Available online at www.sciencedirect.com
SCIENCE DIRECT®

C. R. Acad. Sci. Paris, Ser. I 338 (2004) 271–276

COMPTES RENDUS MATHEMATIQUE

Group Theory/Lie Algebras

New properties of lattices in Lie groups

Fritz Grunewald [a], Vladimir Platonov [b]

[a] *Mathematisches Institut, Heinrich Heine Universität, 40225 Düsseldorf, Germany*
[b] *Max-Planck-Institut für Mathematik, Vivatsgasse 7, 53111 Bonn, Germany*

Received 24 July 2003; accepted after revision 19 December 2003

Presented by Jean-Pierre Serre

Abstract

We study finite extension groups of lattices in Lie groups which have finitely many connected components. We show that every non-cocompact Fuchsian group (these are the non-cocompact lattices in $\mathbf{PSL}(2,\mathbb{R})$) has an extension group of finite index which is not isomorphic to a lattice in a Lie group with finitely many connected components. On the other hand we prove that these are, in an appropriate sense, the only lattices in Lie groups which have extension groups of this kind. We also show that an extension group of finite index of a lattice in a Lie group with finitely many connected components has only finitely many conjugacy classes of finite subgroups. ***To cite this article: F. Grunewald, V. Platonov, C. R. Acad. Sci. Paris, Ser. I 338 (2004).***

Résumé

Nouvelles propriétés des réseaux dans les groupes de Lie. On étudie les extensions finies de réseaux dans les groupes de Lie n'ayant qu'un nombre fini de composantes connexes. Nous démontrons que tout groupe fuchsien (ce sont les réseaux non-cocompacts dans $\mathbf{PSL}(2,\mathbb{R})$) possède une extension finie qui n'est isomorphe à aucun réseau dans un groupe de Lie ayant un nombre fini de composantes connexes. D'autre part, nous démontrons que ces groupes sont les seuls, parmi les réseaux dans les groupes de Lie, pour lesquels il existe de telles extensions finies. Nous montrons aussi qu'une extension finie d'un réseau dans un groupe de Lie ayant un nombre fini de composantes connexes n'a qu'un nombre fini de classes de conjugaison de sous-groupes finis. ***Pour citer cet article : F. Grunewald, V. Platonov, C. R. Acad. Sci. Paris, Ser. I 338 (2004).***

Version française abrégée

Soit G un groupe de Lie réel n'ayant qu'un nombre fini de composantes connexes. Un *réseau dans* G est un sous-groupe discret $\Gamma \leqslant G$ tel que l'espace homogène G/Γ possède une mesure G-invariante finie. Un réseau Γ de G s'appelle *cocompact* si G/Γ est compact. Un groupe abstrait est dit un *réseau* s'il est isomorphe à un réseau dans un groupe de Lie ayant un nombre fini de composantes connexes. Nous donnons une description de tous les réseaux (dans les groupes de Lie ayant un nombre fini de composantes connexes) qui ont la propriété suivante : chaque extension finie est encore un réseau.

E-mail addresses: fritz@math.uni-duesseldorf.de (F. Grunewald), vplatonov@sympatico.ca (V. Platonov).

1631-073X/$ – see front matter
doi:10.1016/j.crma.2003.12.021

Notre premier théorème s'occupe du cas $G = \mathbf{PSL}(2,\mathbb{R})$.

Théorème 0.1. *Soit Γ un réseau dans* $\mathbf{PSL}(2,\mathbb{R})$. *On a* :

(i) *Si Γ est cocompact et Δ est une extension finie de Γ, alors Δ est un réseau.*
(ii) *Si Γ n'est pas cocompact, alors Γ possède une extension finie qui n'est pas un réseau.*

La construction des extensions qui servent à démontrer le Théorème 0.1(ii) se trouve dans la Section 2.
Notre deuxième résultat affirme que ces phénomènes sont uniquement liés au groupe de Lie $\mathbf{PSL}(2,\mathbb{R})$.

