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Lie Algebras/Group Theory

Vanishing of H^1 for Dedekind rings and applications to loop algebras

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Abstract

We establish a version of Serre's 'Conjecture I' for Dedekind domains. As an application, we give a parametrization of twisted loop algebras by means of torsors. ***To cite this article: A. Pianzola, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Résumé

Trivialité de H^1 pour les anneaux de Dedekind, applications aux algèbres de lacets. Nous établissons une version de la « Conjecture I » de Serre pour les anneaux de Dedekind. Ceci nous permet de décrire les algèbres de lacets tordues en termes de torseurs. ***Pour citer cet article : A. Pianzola, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Version française abrégée

Soient k un corps algébriquement clos de caractéristique nulle, et $(\zeta_n)_{n\geqslant 1}$ une famille cohérente de racines primitives n-ième de l'unité. Soit A/k une algèbre munie d'un k-automorphisme σ d'ordre m. La décomposition en espaces propres $A = \bigoplus_{\bar{i}\in\mathbb{Z}_m} A_{\bar{i}}$ avec $A_{\bar{i}} = \{a \in A \mid \sigma(a) = \zeta_m^{\bar{i}} a\}$ permet de définir l'algèbre de lacets

$$L(A,\sigma) := \bigoplus_{i\in\mathbb{Z}} A_{\bar{i}} \otimes t^{i/m} \subset A \otimes S_m,$$

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où $S_m = k[t^{1/m}, t^{-1/m}]$ (avec la convention $\otimes = \otimes_k$). On note $R = k[t, t^{-1}] \subset S_m$. La k-algèbre $L(A, \sigma)$ est en fait une R-algèbre. On dit que $L(A, \sigma)$ est *triviale* si $L(A, \sigma) \cong_{k\text{-alg}} A \otimes_k R$. On a $L(A, \sigma) \cong_{k\text{-alg}} L(A, \sigma^{-1})$.

Théorème 0.1. *Soit A une k-algèbre de dimension finie. Soit* $\mathbf{Aut}\, A \subset \mathbf{GL}(A)$ *le groupe algébrique linéaire des automorphismes de l'algèbre A. On note* F *le groupe fini constant* $\mathbf{Aut}\, A/(\mathbf{Aut}\, A)^0$ *des composantes connexes de* $\mathbf{Aut}\, A$ *et* $^- : \mathbf{Aut}\, A(k) \to \mathrm{F}$ *la surjection canonique.*

(i) $L(A, \sigma) \simeq_{R\text{-alg}} L(A, \tau) \Leftrightarrow \bar\sigma \sim \bar\tau$.
(ii) *Si les centroïdes des k-algèbres $L(A, \sigma)$ et $L(A, \tau)$ coïncident avec R, alors*

$$L(A, \sigma) \simeq_{k\text{-alg}} L(A, \tau) \Leftrightarrow \bar\sigma \sim \bar\tau^{\pm 1}.$$

La preuve repose sur la théorie de la descente. La R-algèbre $L(A, \sigma)$ est une S_m/R-forme de $A \otimes R$ et donne lieu à une classe $[L(A, \sigma)]$ dans l'ensemble pointé de cohomologie étale $H^1(R, \mathbf{Aut}\, A)$. D'autre part, l'ensemble pointé de cohomologie étale $H^1(R, F)$ classifie les revêtements galoisiens finis de R de groupe structural F. Le revêtement universel de R est donné par $\widehat{S} = \varinjlim S_m$ dont le groupe de Galois est le complété profini $\widehat{\mathbb{Z}}$ de $\mathbb{Z}$. Ainsi $H^1(R, F) = \mathrm{Hom}_{\mathrm{cont}}(\widehat{\mathbb{Z}}, F)/\,\mathrm{Int}(F) = F/\,\mathrm{Int}(F)$, l'ensemble des classes de conjugaison de F. De plus, la flèche naturelle

$$H^1(R, \mathbf{Aut}\, A) \to H^1(R, \mathrm{F}) \cong \mathrm{F}/\,\mathrm{Int}(\mathrm{F})$$

envoie $[L(A, \sigma)]$ sur la classe de conjugaison de $\bar\sigma^{-1}$. En particulier, $L(A, \sigma) \cong_R L(A, \tau)$ implique $\bar\tau = \bar\sigma$, c'est le sens facile de l'assertion (i). La classification des R-algèbres $L(A, \sigma)$ est donc un problème de cohomologie étale non-abélienne pour R.

