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Ann. I. H. Poincaré – AN 29 (2012) 737–759

ANNALES DE L'INSTITUT HENRI POINCARÉ
ANALYSE NON LINÉAIRE

www.elsevier.com/locate/anihpc

Global existence and long-time behavior of smooth solutions of two-fluid Euler–Maxwell equations

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Received 28 June 2011; received in revised form 15 March 2012; accepted 3 April 2012

Available online 23 April 2012

Abstract

We consider Cauchy problems and periodic problems for two-fluid compressible Euler–Maxwell equations arising in the modeling of magnetized plasmas. These equations are symmetrizable hyperbolic in the sense of Friedrichs but don't satisfy the so-called Kawashima stability condition. For both problems, we prove the global existence and long-time behavior of smooth solutions near a given constant equilibrium state. As a byproduct, we obtain similar results for two-fluid compressible Euler–Poisson equations.

MSC: 35L45; 35L60; 35Q60

Keywords: Two-fluid flows; Euler–Maxwell equations; Partially dissipative hyperbolic systems; Global smooth solutions; Long-time behavior; Energy estimates

1. Introduction

Euler–Maxwell equations arise in the modeling of magnetized plasmas under conditions on the frequency collision of particles. One example is the modeling of ionospheric plasmas. Let n_e and u_e (respectively, n_i and u_i) be the density and velocity vector of the electrons (respectively, ions), E and B be, respectively, the electric field and magnetic field of a magnetized plasma. The fields E and B are coupled to (n_ν, u_ν), $\nu = e, i$, through the Maxwell equations and act on them via the Lorentz force. These variables are functions of the time $t > 0$ and the position $x \in \Omega \subset \mathbb{R}^3$. In this paper, we consider the Cauchy problem in $\Omega = \mathbb{R}^3$ and the periodic problem in a three-dimensional torus $\Omega = \mathbb{T}^3 \stackrel{def}{=} (\mathbb{R}/\mathbb{Z})^3$.

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0294-1449/$ – see front matter
http://dx.doi.org/10.1016/j.anihpc.2012.04.002

The two-fluid compressible Euler–Maxwell system reads (see [4,7,30]):

$$\begin{cases} \partial_t n_\nu + \operatorname{div}(n_\nu u_\nu) = 0, \\ \partial_t (n_\nu u_\nu) + \operatorname{div}(n_\nu u_\nu \otimes u_\nu) + \nabla p_\nu(n_\nu) = q_\nu n_\nu (E + u_\nu \times B) - n_\nu u_\nu, \\ \partial_t E - \nabla \times B = -(q_e n_e u_e + q_i n_i u_i), \quad \operatorname{div} E = n_i - n_e, \\ \partial_t B + \nabla \times E = 0, \quad \operatorname{div} B = 0, \quad \nu = e, i, \end{cases} \tag{1.1}$$

for $(t, x) \in (0, \infty) \times \Omega$. Here $\otimes$ stands for the tensor product, $q_e = -1$ (respectively, $q_i = 1$) is the charge of the electron (respectively, ion), and p_ν is the pressure function. Throughout this paper, we suppose that p_ν is smooth and strictly increasing on $(0, +\infty)$. This includes the usual state equation of γ-law:

$$p_\nu(n) = n^\gamma, \quad \text{with } \gamma \geqslant 1.$$

System (1.1) is complemented by initial conditions:

$$t = 0: \quad (n_\nu, u_\nu, E, B) = \left(n_\nu^0, u_\nu^0, E^0, B^0\right), \quad \nu = e, i, \quad \text{in } \Omega. \tag{1.2}$$

In (1.1), all physical parameters are set equal to unity. This is not an essential restriction in the study of global existence of smooth solutions. Otherwise, the smallness conditions on the initial data in the main results Theorems 1.1–1.2 would depend on the parameters. We refer to [27] for descriptions and formal asymptotic analysis for (1.1) with various physical parameters. In the momentum equations (1.1), the quantity $n_\nu(E + u_\nu \times B)$ stands for the Lorentz force and $-n_\nu u_\nu$ is the relaxation damping. For smooth solutions with $n_\nu > 0$, these equations are equivalent to

$$\partial_t u_\nu + (u_\nu \cdot \nabla) u_\nu + \nabla h_\nu(n_\nu) = q_\nu (E + u_\nu \times B) - u_\nu,$$

where $\cdot$ denotes the inner product of $\mathbb{R}^3$ and h_ν is the enthalpy function defined by

$$h_\nu'(n) = \frac{p_\nu'(n)}{n}.$$

Since p_ν is strictly increasing on $(0, +\infty)$, so is h_ν. It is well known that the constraint equations

$$\operatorname{div} E = n_i - n_e, \qquad \operatorname{div} B = 0 \tag{1.3}$$

are compatible with another equations in (1.1). They hold for $t > 0$ if and only if the prescribed initial data satisfy (1.3):

$$\operatorname{div} E^0 = n_i^0 - n_e^0, \qquad \operatorname{div} B^0 = 0. \tag{1.4}$$

