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Probability Theory

A generalized existence theorem of BSDEs

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Abstract

In this Note, we deal with one-dimensional backward stochastic differential equations (BSDEs) where the coefficient is left-Lipschitz in y (may be discontinuous) and Lipschitz in z, but without explicit growth constraint. We prove, in this setting, an existence theorem for backward stochastic differential equations. ***To cite this article: G. Jia, C. R. Acad. Sci. Paris, Ser. I 342 (2006).***

Résumé

Un théorème d'existence généralisé des EDSRs. Dans cette Note, nous traitons l'équation différentielle stochastique rétrograde en une dimension, où le coéfficient est Lipschitzien à gauche en y (peut-être discontinu) et Lipschitzien en z, sans croissance contrainte explicite. Nous montrons, dans ce cas, un théorème d'existence de la solution pour équation différentielle stochastique rétrograde. ***Pour citer cet article : G. Jia, C. R. Acad. Sci. Paris, Ser. I 342 (2006).***

1. Introduction

One-dimensional BSDEs are equations of the following type defined on $[0,T]$:

$$y_t = \xi + \int_t^T g(s, y_s, z_s)\,\mathrm{d}s - \int_t^T z_s\,\mathrm{d}W_s, \quad 0 \leqslant t \leqslant T, \tag{1}$$

where $(W_t)_{0\leqslant t\leqslant T}$ is a standard d-dimensional Brownian motion on a probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\leqslant t\leqslant T}, P)$ with $(\mathcal{F}_t)_{0\leqslant t\leqslant T}$ the filtration generated by W. The function $g : \Omega \times [0,T] \times \mathbb{R} \times \mathbb{R}^d \to \mathbb{R}$ is generally called a generator of (1), here T is the terminal time, and the $\mathbb{R}$-valued $\mathcal{F}_T$-adapted random variable ξ is a terminal condition; (g, T, ξ) are the parameters of (1).

A solution is a couple $(y_t, z_t)_{0\leqslant t\leqslant T}$ of processes adapted to filtration $(\mathcal{F}_t)_{0\leqslant t\leqslant T}$ which have some integrability properties, depending on the framework imposed by the type of assumptions on g.

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Nonlinear BSDEs were first introduced by Pardoux and Peng [5], who proved the existence and uniqueness of a solution under assumptions on g and ξ, the most important of which are the Lipschitz continuity of g on (y,z) and the square integrability of ξ. Since then, BSDEs have been studied with great interest. In particular, many efforts have been made to relax the assumption on the generator g; for instance, Lepeltier and San Martin [3] have proved the existence of a solution for (1) when g is only continuous in (y,z) with linear growth, and Kobylanski [2] obtained the existence and uniqueness of a solution when g is continuous and has a quadratic growth in z and the terminal condition ξ is bounded.

In this Note, we mainly deal with one-dimensional BSDEs associated with coefficient which may be discontinuous in y, and without explicit growth constraint. In fact, we show that the one-dimensional BSDE associated with (g,T,ξ) has at least a solution if g satisfies the following conditions:

- H1. $g(t,\cdot,z)$ is left-continuous, and $g(t,y,\cdot)$ is Lipschitz continuous, i.e., there exists a positive constant A, such that $|g(t,y,z_1)-g(t,y,z_2)|\leqslant A|z_1-z_2|$, for all $t\in[0,T],\ y\in\mathbb{R},\ z_1,z_2\in\mathbb{R}^d$.
- H2. there exist two BSDEs with generators g_1, g_2 respectively, such that $g_1(t,y,z)\leqslant g(t,y,z)\leqslant g_2(t,y,z)$, for all $t\in[0,T]$, $y\in\mathbb{R}$, $z\in\mathbb{R}^d$, and for given T and ξ, the equations (g_1,T,ξ) and (g_2,T,ξ) have at least one solution respectively, denoted by $\{Y_t^i,Z_t^i\}$, $i=1,2$, where $Y_t^1\leqslant Y_t^2$, for $t\in[0,T]$, a.s., a.e. Moreover, the processes $g_i(t,Y_t^i,Z_t^i)$ are square integrable.
- H3. $g(t,\cdot,z)$ satisfies left Lipschitz condition in y, i.e., $g(t,y_1,z)-g(t,y_2,z)\geqslant -A(y_1-y_2)$, for all $y_1\geqslant y_2$, $z\in\mathbb{R}^d$ and $t\in[0.T]$.