Suivant la conjecture I de Serre établie par Steinberg et par Borel et Springer, on sait que l'application $H^1(K, G) \to H^1(K, G/G^0)$ est injective pour tout groupe algébrique linéaire réductif G/K défini sur un corps K de dimension $\leqslant 1$. Utilisant des techniques de géométrie algébrique [4], on montre alors l'énoncé plus précis suivant :

Théorème 0.2. *Soit* $\mathfrak{X} = \mathrm{Spec}\, D$ *un schéma de Dedekind dont la fibre générique* $\mathrm{Spec}\, K$ *est donné par un corps K de dimension* $\leqslant 1$.

(i) *Les assertions suivantes sont équivalentes* :
 (a) $H^1(\mathfrak{X}, \mathbf{G}) = 1$ *pour tout $\mathfrak{X}$-groupe réductif* $\mathbf{G}$.
 (b) $\mathrm{Pic}(\mathfrak{X}') = 1$ *pour tout revêtement étale fini et connexe $\mathfrak{X}'$ de $\mathfrak{X}$.*
(ii) *Sous les conditions de* (i)*, on a* :
 (a) $H^2(\mathfrak{X}, \mathbf{M}) = 1$ *pour tout $\mathfrak{X}$-groupe multiplicatif* $\mathbf{M}$ *de type fini.*
 (b) *Tout $\mathfrak{X}$-groupe réductif est quasi-déployé.*

L'anneau R satisfait l' hypothèse (i)(b). Le théorème s'applique donc dans ce cas et on en tire l'injectivité de l'application $H^1(R, \mathbf{Aut}\, A) \to H^1(R, F)$. Ainsi $L(A, \sigma) \cong_R L(A, \tau)$ si et seulement si $\bar\sigma$ et $\bar\tau$ sont conjugués dans F. On a en fait une bijection $H^1(R, \mathbf{Aut}\, A) \cong H^1(R, F)$. On conclut que toutes les R-formes de A sont des algèbres de lacets.

1. A principle for loop algebras

Twisted loop algebras were introduced by V. Kac to provide concrete realization of affine Kac–Moody Lie algebras. Let us begin by recalling the definition of loop algebras, and introducing some pertinent notation.

Throughout this Note k will denote an algebraically closed field of characteristic 0. Unless explicitly mentioned or noted otherwise, all algebras and tensor products are over k. We fix once and for all a set $(\zeta_n)_{n\geqslant 1}$ of compatible primitive n-th roots of unity in $k^\times$ (thus $\zeta_{ln}^l=\zeta_n$). Set $R=k[t,t^{-1}]$.

Let A be an algebra over k (there is no restriction on A, but one may be particularly interested in a certain 'kind' of algebras like Lie algebras, Jordan algebras, ...). If σ is an automorphism of A of finite order $m=m(\sigma)$ we have the eigenspace decomposition $A=\bigoplus_{i=0}^{m-1}A_{\bar{i}}$, where $^-:\mathbb{Z}\to Z_m=\mathbb{Z}/m\mathbb{Z}$ is the canonical map and $A_{\bar{i}}:=\{a\in A\mid \sigma(a)=\zeta_m^i a\}$. We then define[1] the *loop algebra* $L(A,\sigma)$ of the pair (A,σ) as follows:

$$L(A,\sigma)=\bigoplus_{i\in\mathbb{Z}}A_{\bar{i}}\otimes t^{i/m}\subset A\otimes S_m,$$

where $S_m=k[t^{1/m},t^{-1/m}]$. It is clear that, thus defined, $L(A,\sigma)$ has a natural k-algebra structure (infinite dimensional unless A is the zero algebra).