Théorème 0.2. *Soit Γ un réseau dans un groupe de Lie réel ayant un nombre fini de composantes connexes. On suppose que la composante connexe de l'identité G° n'a aucun facteur simple isomorphe à* $\mathbf{PSL}(2,\mathbb{R})$ *tel que la projection de $\Gamma \cap G^\circ$ dans ce facteur soit discrète mais non cocompacte. Si Δ est une extension finie de Γ, alors Δ est un réseau.*

Nous démontrons aussi que tout groupe qui est extension finie d'un réseau n'a qu'un nombre fini de classes de conjugaison de sous-groupes finis. Ce résultat se déduit de l'existence de domaines fondamentaux pour un réseau ayant des propriétés de finitude convenables. Soit $\mathbf{g}$ un groupe fini opérant sur un groupe Γ ; alors le premier ensemble de cohomologie $H^1(\mathbf{g},\Gamma)$ est défini par Borel et Serre dans [1]. Nous obtenons le théorème de finitude suivant :

Théorème 0.3. *Soit Γ un réseau et soit* $\mathbf{g}$ *un groupe fini opérant sur Γ. Alors $H^1(\mathbf{g},\Gamma)$ est fini.*

Ce résultat a été démontré par Borel et Serre dans [1] lorsque Γ est un $\mathbf{g}$-groupe arithmétique, et plus généralement pour Γ un groupe arithmétique dans [5].

1. Introduction

Let G be a real Lie group with finitely many connected components. A *lattice in G* is a discrete subgroup $\Gamma \leqslant G$ so that the homogeneous space G/Γ carries a finite G-invariant measure. A lattice Γ in G is called *cocompact* if G/Γ is compact. An abstract group is said to be a *lattice* if it is isomorphic to a lattice in a real Lie group which has finitely many connected components. The main aim of this work is concerned with the structure of finite extension groups of lattices. We call a group Δ a *finite extension* of a group Γ if Δ contains an isomorphic copy of Γ of finite index. The following surprising result shows that not every finite extension of a lattice is again a lattice.

Theorem 1.1. *Let Γ be a lattice in* $\mathbf{PSL}(2,\mathbb{R})$. *Then the following hold*:

(i) *If Γ is cocompact and Δ is a finite extension of Γ then Δ is a lattice.*
(ii) *If Γ is not cocompact then Γ has a finite extension which is not a lattice.*

Part (i) of Theorem 1.1 is proved in [5]. The proof uses Kerkhoff's positive solution of the Nielsen realisation problem, see [7]. This method can be used to describe more specifically the Lie group in which a given finite extension of a cocompact lattice in $\mathbf{PSL}(2,\mathbb{R})$ can be realised as a lattice, see Section 2. The finite extensions which we use to prove part (ii) of Theorem 1.1 can be found in Section 2.

By the following theorem the lattices mentioned in part (ii) of Theorem 1.1 are in an appropriate sense the only lattices which have finite extensions which are not lattices.

Theorem 1.2. *Let Γ be a lattice in a real Lie group G which has finitely many connected components. Assume that the connected component of the identity G° does not have a simple factor isomorphic to $\mathbf{PSL}(2,\mathbb{R})$ such that the projection of $\Gamma \cap G(\mathbb{R})^\circ$ into this factor is discrete but not cocompact. If Δ is a finite extension group of Γ, then Δ is a lattice.*

We have the following generalization of Theorem 1.1(i):

Corollary 1.3. *Let Γ be a cocompact lattice in a real Lie group G which has finitely many connected components. Then every finite extension of Γ is a lattice.*

A group Δ is called polycyclic by finite if it contains a polycyclic subgroup of finite index. It is well known that every polycyclic by finite group contains a subgroup of finite index which is a lattice in a simply connected Lie group (see [10], Chapter 3). This result together with Theorem 1.2 leads to:

Theorem 1.4. *Every polycyclic by finite group is a lattice in a finite extension of a simply connected solvable Lie group.*

One of the basic finiteness properties satisfied by lattices is that they have only finitely many conjugacy classes of finite subgroups. To prove this result one first considers the case in which Γ is a lattice in a semi-simple Lie group G. Then Γ acts discontinuously on the the symmetric space X of G. The finiteness property is implied by the existence of a suitable fundamental domain for the action of Γ on X. The general case follows by a simple reduction argument. Next we ask about the generalization of this finiteness result to groups Δ which are finite extensions of lattices Γ. There is no direct way to deduce the finiteness property for Δ from that of Γ since there is a finitely generated subgroup Δ of $\mathbf{SL}(4,\mathbb{Z})$ which contains infinitely many conjugacy classes of elements of order 4 (see [5]). This group Δ even contains a torsion-free subgroup of finite index. We prove as one of our main results:

Theorem 1.5. *Let Δ be a group which contains a lattice of finite index. Then Δ has only finitely many conjugacy classes of finite subgroups.*

If $\mathbf{g}$ is a finite group acting by group automorphisms on a group Γ then the first cohomology set $H^1(\mathbf{g},\Gamma)$ is defined by Borel and Serre in [1]. Our Theorem 1.5 implies:

Theorem 1.6. *Let Γ be a lattice and $\mathbf{g}$ a finite group acting on Γ by group automorphisms. Then $H^1(\mathbf{g},\Gamma)$ is finite.*

Problems like the above were already studied in the following formally similar but different situation. Let $G < \mathbf{GL}(n,\mathbb{C})$ be a linear algebraic group G defined over $\mathbb{Q}$ and $\Gamma \leqslant G$ an arithmetic subgroup. The paper [5] contains a criterion similar to Theorem 1.2 for a finite extension group Δ of Γ to be an arithmetic group. This paper also gives examples in which Δ is not an arithmetic group. The paper [6] is devoted to the study of the solvable case. Note here that in contrast to our Theorem 1.4 there are interesting polycyclic groups which are not arithmetic groups. Theorem 1.5 was proved by Borel and Serre in [1] for split extensions $\Gamma \rtimes \mathbf{g}$ where $\mathbf{g}$ is a finite group and Γ is a $\mathbf{g}$-arithmetic group. Its generalization to any finite extension group of an arithmetic group is contained in [5]. Also Theorem 1.6 was proved by Borel and Serre in [1] for a $\mathbf{g}$-arithmetic group Γ and generalized in [5].

2. Proof of Theorem 1.1

This section mainly contains the constructions which are necessary to prove part (ii) of Theorem 1.1.

As already mentioned part (i) is proved in [5]. We start off with a finite extension Δ of a cocompact lattice Γ in $\mathbf{PSL}(2,\mathbb{R})$. Assuming that Γ is torsion-free and normal in Δ we find that the centraliser $\mathbf{Z}_\Delta(\Gamma)$ of Γ in Δ is a finite normal subgroup of Δ. Moreover the finite subgroup $\Delta/(\Gamma\cdot\mathbf{Z}_\Delta(\Gamma)$ of the outer automorphism group of Γ has a fixed point acting on the Teichmüller space of Γ by Kerkhoff's result. This then implies that $\Delta/\mathbf{Z}_\Delta(\Gamma)$ is isomorphic to a cocompact discrete subgroup in $\mathbf{PSL}(2,\mathbb{R})$. In general we can prove that there is a finite extension H of $\mathbf{PSL}(2,\mathbb{R})$ so that Δ is isomorphic to a cocompact lattice in H.

Let now Γ be a lattice in $\mathbf{PSL}(2,\mathbb{R})$ which is not cocompact in $\mathbf{PSL}(2,\mathbb{R})$. It is well known that Γ is the free product of $\ell\in\mathbb{N}\cup\{0\}$ infinite cyclic groups and $n\in\mathbb{N}\cup\{0\}$ finite cyclic groups that is

$$\Gamma=\left\langle h_1,\dots,h_\ell,\ e_1,\dots,e_n \mid e_1^{m_1}=\dots=e_n^{m_n}=1\right\rangle\cong\mathbb{Z}*\dots*\mathbb{Z}*\mathbf{C}_{m_1}*\dots*\mathbf{C}_{m_n} \tag{1}$$

see [4]. Here $\mathbf{C}_m$ stands for the finite cyclic group of order m. The condition that the group Γ defined in (1) is isomorphic to a (non-cocompact) lattice in $\mathbf{PSL}(2,\mathbb{R})$ is:

$$\ell-1+\sum_{i=1}^{n}\left(1-\frac{1}{m_i}\right)>0. \tag{2}$$

Also it is well-known that a lattice in $\mathbf{PSL}(2,\mathbb{R})$ is not cocompact if and only if it contains a free subgroup of finite index.

For each of the groups listed in (1) we shall construct an extension group which we prove not to be a lattice. In these proofs we shall use the following result as a first reduction step.

Proposition 2.1. *Let Δ be a finite extension of a lattice Γ in* $\mathbf{PSL}(2,\mathbb{R})$. *Assume that Δ is a lattice in the real Lie group G. Then G can be expressed as $G=H\cdot K$ where H, K are closed normal subgroups of G with $H\cap K$ being a finite subgroup of the center of G and where H is a finite extension of* $\mathbf{PSL}(2,\mathbb{R})$ *and K is compact. If Δ has no nontrivial finite normal subgroups then Δ is isomorphic to a lattice in* $\mathbf{PGL}(2,\mathbb{R})$.