Remark 1. We can find an inverse version of H3 in [4], where Pardoux studied multidimensional BSDEs, and he assumed that g satisfies

$$\big\langle x-y, g(t,x,z)-g(t,y,z)\big\rangle\leqslant A|x-y|^2. \tag{2}$$

In 1-dimensional case, (2) can be rewritten as $g(t,x,z)-g(t,y,z)\leqslant A(x-y)$ for all $t\in[0,T], z\in\mathbb{R}^d$ and $x\geqslant y$. Clearly, (2) implies uniqueness of solution. By Theorem 5, we will prove that H3 implies existence of solution. The combination of H3 with (2) is Lipschitz continuity of g in y in 1-dimensional case.

Remark 2. Obviously, we do not know whether Eq. (1) satisfying H1–H3 has solution or not by the results of [2,3] or [4], even if g is continuous in y, for example,

$$y_t=\xi+\int_t^T\big(\mathbf{sgn}(y_s)y_s^2+\sin\big(|z_s|\big)\big)\,\mathrm{d}s-\int_t^T z_s\,\mathrm{d}W_s,\quad t\in[0,T], \tag{3}$$

where $\mathbf{sgn}(y)=-1$ when $y\leqslant 0$, otherwise $\mathbf{sgn}(y)=1$. But we know that (3) has solution for some ξ and T by Theorem 5.

This Note is organized as follows. In Section 2 we formulate the problem accurately and give some preliminary results. Finally, Section 3 is devoted to the proof of the main theorem.

2. Preliminaries

Let $T>0$ be a fixed terminal time and $W=(W_t)_{0\leqslant t\leqslant T}$ a d-dimensional Brownian motion defined on a probability space $(\Omega,\mathcal{F},P)$, whose natural filtration is denoted $(\mathcal{F}_t)_{0\leqslant t\leqslant T}$, where $\mathcal{F}_t=\sigma\{W_s, s\leqslant t\}$. Let $\mathcal{P}$ be the σ-field on $\Omega\times[0,T]$ of $\mathcal{F}_t$-progressively measurable sets. Let $\mathcal{H}_n^2$ be the set of $\mathcal{P}$-measurable processes $V=(V_t)_{0\leqslant t\leqslant T}$ with values in $\mathbb{R}^n$ such that $E[\int_0^T|V_s|^2\,\mathrm{d}s]<\infty$, and let $\mathcal{S}^2$ be the set of continuous $\mathcal{P}$-measurable processes $V=(V_t)_{0\leqslant t\leqslant T}$ with values in $\mathbb{R}$ such that $E[\sup_{t\in[0,T]}|V_t|^2]<\infty$.

Now, let $\xi\in L^2(\Omega,\mathcal{F}_T,P)$ be a terminal value, $g:\Omega\times[0,T]\times\mathbb{R}\times\mathbb{R}^d\to\mathbb{R}$ the generator, such that the process $(g(\omega,t,0,0))_{t\in[0,T]}\in\mathcal{H}_1^2$ and, for any $(y,z)\in\mathbb{R}\times\mathbb{R}^d$, $(g(\omega,t,y,z))_{t\in[0,T]}$ is $\mathcal{P}$-measurable.

A solution of such an equation (g, T, ξ) is a $\mathcal{P}$-measurable process $(y, z) = (y_t, z_t)_{t\in[0,T]}$ valued in $\mathbb{R} \times \mathbb{R}^d$ such that

$$y_t = \xi + \int_t^T g(s, y_s, z_s)\,\mathrm{d}s - \int_t^T z_s\,\mathrm{d}W_s, \quad 0 \leqslant t \leqslant T, \tag{4}$$

where $(y, z) \in \mathcal{S}^2 \times \mathcal{H}_d^2$. Also we need one lemma, which is a special case of the well-known Comparison Theorem (see [6,1]).