Examples. (i) $L(A,\mathrm{Id})=A\otimes R$. We say that $L(A,\sigma)$ is *trivial* if $L(A,\sigma)\simeq_{k\text{-alg}}A\otimes R$.

(ii) (V. Kac). Every (derived modulo its centre) affine Kac–Moody Lie algebras is isomorphic to a loop algebra of some finite dimensional simple Lie algebra over k.

Empirical evidence suggest that loop algebras tend to behave according to the following:

Loop Algebra Principle. *Let A be a k-algebra. There exists a normal subgroup* $\operatorname{Int}A$ *of* $\operatorname{Aut}A$ *with the property that for any two finite order automorphisms σ and τ of A*

$$L(A,\sigma)\simeq_{k\text{-alg}}L(A,\tau)\Leftrightarrow\bar{\sigma}\sim\bar{\tau}^{\pm 1},$$

where $^-:\operatorname{Aut}A\to\operatorname{Aut}A/\operatorname{Int}A$ *is the canonical map.*

Here and in what follows $\sim$ stands for conjugacy in the abstract group in question ($\operatorname{Aut}A/\operatorname{Int}A$ in the present case).

In formulating this statement, I have benefited from several conversations with V. Kac on the subject. It is interesting to point out that the Principle can also be turned around to provide the 'right' definition of the group $\operatorname{Int}A$ of inner automorphisms (see example (3) below).

Here are examples where the Principle is known to hold:

(1) A is finite dimensional and either:
  (a) The (algebraic) group of automorphisms of A is connected (in which case all loop algebras are trivial [10]).
  (b) A is a simple Lie algebra. (Mostly Kac's work and the origin of the subject [9]. Complete proofs by different methods appear in [1] and [10].)
  (c) A is a simple Lie superalgebra. (V. Serganova's unpublished thesis. The proof is computational. Another proof along the spirit of this note is given in [7].)
(2) A is an indecomposable symetrizable Kac–Moody algebra [1].
(3) Some Lie N-conformal superalgebras. (Strictly speaking, these are not algebras; but the Principle still seems to apply. The $N=4$ case is particularly striking. Here the automorphism group is $(\mathrm{SL}_2\times\mathrm{SL}_2)/\pm 1$, but only one the SL_2 behaves as inner automorphisms.)

The Lie theoretical flavour of A in all of the above examples is somehow misleading. We shall see that the classification of loop algebras depends *solely* on the structure of $\mathbf{Aut}\,A$, and *not* on the nature of A itself. (For

[1] One can define loop algebras in arbitrary characteristic by means of gradings. However the strongest and most interesting results take place under our present assumption on k.

example, *the same proof* shows that all loop algebras of the 27 dimensional exceptional Jordan algebra and of the simple Lie algebra of type F_4 are trivial.)

2. The torsor viewpoint

In [1] and [10] it is shown that the study of loop algebras is in fact a problem in Galois cohomology. The crucial observation is that $L(A,\sigma)$ is also in a natural way an R-algebra and

$$L(A,\sigma)\otimes_R S_m \simeq_{S_m\text{-alg}} A\otimes S_m \simeq_{S_m\text{-alg}} (A\otimes R)\otimes_R S_m \tag{1}$$

(see Section 1 for notation). Since S_m/R is étale (in fact Galois, see below), we conclude that $L(A,\sigma)$ is, *as an algebra over R*, locally isomorphic to $A\otimes R$ (on the étale site of Spec R). Thus to $L(A,\sigma)$ we can attach a torsor $\mathcal{L}(A,\sigma)$ over Spec R under the R-group (sheaf) $\mathbf{Aut}\,A_R$.

$$L(A,\sigma)\rightsquigarrow \big[\mathcal{L}(A,\sigma)\big]\in H^1(R,\mathbf{Aut}\,A_R).$$

The Zen of torsors[2] then yields

$$L(A,\sigma)\sim_{R\text{-alg}} L(A,\tau)\Leftrightarrow \big[\mathcal{L}(A,\sigma)\big]=\big[\mathcal{L}(A,\tau)\big].$$

As a very concrete application of this highly abstract point of view we will establish the following version of the Loop Algebra Principle.