The first statement of Proposition 2.1 follows from results of Prasad [9] (see [5], Section 2 for more details). If Δ has no nontrivial finite normal subgroups we can choose the compact subgroup K to be trivial and Δ is a lattice in the finite extension H of $\mathbf{PSL}(2,\mathbb{R})$. Consider then the homomorphism

$$\Phi: H\to \mathrm{Aut}(H^\circ)=\mathbf{PGL}(2,\mathbb{R}),\qquad \Phi(h)(g):=hgh^{-1}\quad (h\in H,\ g\in H^\circ).$$

The kernel of Φ is a finite normal subgroup of H. This shows that $\Phi(\Delta)$ is a lattice in $\mathbf{PGL}(2,\mathbb{R})$ isomorphic to Δ.

In constructing the finite extension groups we make the following case distinction:

1. $\ell=2,\ n=0.$

Here $\Gamma=\langle h_1,\ h_2\rangle$ is a free group on 2 generators. Consider the following three automorphisms of Γ:

$$\sigma_1(h_1):=h_1^{-1},\quad \sigma_2(h_2):=h_2^{-1},\quad \sigma_3(h_1):=h_2,\quad \sigma_3(h_2):=h_1 \tag{3}$$

with the convention that σ_i acts identically on the generators not mentioned. Let A be the subgroup of the automorphism group of Γ generated by σ_1, σ_2, σ_3 and put $\Delta:=\Gamma\rtimes A$. The group A is isomorphic to the dihedral group of order 8, hence Δ contains Γ as a subgroup of index 8. The group Δ is not a lattice. To see this notice first that Δ has no nontrivial finite normal subgroups. We infer from the Proposition 2.1 that if Δ is a lattice then it is a lattice in $\mathbf{PGL}(2,\mathbb{R})$. We identify Δ with its image in $\mathbf{PGL}(2,\mathbb{R})$ and put $\Delta^\circ:=\Delta\cap\mathbf{PSL}(2,\mathbb{R})$. By the above Δ° is a noncocompact lattice in $\mathbf{PSL}(2,\mathbb{R})$ and has a decomposition (1). It follows from Kurosh's theorem that every Abelian subgroup of Δ° is cyclic. Hence Δ° is of index 2 in Δ. We also find that $\Delta^\circ\cap A$ is cyclic of order 4 and hence is generated by $\sigma_3\sigma_1$. This also implies $\sigma_2\sigma_1=(\sigma_3\sigma_1)^2$ is in Δ° but σ_1 is not in Δ°. Notice that $\sigma_2\sigma_1$ is an involution centralized by σ_1. A simple computation shows that the centralizer in $\mathbf{PGL}(2,\mathbb{R})$ of every involution from $\mathbf{PSL}(2,\mathbb{R})$ is contained in $\mathbf{PSL}(2,\mathbb{R})$. This finishes the proof that Δ is not a lattice.

From a presentation of Δ it is easy to see that there is a unique subgroup of index 2 in Δ which is isomorphic to a lattice in $\mathbf{PSL}(2,\mathbb{R})$. This subgroup is isomorphic to $\mathbf{C}_2 * \mathbf{C}_4$.

2. $\ell \geqslant 2,\ \ell + n \geqslant 3$.

In this case the lattice Γ needs generators $h_1,\ h_2$ and at least one more generator f (equal to h_3 or e_1) in the product decomposition (1). Let A be the subgroup of the automorphism group of Γ generated by $\sigma_1,\ \sigma_2,\ \sigma_3$ given in (3) (with the convention that σ_i acts identically on the generators not mentioned) and put $\Delta := \Gamma \rtimes A$. Then Δ contains Γ as a subgroup of index 8 and Δ is not a lattice. To see this notice again that Δ has no nontrivial finite normal subgroups. We infer from the Proposition 2.1 that if Δ is a lattice then it is a lattice in $\mathbf{PGL}(2,\mathbb{R})$. Let B be the Zariski closure of the subgroup $T := \langle h_2,\ f\rangle$ in $\mathbf{PGL}(2,\mathbb{R})$. Since T is not solvable B contains $\mathbf{PSL}(2,\mathbb{R})$. The element σ_1 centralizes T and hence B which is a contradiction.

3. $\ell = 1$.

