Theorem 1.2. The main idea is to apply a Gauss reduction to system (5). Let $z = a_{12}v + a_{13}w$ defined in ω_T. Suppose, for example, that the assumptions of Theorem 1.2 are satisfied for $j = 3$. If (u,v,w) is a solution of system (5), then u, z satisfy

$$\begin{cases} \partial_t u = \operatorname{div}(H_1 \nabla u) + a_{11} u + z + f & \text{in } \omega_T, \\ \partial_t z = \operatorname{div}(H_2 \nabla z) + A \cdot \nabla z + az + eu + B \cdot \nabla v + bv + G & \text{in } \omega_T, \end{cases}$$

with $B = B_2$. By applying results of [6], one can observe z by u from the first equation. The previous Theorem 1.1 is used to observe v by z (in ω_T). The conclusion follows assuming $K \geqslant \max\{\frac{3\ln 2}{\|\beta|_\infty}, \|\beta\|_\infty\}$.

4. Applications

The following results will be detailed in [5].

• **Controllability of semilinear systems.** The previous Carleman estimates allow us to prove controllability to trajectories for a class of semilinear reaction–diffusion–convection systems of order 2 and reaction–diffusion systems of order 3.

• **Identification of coefficients.** The Carleman estimate in Theorem 1.2, allows us to obtain the identification of one coefficient in each equation of system (5) by observing u on ω_T and by the knowledge of these coefficients on ω. An L^2-stability of this identification is also derived.

• **Generalization to systems of order *n*.** All the previous results can be generalized to a class of $n \times n$ parabolic systems with $n - 2$ observations (or controls).

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