Theorem 2.1. *Let $\mathbf{Aut}\,A\subset \mathbf{GL}(A)$ be the linear algebraic group of automorphisms of a finite dimensional k-algebra A. Let $\mathrm{F}_k:=\mathbf{Aut}\,A/\mathbf{Aut}^0\,A$ be the* (*finite constant*) *group of connected components of $\mathbf{Aut}\,A$ and $^-:\mathbf{Aut}\,A(k)\to \mathrm{F}:=\mathrm{F}_k(k)$ the corresponding canonical* (*abstract*) *group homomorphism. Finally, let σ and τ be two elements of $\mathbf{Aut}\,A(k)$ of finite order. Then*:

(i) $L(A,\sigma)\simeq_{R\text{-alg}} L(A,\tau)\Leftrightarrow \bar\sigma\sim\bar\tau.$
(ii) *If the centroids of the k-algebras $L(A,\sigma)$ and $L(A,\tau)$ coincide with R* (*with R acting naturally by scalar multiplication. For example, if A is central and perfect*), *then*

$$L(A,\sigma)\simeq_{k\text{-alg}} L(A,\tau)\Leftrightarrow \bar\sigma\sim\bar\tau^{\pm 1}.$$

For convenience we will denote the connected component of the identity $\mathbf{Aut}^0\,A$ of $\mathbf{Aut}\,A$ by $\mathbf{G}$. We then have the exact sequence

$$1\to \mathbf{G}\to \mathbf{Aut}\,A\to \mathrm{F}_k\to 1. \tag{2}$$

Applying the base change R/k to this sequence and passing to étale cohomology yields

$$H^1(R,\mathbf{G}_R)\to H^1(R,\mathbf{Aut}\,A_R)\xrightarrow{\psi} H^1(R,\mathrm{F}_R). \tag{3}$$

From Grothendieck's work ([8] XI 5), we know that $H^1(R,\mathrm{F}_R)$ agrees with the (continuous) cohomology $H^1(\pi_1(R),\mathrm{F})$ where $\pi_1(R)$ is the algebraic fundamental group of Spec R based on the geometric point $(t-1)$. We have $\pi_1(R)=\widehat{\mathbb{Z}}:=\varprojlim \mathbb{Z}_n$ where the $\mathbb{Z}_n$ are compatibly identified, via our fixed choice of compatible roots of unity, with the Galois group of the Kummer extensions S_n/R (thus $\bar 1\in\mathbb{Z}_n$ acts on S_n via $t^{1/n}\mapsto \zeta_n t^{1/n}$). In particular, $\pi_1(R)$ is a cyclic group (in the profinite sense) with canonical generator $1\in\mathbb{Z}\subset\widehat{\mathbb{Z}}$. A 1-cocycle $v\in Z^1(\pi_1(R),\mathrm{F})$

[2] Allegorical allusion to the essence of torsors as forms.

is thus completely determined by (and is henceforth identified with) its value v_1 at 1. Because F_R is constant, two 1-cocycles are cohomologous if their values at 1 are conjugate. Thus

$$H^1(R, \mathrm{F}_R) \simeq \text{Set of conjugacy classes of F},$$

which is suggestively related to the right-hand side of the Principle! In fact part (i) of Theorem 2.1, which is nothing but the R-version of the Principle, follows from the following crucial fact.

$$\text{The canonical map } \psi : H^1(R, \mathbf{Aut}\, A_R) \to H^1(R, \mathrm{F}_R) \text{ is injective.} \tag{4}$$

Indeed, this map is given (with the identifications described above) by $[\mathcal{L}(A,\sigma)] \mapsto [\sigma^{-1}]$ where this last $[\cdot]$ stands for the conjugacy class in F of the element in question (see [1] and [10] for details).