In this case we have $n \geqslant 1$. We consider first the case $n = 1$. Let Γ_1 be the kernel of the obvious homomorphism $\Gamma \to \mathbf{C}_{m_1}$, it is the subgroup (freely) generated by

$$h_1,\ h_2 := e_1 h_1 e_1^{-1}, \dots, h_{m_1} := e_1^{m_1-1} h_1 e_1^{1-m_1}. \tag{4}$$

The symmetric group $\mathcal{S}_{m_1}$ embedds in an obvious way into the automorphism group of Γ_1 by permuting the generators (4). Further we can embed $\mathbf{C}_2^{m_1}$ as a group A_0 of automorphism group of Γ_1 by letting $\varepsilon = (\varepsilon_1, \dots, \varepsilon_{m_1})$ ($\varepsilon_i = \pm 1$) act as $\varepsilon(h_i) := h_i^{\varepsilon_i}$. We let A be the subgroup of the automorphism group of Γ_1 generated by A_0 and $\mathcal{S}_{m_1}$. The group A is isomorphic to the permutational wreath product $A_0 \wr \mathcal{S}_{m_1}$ and has order $2^{m_1} m_1!$. Put $\Delta := \Gamma_1 \rtimes A$, then Γ is obviously isomorphic to a subgroup of index $2^{m_1}(m_1 - 1)!$ in Δ. The group Δ is not a lattice. For $m_1 = 2$ this group is isomorphic to the group, also called Δ, in the case $\ell = 0,\ n = 0$ above and we already know that Δ is not a lattice. For $m_1 \geqslant 3$ we note that Δ has no nontrivial finite normal subgroups and hence would embed as a lattice into $\mathbf{PGL}(2,\mathbb{R})$. But all finite subgroups of this group are either cyclic or dihedral groups.

If $n > 1$ we use analogously to the case $\ell \geqslant 2,\ \ell + n \geqslant 3$ a similar construction to find an extension group Δ of Γ which is not a lattice.

4. $\ell = 0$.

We first treat the case $n = 2$. In this case m_1 or m_2 is bigger than 2. If $m_1 = 2,\ m_2 = 3$ we let Γ_1 be the kernel of the obvious homorphism $\Gamma \to \mathbf{C}_2$. This group is isomorphic to $\mathbf{C}_3 * \mathbf{C}_3$ which has (similar to the case $\ell = 2,\ n = 0$) the dihedral group $\mathbf{D}_4$ of order 8 in its automorphism group. We put $\Delta := \Gamma_1 \rtimes \mathbf{D}_4$. The group Δ contains Γ as a subgroup of index 4. By an argument analogous to that in the case $\ell = 2,\ n = 0$ the group Δ is not a lattice. If $m_1 = 2,\ m_2 = 4$ we noted in the case $\ell = 2,\ n = 0$ that there is an extension group of index 2 which is not a lattice. If $m_1 = 3,\ m_2 = 3$ or $m_1 = 3,\ m_2 = 4$ we let Γ_1 be the kernel of the obvious homorphism $\Gamma \to \mathbf{C}_{m_2}$ and find an appropriate permutational wreath product A in the automorphism group of Γ_1 (see $\ell = 1$). The split extension $\Delta := \Gamma_1 \rtimes A$ contains a copy of Γ as a subgroup of finite index and is not a lattice. In all other cases we reorder $m_1,\ m_2$ so that $m_2 \geqslant m_1$ and let Γ_1 be the kernel of the obvious homorphism $\Gamma \to \mathbf{C}_{m_2}$. This group is isomorphic to the free product of m_2 copies of $\mathbf{C}_{m_1}$. Hence Γ_1 contains the symmetric group $\mathcal{S}_{m_2}$ in its automorphism group. The group $\Delta := \Gamma_1 \rtimes \mathcal{S}_{m_2}$ contains Γ as a subgroup of index $(m_2 - 1)!$. By arguments used before Δ is not a lattice. All cases $n \geqslant 3$ can be treated similarly to the last case. Let m_n satisfy $m_n \geqslant m_i$ for $i = 1, \dots, n$ and let Γ_1 be the kernel of the obvious homorphism $\Gamma \to \mathbf{C}_{m_n}$. This group is isomorphic to a free product of finite cyclic groups and contains an appropriate permutation group $\mathcal{S}$ in its automorphism group. The group $\Delta := \Gamma_1 \rtimes \mathcal{S}$ contains a copy of Γ as a subgroup of finite index. By our usual arguments Δ is not a lattice.

Let Γ be a lattice in $\mathbf{PSL}(2,\mathbb{R})$ which is not cocompact. Let $m(\Gamma)$ be the minimal index of an extension group Δ of Γ which is not a lattice. The question whether $m(\Gamma) = 2$ (in all cases) leads, for small ℓ and n in the decomposition (1), to very interesting problems about the existence of lattices in $\mathbf{PSL}(2,\mathbb{R})$.