The Euler–Maxwell system (1.1) is nonlinear and symmetrizable hyperbolic for $n_\nu > 0$ in the sense of Friedrichs (see [12] and Section 2). Then, according to the result of Kato [18], the Cauchy problem or the periodic problem (1.1)–(1.2) has a unique local smooth solution when the initial data are smooth. Here we are concerned with stabilities of global smooth solutions to (1.1)–(1.2) near a constant state being a particular solution of (1.1). It is easy to see that this constant state is necessarily given by

$$(n_e, n_i, u_e, u_i, E, B) = (\bar{n}, \bar{n}, 0, 0, 0, \bar{B}) \in \mathbb{R}^{14}.$$

For simplicity, in what follows we set $\bar{n} = 1$. In the study of the long-time behavior of solutions in the periodic case, we further assume

$$\int_{\mathbb{T}^3} n_\nu^0(x)\, dx = 1, \qquad \int_{\mathbb{T}^3} B^0(x)\, dx = \bar{B}, \quad \nu = e, i. \tag{1.5}$$

Using the equations for n_ν and B, we see that for $\nu = e, i$,

$$\int_{\mathbb{T}^3} n_\nu(t, x)\, dx \quad \text{and} \quad \int_{\mathbb{T}^3} B(t, x)\, dx$$

are conservative quantities for all time $t \geqslant 0$.

Proposition 1.1 *(Local existence of smooth solutions). (See [18,20].) Let $\Omega = \mathbb{R}^3$ or $\Omega = \mathbb{T}^3 = (\mathbb{R}/\mathbb{Z})^3$. Let $\bar{B} \in \mathbb{R}^3$ be any given constant and $s \geqslant 3$ be an integer. Suppose (1.4) holds and for $\nu = e, i$, $(n_\nu^0 - 1, u_\nu^0, E^0, B^0 - \bar{B}) \in H^s(\Omega)$ with $n_\nu^0 \geqslant 2\kappa$ for some given constant $\kappa > 0$. Then there exists $T > 0$ such that problem (1.1)–(1.2) has a unique smooth solution satisfying $n_\nu \geqslant \kappa$ in $[0, T] \times \Omega$ and*

$$(n_\nu - 1, u_\nu, E, B - \bar{B}) \in C^1\big([0, T); H^{s-1}(\Omega)\big) \cap C\big([0, T); H^s(\Omega)\big), \quad \nu = e, i,$$

where $H^s(\Omega) = W^{s,2}(\Omega)$ is the usual Sobolev space.

In a plasma for which the ions are non-moving and become a uniform background with a fixed unit density, the evolution of electrons obeys a one-fluid Euler–Maxwell system:

$$\begin{cases} \partial_t n + \operatorname{div}(nu) = 0, \\ \partial_t u + (u \cdot \nabla)u + \nabla h(n) = -E - u \times B - u, \\ \partial_t E - \nabla \times B = nu, \quad \operatorname{div} E = 1 - n, \\ \partial_t B + \nabla \times E = 0, \quad \operatorname{div} B = 0. \end{cases} \tag{1.6}$$

For a simplified version of (1.6) in one-dimensional case, the global existence of entropy solutions was proved in [8] by the compensated compactness method. In the multi-dimensional case, the mathematical analysis of (1.6) was carried out only recently and the global existence of smooth solutions near constant states was obtained independently with different techniques in [10,28,33]. In the case without damping, an assumption of potential flows linked by u and B was made in [13] to yield such a global existence result. Let us mention also results for system (1.6) in [24–26,32] on the justification of asymptotic limits and in [9] on the numerical investigation.