Lemma 3. *Suppose that* $f_1(s, y, z) = ly + m\|z\|$, $f_2(s, y, z) = l|y| + m\|z\|$ *for some constants* $l, m \in \mathbb{R}$, *and* $\phi_s \in \mathcal{H}_1^2$ *is a non-negative process, moreover,* (y_t^i, z_t^i) *are the solutions of the BSDEs* $(f_i + \phi_t, T, \xi)$ *for* $i = 1, 2$. *If* $\xi \in L^2(\Omega, \mathcal{F}_T, P)$, *and* $\xi \geqslant 0$ *a.s., then* $y_t^i \geqslant 0$, P*-a.s.,* $i = 1, 2$.

3. Existence

In this section we consider the existence of BSDE (4) under the assumption H1–H3. At first, we denote that (Y_t^j, Z_t^j) are the solutions of (g_j, T, ξ), where $j = 1, 2$, that is

$$Y_t^j = \xi + \int_t^T g_j\big(s, Y_s^j, Z_s^j\big)\,\mathrm{d}s - \int_t^T Z_s^j\,\mathrm{d}W_s, \tag{5}$$

where g_j satisfies H2 and $g_j(t, Y_t^j, Z_t^j) \in \mathcal{H}_1^2$. Now we construct a sequence of BSDEs as follows:

$$\underline{y}_t^i = \xi + \int_t^T \big(g\big(s, \underline{y}_s^{i-1}, \underline{z}_s^{i-1}\big) - A\big(\underline{y}_s^i - \underline{y}_s^{i-1}\big) - A\big\|\underline{z}_s^i - \underline{z}_s^{i-1}\big\|\big)\,\mathrm{d}s - \int_t^T \underline{z}_s^i\,\mathrm{d}W_s, \tag{6}$$

where $i = 1, \ldots$ and $(\underline{y}_t^0, \underline{z}_t^0) = (Y_t^1, Z_t^1)$. Obviously, Eqs. (6) $(i = 1, 2, \ldots)$ have an unique adapted solution respectively if $g(s, \underline{y}_s^{i-1}, \underline{z}_s^{i-1}) \in \mathcal{H}_1^2$. For these equations, we have:

Lemma 4. *Under Assumption* H1–H3, *the following properties hold true* (1) *For any positive integer* i, *Eq.* (6) *has a unique adapted solution* $(\underline{y}_t^i, \underline{z}_t^i) \in \mathcal{S}^2 \times \mathcal{H}_d^2$. (2) *For any positive integer i,* $Y_t^1 \leqslant \underline{y}_t^i \leqslant \underline{y}_t^{i+1} \leqslant Y_t^2$.

Proof. Firstly, we prove that (1) and (2) hold true for i=1. By $Y_t^2 \geqslant Y_t^1$ and H2, it follows that $g(t, Y_t^2, Z_t^2) - g(t, Y_t^1, Z_t^1) \geqslant -A(Y_t^2 - Y_t^1) - A\|Z_t^2 - Z_t^1\|$. Thus $g_2(t, Y_t^2, Z_t^2) + A(Y_t^2 - Y_t^1) + A\|Z_t^2 - Z_t^1\| \geqslant g(t, Y_t^2, Z_t^2) + A(Y_t^2 - Y_t^1) + A\|Z_t^2 - Z_t^1\| \geqslant g(t, Y_t^1, Z_t^1) \geqslant g_1(t, Y_t^1, Z_t^1)$. This implies that $g(t, Y_t^1, Z_t^1) \in \mathcal{H}_1^2$ and Eq. (6) has a unique adapted solution $(\underline{y}_t^1, \underline{z}_t^1)$.

Now, by (6) and (5) when $i = 1$ and $j = 1$, $\underline{y}_t^1 - Y_t^1 = \int_t^T (-A(\underline{y}_s^1 - Y_s^1) - A\|\underline{z}_s^1 - Z_s^1\| + \phi_s^1)\,\mathrm{d}s - \int_t^T (\underline{z}_s^1 - Z_s^1)\,\mathrm{d}W_s$, where $\phi_s^1 := g(s, Y_s^1, Z_s^1) - g_1(s, Y_s^1, Z_s^1) \geqslant 0$ and $\phi_s^1 \in \mathcal{H}_1^2$, using Lemma 3, we have $\underline{y}_t^1 \geqslant Y_t^1$.