To establish (4) one must show that $H^1(R, {}_{\mathfrak{Y}}\mathbf{G}_R) = 1$ vanishes for all $[\mathfrak{Y}] \in H^1(R, \mathbf{Aut}\, A_R)$. A complication arises from the fact that for $[\mathfrak{Y}] \neq 1$ the twisted R-groups ${}_{\mathfrak{Y}}\mathbf{G}_R$ do not come from k-groups. We will establish this result for $\mathbf{G}$ reductive in the next section. For $\mathbf{G}$ unipotent, one reduces to the abelian case and concludes from $H^1(R, \mathbf{GL}_{n\,R}) = 1$.

The passage from R-isomorphism to k-isomorphism that establishes part (ii) of the theorem goes as follows. Assume without loss of generality that A is not the zero algebra. Any k-algebra isomorphism between two loop algebras $L(A,\sigma)$ and $L(A,\tau)$ induces an isomorphism between their centroids. Under our assumptions, these centroids are canonically isomorphic to R. As a consequence (see Section 4 of [1] for details) one obtains

$$L(A,\sigma) \simeq_{k\text{-alg}} L(A,\tau) \Leftrightarrow L(A,\sigma) \simeq_{R\text{-alg}} L(A,\tau^{\pm 1}) \Leftrightarrow \bar{\sigma} \sim \bar{\tau}^{\pm 1}, \tag{5}$$

where this last equivalence comes again from the torsor interpretation.

Remark 1. The k-algebra A is an *invariant* of the loop construction. For if $L(A_1,\sigma_1) \simeq_{k\text{-alg}} L(A_2,\sigma_2)$ we use (1) and (5) to conclude that $A_1 \otimes S_m \simeq A_2 \otimes S_m$ as S_m-algebras when m is the product of the orders of σ_1 and σ_2. A specialization argument now readily yields $A_1 \simeq A_2$.

Remark 2. Theorem 2.1 remains valid for arbitrary algebras as long as the R-group $S \mapsto \mathrm{Aut}_{S\text{-alg}}\, A \otimes S$ is representable by a linear algebraic group.

3. Vanishing of H^1 for Dedekind rings

As we saw above, the validity of the Principle for certain loop algebras hinges around the vanishing of H^1 for certain reductive group schemes over R. That such vanishing does take place is a consequence of the following result (a version of 'Serre's Conjecture I' for Dedekind domains which we hope may be of some independent interest).

Theorem 3.1. *Let $\mathfrak{X} = \mathrm{Spec}\, D$ be a Dedekind scheme whose generic fiber is the spectrum of a field K of dimension* 1.

(i) *The following two conditions are equivalent*:
  (a) $H^1(\mathfrak{X}, \mathbf{G}) = 1$ *for every reductive $\mathfrak{X}$-group* $\mathbf{G}$.
  (b) $\mathrm{Pic}(\mathfrak{X}') = 1$ *for every finite connected étale cover $\mathfrak{X}'$ of $\mathfrak{X}$.*

(ii) *If the conditions of* (i) *above hold then*:
  (a) $H^2(\mathfrak{X}, \mathbf{M}) = 1$ *for every multiplicative $\mathfrak{X}$-group* $\mathbf{M}$ *of finite type.*
  (b) *Every reductive $\mathfrak{X}$-group is quasisplit.*

Proof. (i) From basic properties of Weil restrictions we have $H^1(\mathfrak{X}, \mathcal{R}_{\mathfrak{X}'/\mathfrak{X}}\mathbf{G}_{m_{\mathfrak{X}'}}) = \mathrm{Pic}(\mathfrak{X}')$ so clearly (a) $\Rightarrow$ (b).