3. Proof of Theorem 1.2

The strategy of the proof of Theorem 1.2 is the same as for the proof of Theorem 1.2 in [5]. In the arguments $\mathbb{Q}$-defined algebraic maps have to be replaced by analytic maps and arithmetic groups have to be replaced by lattices. The results then translate more or less word for word. We give a brief sketch.

Let Γ be a lattice in the Lie group G and Δ a finite extension of Γ. We may assume that Γ is a normal subgroup of Δ. If the maps from $\Gamma \to \Gamma$ which are induced by conjugation with elements from Δ can be obtained by restriction of analytic maps from G to G then Δ can be proved to be a lattice in a finite extension of the Lie group G (see [5], Section 2). If G is semisimple and Γ is an irreducible lattice in G this follows from standard rigidity results. If Γ is not irreducible Lemma 2.7 and Corollary 2.8 from [5] are easily adapted. For Lie groups G which have a nontrivial solvable radical the rigidity results of Section 3 of [5] can be translated almost word for word from the algebraic-arithmetic to the analytic-lattice world.

4. Conjugacy classes of finite subgroups

For the proof of Theorem 1.5 a special case has to be treated beforehand.

Proposition 4.1. *Let $\Gamma = \Gamma_1 \times \cdots \times \Gamma_k$ be a direct product of finitely many groups which are lattices in $\mathbf{PSL}(2,\mathbb{R})$. Then every finite extension group Δ of Γ has only finitely many conjugacy classes of finite subgroups.*

If $k = 1$ and if Γ_1 is not cocompact in $\mathbf{PSL}(2,\mathbb{R})$ then Δ is a finite extension of a finitely generated free group. We then use the fact that such a group is the fundamental group of a finite graph of groups (see [3]) together with the fixed point theorem for finite groups acting on trees (see [2]). If $k = 1$ and if Γ_1 is cocompact in $\mathbf{PSL}(2,\mathbb{R})$ then Theorem 1.1 implies our result. Direct products are treated by an induction technique using the fixed point theorem for finite groups acting on CAT(0)-spaces (see [2], Section 1).

A statement similar to Proposition 4.1 but for finite extensions of free groups of infinite rank does not hold. The minimal counterexample (that is $[\Delta : F_\infty] = 2$) is given in [8].

Sketch of the proof of Theorem 1.5. By factoring out the (connected) solvable radical of G we reduce to a case when G is semisimple. The above Proposition 4.1 allows us to reduce our proof to the case of a semisimple Lie group without factors isomorphic to $\mathbf{PSL}(2,\mathbb{R})$. We then use Theorem 1.2 and the existence of fundamental domains with Siegel's property for a lattice in a semisimple Lie group. □

References

[1] A. Borel, J.-P. Serre, Théorèmes de finitude en cohomologie galoisienne, Comment. Math. Helv. 39 (1964) 111–164; A. Borel, Collected Papers, vol. II, Springer-Verlag, 1983, pp. 362–415.
[2] M.R. Bridson, Geodesics and curvature in metric simplicial complexes, in: E. Ghys, A. Haefliger, A. Verjovsky (Eds.), Group Theory From a Geometric Viewpoint, World Scientific, 1991, pp. 373–463.
[3] W. Dicks, M.J. Dunwoody, Groups Acting on Graphs, Cambridge University Press, 1989.
[4] L. Greenberg, Finiteness theorems for Fuchsian and Kleinian groups, in: E.W. Harvey (Ed.), Discrete Groups and Automorphic Functions, Academic Press, 1977, pp. 199–255.
[5] F. Grunewald, V. Platonov, Rigidity results for groups with radical, cohomology of finite groups and arithmeticity problems, Duke Math. J. 100 (1999) 321–358.
[6] F. Grunewald, V. Platonov, Solvable arithmetic groups and arithmeticity problems, Int. J. Math. 10 (1999) 327–366.
[7] S.P. Kerckhoff, The Nielsen realization problem, Ann. Math. 117 (1983) 235–265.
[8] V. Platonov, The theory of algebraic linear groups and periodic groups, Amer. Math. Soc. Transl. 69 (1968) 61–110.
[9] G. Prasad, Discrete subgroups isomorphic to lattices in Lie groups, Amer. J. Math. 98 (1976) 853–863.
[10] M.S. Raghunathan, Discrete Subgroups of Lie Groups, in: Ergeb. Math. Grenzgeb., vol. 68, Springer-Verlag, 1972.