Again we consider Eqs. (6) and (5) when $i = 1$ and $j = 2$, $Y_t^2 - \underline{y}_t^1 = \int_t^T (-A(Y_s^2 - \underline{y}_s^1) - A\|Z_s^2 - \underline{z}_s^1\| + \psi_s^1)\,\mathrm{d}s - \int_t^T (Z_s^2 - \underline{z}_s^1)\,\mathrm{d}W_s$, where $\psi_s^1 := A(Y_s^2 - Y_s^1) + A\|Z_s^2 - \underline{z}_s^1\| + g_2(s, Y_s^2, Z_s^2) - g(s, Y_s^1, Z_s^1) + A\|\underline{z}_s^1 - Z_s^1\| \geqslant g(s, Y_s^2, Z_s^2) - g(s, Y_s^1, Z_s^1) + A(Y_s^2 - Y_s^1) + A\|Z_s^2 - Z_s^1\| \geqslant 0$. Obviously, $\psi_s^1 \in \mathcal{H}_1^2$, by the comparison theorem, we have $Y_t^2 \geqslant \underline{y}_t^1$. Thus, we get (1), (2) whenever $i = 1$. That is, $Y_t^1 \leqslant \underline{y}_t^1 \leqslant Y_t^2$ and $g(t, Y_t^1, Z_t^1) \in \mathcal{H}_1^2$.

Similarly, $g_2(t, Y_t^2, Z_t^2) - g(t, \underline{y}_t^1, \underline{z}_t^1) \geqslant g(t, Y_t^2, Z_t^2) - g(t, \underline{y}_t^1, \underline{z}_t^1) \geqslant -A(Y_t^2 - \underline{y}_t^1) - A\|Z_t^2 - \underline{z}_t^1\|$. So, we have $g_2(t, Y_t^2, Z_t^2) + A(Y_t^2 - \underline{y}_t^1) + A\|Z_t^2 - \underline{z}_t^1\| \geqslant g(t, \underline{y}_t^1, \underline{z}_t^1)$. But $g(t, \underline{y}_t^1, \underline{z}_t^1) \geqslant g_1(t, Y_t^1, Z_t^1) - A(\underline{y}_t^1 - Y_t^1) - A\|\underline{z}_t^1 - Z_t^1\|$, this implies that $g(t, \underline{y}_t^1, \underline{z}_t^1) \in \mathcal{H}_1^2$ and Eq. (6) has a unique adapted solution when $i = 2$. Using the similar method, we get $\underline{y}_t^2 \geqslant \underline{y}_t^1$ and $\underline{y}_t^2 \leqslant Y_t^2$.

Now we assume that $Y_t^1 \leqslant \underline{y}_t^{i-1} \leqslant \underline{y}_t^i \leqslant Y_t^2$ and $g(t, \underline{y}_t^{i-1}, \underline{z}_t^{i-1}) \in \mathcal{H}_1^2$, we consider Eq. (6) for $i + 1$, which can be written as

$$\underline{y}_t^{i+1} = \xi + \int_t^T \big(g\big(s, \underline{y}_s^i, \underline{z}_s^i\big) - A\big(\underline{y}_s^{i+1} - \underline{y}_s^i\big) - A\big\|\underline{z}_s^{i+1} - \underline{z}_s^i\big\|\big)\,\mathrm{d}s - \int_t^T \underline{z}_s^{i+1}\,\mathrm{d}W_s, \tag{7}$$