Assume (b) holds. Let $\mathfrak{B}/\mathfrak{X}$ be the (projective and smooth) scheme of Borel subgroups of **G**. By passing to the generic fiber and applying the usual 'Conjecture I' ([11] Chapter III Theorem 2.3.1′), we conclude that $\mathfrak{B}$ admits a section over a nonempty open set $\mathfrak{U} \subset \mathfrak{X}$. Since $\mathfrak{X}$ is regular of dimension 1 this section extends to $\mathfrak{X}$ and therefore **G** has a Borel subgroup **B**.

Let $[\mathfrak{Y}] \in H^1(\mathfrak{X}, \mathbf{G})$. By [4] Exp. XXIV Proposition 4.2.1 $\mathfrak{Y}/\mathbf{B} \simeq \mathfrak{Y} \times^{\mathbf{G}} \mathbf{G}/\mathbf{B}$ is a projective and smooth scheme over $\mathfrak{X}$. Again by passing to the generic fiber and reasoning as above, we see that the structure morphism $q : \mathfrak{Y} \times^{\mathbf{G}} \mathbf{G}/\mathbf{B} \to \mathfrak{X}$ admits a section. From this it follows that $\mathfrak{Y}$ comes from a **B**-torsor ([3] Chapter III Proposition 4.4.6(b)), hence by dévissage from a **T**-torsor where $\mathbf{T} \subset \mathbf{B}$ is a maximal torus of **G** ([4] Exp. XXII, Corollary 5.9.7). Thus, we must show that $H^1(\mathfrak{X}, \mathbf{T}) = 1$. Now since $\mathfrak{X}$ is normal and Noetherian one knows that **T** is isotrivial ([4] Exp. X Théorème 5.16). One can therefore apply Theorem 4.1(iii) of [2] to conclude that the restriction map $H^1(\mathfrak{X}, \mathbf{T}) \to H^1(K, \mathbf{T}_K)$ is injective. Since by the usual Conjecture I this last H^1 is trivial, (i) follows.

(ii) As before, our **M** in (a) is isotrivial. From [2] Proposition 1.3 we obtain an exact sequence

$$H^1(\mathfrak{X}, \mathbf{Q}) \to H^2(\mathfrak{X}, \mathbf{M}) \to H^2(\mathfrak{X}, \mathbf{P})$$

where **Q** and **P** are $\mathfrak{X}$-tori with **P** quasitrivial. The two external terms of this sequence vanish (the left H^1 by part (i) of the theorem, the right H^2 by Shapiro's Lemma and basic properties of the Brauer group). Now (a) follows.

Finally, a reductive $\mathfrak{X}$-group **G** is an étale twisted form of a (split) Chevalley group $\mathbf{G}_{\mathrm{dep}}$. By part (i) of the theorem and [4] Exp. XXIV Théorèmes 1.3(a) and 3.11, such twisted forms are parametrized by $H^1(X, \mathbf{AutExtG}_{\mathrm{dep}})$ and are all quasisplit.

This completes the proof of Theorem 3.1. If k is algebraically closed of characteristic 0, the Dedekind scheme $\mathfrak{X} = \operatorname{Spec} k[t, t^{-1}]$ satisfies condition (i)(b) of Theorem 3.1. The proof of Theorem 2.1 is therefore also complete.

Remark 3. Under the assumptions of Theorem 3.1, the map in (4) would appear to be surjective. The obstructions lie on various $H^2(\mathfrak{X}, \mathcal{L})$ for liens based on our group **G**. Because **G** is reductive, these $H^2(\mathfrak{X}, \mathcal{L})$ contains neutral classes ([5] Proposition 1.2). On the other hand $H^2(\mathfrak{X}, \mathcal{L})$ is a principal homogeneous spaces under $H^2(\mathfrak{X}, \mathbf{Z}(\mathbf{G})) = 1$. It follows that each $H^2(\mathfrak{X}, \mathcal{L})$ consists of a single neutral class. All obstructions to surjectivity therefore vanish (see [6] for details). The interpretation of this result within our context is that on the étale site of R, *any* form of the R-algebra $A \otimes R$ is a loop algebra.

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