here $g_2(t,Y_t^2,Z_t^2)-g(t,\underline{y}_t^i,\underline{z}_t^i)\geqslant g(t,Y_t^2,Z_t^2)-g(t,\underline{y}_t^i,\underline{z}_t^i)\geqslant -A(Y_t^2-\underline{y}_t^i)-A\|Z_t^2-\underline{z}_t^i\|$, $g_2(t,Y_t^2,Z_t^2)+A(Y_t^2-\underline{y}_t^i)+A\|Z_t^2-\underline{z}_t^i\|\geqslant g(t,\underline{y}_t^i,\underline{z}_t^i)$ and $g(t,\underline{y}_t^i,\underline{z}_t^i)\geqslant g(t,Y_t^1,Z_t^1)-A(\underline{y}_t^i-Y_t^1)-A\|\underline{z}_t^i-Z_t^1\|\geqslant g_1(t,Y_t^1,Z_t^1)-A(\underline{y}_t^i-Y_t^1)-A\|\underline{z}_t^i-Z_t^1\|$, this implies that $g(t,\underline{y}_t^i,\underline{z}_t^i)\in\mathcal{H}_1^2$, and Eq. (7) has a unique adapted solution. By the similar procedure, we have $\underline{y}_t^i\leqslant\underline{y}_t^{i+1}\leqslant Y_t^2$. The proof is complete. □

Now, we introduce our main result:

Theorem 5. *Under Assumption* H1–H3*, and* $\{\underline{y}_t^i,\underline{z}_t^i\}_{i=1}^\infty$ *are the solutions of* (6)*, then* $\{\underline{y}_t^i,\underline{z}_t^i\}_{i=1}^\infty$ *converges in* $\mathcal{S}^2\times\mathcal{H}_d^2$ *to* $(\underline{y}_t,\underline{z}_t)$*,* $(\underline{y}_t,\underline{z}_t)$ *is a solution of Eq.* (4)*.*

Proof. The inequality in Lemma 4 leads to the fact that $\{\underline{y}_t^i\}_{i=1}^\infty$ converges to a limit $\underline{y}_t$ in $\mathcal{S}^2$, and we have $\sup_i E\sup_{0\leqslant t\leqslant T}|\underline{y}_t^i|^2\leqslant E\sup_{0\leqslant t\leqslant T}|Y_t^1|^2+E\sup_{0\leqslant t\leqslant T}|Y_t^2|^2<\infty$. Applying the Itô formula to $|\underline{y}_t^{i+1}|^2$, we have $E[|\underline{y}_T^{i+1}|^2]=|\underline{y}_0^{i+1}|^2+E\int_0^T(\|\underline{z}_t^{i+1}\|^2-2\underline{y}_t^{i+1}(g(t,\underline{y}_t^i,\underline{z}_t^i)-A(\underline{y}_t^{i+1}-\underline{y}_t^i)-A\|\underline{z}_t^{i+1}-\underline{z}_t^i\|)\,\mathrm{d}s$. Then $|g(t,\underline{y}_t^i,\underline{z}_t^i)|\leqslant|g_2(t,Y_t^2,Z_t^2)+A(Y_t^2-\underline{y}_t^i)+A\|Z_t^2-\underline{z}_t^i\||+|g_1(t,Y_t^1,Z_t^1)-A(\underline{y}_t^i-Y_t^1)-A\|\underline{z}_t^i-Z_t^1\||\leqslant\sum_{j=1}^2[|g_j(t,Y_t^j,Z_t^j)|+A(|Y_t^j|+|Z_t^j|)]+2A(|\underline{y}_t^i|+|\underline{z}_t^i|)$. So $E\int_0^T\|\underline{z}_t^{i+1}\|^2\,\mathrm{d}t=2E\int_0^T(\underline{y}_t^{i+1}[g(t,\underline{y}_t^i,\underline{z}_t^i)-A(\underline{y}_t^{i+1}-\underline{y}_t^i)-A\|\underline{z}_t^{i+1}-\underline{z}_t^i\|])\,\mathrm{d}t+E|\xi|^2-|\underline{y}_0^{i+1}|^2\leqslant C+\frac{1}{8}E\int_0^T(|\underline{z}_t^{i+1}|^2+|\underline{z}_t^i|^2)\,\mathrm{d}t$. That is $E\int_0^T\|\bar{z}_t^{i+1}\|^2\,\mathrm{d}t\leqslant\overline{C}+\frac{1}{7}E\int_0^T\|\bar{z}_t^i\|^2\,\mathrm{d}t$ where $\overline{C}=\frac{8}{7}(C+E|\xi|^2)$, and $C=2\sup_i\{E\int_0^T(2A|\bar{y}_t^{i+1}||\bar{y}_t^i|+|g(s,0,0)||\bar{y}_t^{i+1}|+(32A^2+2A)|\bar{y}_t^{i+1}|^2)\,\mathrm{d}t\}$. This implies that $\sup_i E\int_0^T\|\underline{z}_t^i\|^2\,\mathrm{d}t<\infty$, which yields that the quantities $\psi_t^{i+1}=g(t,\underline{y}_t^i,\underline{z}_t^i)-A(\underline{y}_t^{i+1}-\underline{y}_t^i)-A\|\underline{z}_t^{i+1}-\underline{z}_t^i\|$ are uniformly bounded in $\mathcal{H}_1^2$. Set $C_0=\sup_i E\int_0^T|\psi_t^i|^2\,\mathrm{d}t$. Now apply Itô's formula to $|\underline{y}_t^p-\underline{y}_t^q|^2$, $E|\underline{y}_t^p-\underline{y}_t^q|^2+E\int_t^T\|\underline{z}_t^p-\underline{z}_t^q\|^2\,\mathrm{d}s=E\int_t^T 2(\underline{y}_s^p-\underline{y}_s^q)(\psi_s^p-\psi_s^q)\,\mathrm{d}s\leqslant 4C_0[E(\int_0^T|\bar{y}_t^p-\bar{y}_t^q|^2\,\mathrm{d}s)]^{1/2}$. It follows that $\{\underline{z}_t^i\}_{i=1}^\infty$ is a Cauchy sequence in $\mathcal{H}_d^2$, therefore $\{\underline{z}_t^i\}_{i=1}^\infty$ converges in $\mathcal{H}_d^2$, we denote the limit by $\underline{z}_t$. We now pass to the limit, as $i\to\infty$ on both sides of (6), it follows that $\underline{y}_t=\xi+\int_t^T g(s,\underline{y}_s,\underline{z}_s)\,\mathrm{d}s-\int_t^T\underline{z}_s\,\mathrm{d}Ws$. Obviously, $(\underline{y}_t,\underline{z}_t)$ solves Eq. (4). The proof is complete. □

Remark 6. By Theorem 5, we know that Eq. (3) in Remark 2 has at least one solution when $\xi=1$ and $T=\frac{1}{2}$, because $g=\mathbf{sgn}(y)y^2+\sin(|z|)$ satisfies H1–H3, where $g_2=y^2+1$, $g_1=-y^2-1$, and $g_1\leqslant g\leqslant g_2$, moreover, the solutions of $(g_i,\frac{1}{2},1)$, $i=1,2$, are $(\tan(\frac{\pi}{4}-\frac{1}{2}+t),0)$ and $(\tan(\frac{\pi}{4}+\frac{1}{2}-t),0)$, respectively, and $\tan(\frac{\pi}{4}+\frac{1}{2}-t)\geqslant\tan(\frac{\pi}{4}-\frac{1}{2}+t)$ when $t\in[0,\frac{1}{2}]$. But, we cannot assert that (g,T,ξ) have solution for the case $T\neq\frac{1}{2}$ or $\xi\neq 1$ since, in these cases, (g_i,T,ξ) may have no solution, or blow-up solution. But the solutions may be non-unique, for example, consider the BSDE $(\mathbf{1}(y)\sqrt{|y|},1,0)$, where $\mathbf{1}(y)=0$ when $y\leqslant 0$, otherwise $\mathbf{1}(y)=1$. Clearly, $(\mathbf{1}(y)\sqrt{|y|},1,0)$ satisfies H1–H3 where $g_1=-\frac{1}{2}-\frac{|y|}{2}$ and $g_2=\frac{1}{2}+\frac{|y|}{2}$, moreover, $(y_t,z_t)=(0,0)$ and for any $c\in[0,T]$, $(y_t,z_t)=([\max\{\frac{c-t}{2},0\}]^2,0)$ are both the solutions of $(\mathbf{1}(y)\sqrt{|y|},1,0)$.

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