In [28], the author of this paper (with S. Wang and Q.L. Gu) considered the periodic problem in (1.6). Using energy estimates, the global existence of smooth solutions was established for periodic initial data in $H^s(\mathbb{T}^3)$ for all integers $s \geqslant 3$ near a constant state $(n, u, E, B) = (1, 0, 0, \bar{B})$. The solution satisfies

$$(n, u, E, B) \in C^1\big([0, +\infty); H^{s-1}\big(\mathbb{T}^3\big)\big) \cap C\big([0, +\infty); H^s\big(\mathbb{T}^3\big)\big)$$

with a long-time asymptotic property for variables (n, u):

$$\lim_{t \to +\infty} \big\| \big(n(t) - 1, u(t)\big) \big\|_{s-1} = 0.$$

Here and in what follows $\| \cdot \|_s$ stands for the norm of $H^s(\mathbb{T}^3)$ (or $H^s(\Omega)$ with $\Omega = \mathbb{R}^3$ or $\Omega = \mathbb{T}^d$). Although these results are proved in the periodic problem they also hold in the whole space case in $\mathbb{R}^3$ without any difficulty in the proof. See the proofs in Sections 2 and 3 in the two-fluid case. However, the long-time asymptotic property for variables (E, B) has not been obtained and so far no results are available on the global existence of solutions to the two-fluid Euler–Maxwell system.

In this paper we prove that problem (1.1)–(1.2) admits a global smooth solution if the initial data are close enough to a constant state. Moreover, we establish the long-time behavior of solutions for all variables (n_ν, u_ν, E, B). These results are stated in Theorems 1.1 and 1.2.

For first order nonlinear symmetrizable hyperbolic systems, it is well known that, generically speaking, smooth solutions exist only locally in time and singularities may appear in a finite time (see [20] and the references therein). In the presence of dissipation terms in the system, however, the global existence of smooth solutions can be obtained. The dissipation terms are related to stability conditions of the system. Among them Kawashima stability condition plays an important role and is fulfilled by many physical models (see [16]). It was introduced in [31] in the study of asymptotic behavior of solutions for parabolic equations. Under this condition, the global existence of smooth solutions near a constant state of nonlinear symmetrizable hyperbolic systems of balance laws was proved in [16] in one space dimension and was extended in [34] to several space dimensions. The long-time asymptotics were given in [5]. We also refer to [3] for improved results and references therein on these questions. All these results show the stability of solutions near constant states. On the other hand, it is known that the Kawashima condition is sufficient but not necessary to the global existence of solutions, as showed by the examples given in [35,6] in one space dimension. In [28] we have proved that the one-fluid Euler–Maxwell system (1.6) does not satisfy this condition. For the sake of completeness, we check in Appendix A that the Kawashima condition also fails for system (1.1). Thus, the results in Theorems 1.1 and 1.2 are not trivial.

Now let us explain the main difference of proofs in the one and two-fluid Euler–Maxwell systems. From (1.1) it is easy to see that variable u_ν is dissipative. Using the classical energy method for symmetrizable hyperbolic systems, we get easily an energy estimate for u_ν in $L^2(0,T;H^s(\Omega))$ (see Lemma 2.4). Then the key step for proving the global existence with asymptotic properties of solutions is to control $n_\nu - 1$ in $L^2(0,T;H^s(\Omega))$. In the one-fluid Euler–Maxwell system (1.6), this is achieved in estimate

$$\begin{aligned}&\|w(t)\|_s^2 + \int_0^t \left(\|n(\xi)-1\|_s^2 + \|u(\xi)\|_s^2\right) d\xi\\ &\quad\leqslant C\|w(0)\|_s^2 + C\int_0^t \|w(\xi)\|_s\left(\|n(\xi)-1\|_s^2 + \|u(\xi)\|_s^2\right) d\xi, \qquad (1.7)\end{aligned}$$

provided that

$$\sup_{t\in[0,T]} \|w(t)\|_s \leqslant D,$$

where $w = (n-1, u, E, B-\bar{B})$, $C>0$ and $D>0$ are appropriate constants independent of T. In the two-fluid case, due to coupling terms, the proof of such an estimate is more technique. It is divided into two steps. In the first step, we show a weaker estimate than (1.7) (see (2.46) of Lemma 2.9) which is sufficient to prove the global existence and long-time behavior for (n_ν, u_ν). In the second step, we establish estimates for E and ∇B, respectively, in $L^2(0,T;H^{s-1}(\Omega))$ and $L^2(0,T;H^{s-2}(\Omega))$. Thus, by a classical argument together with the Sobolev and the Poincaré inequalities yields the long-time behavior for (E,B). We remark that in the whole space case, the long-time behavior of $n_\nu - 1$ holds only in a weaker space than that of u_ν, due to the absence of time dissipation estimates for $n_\nu - 1$. This is different from the one-fluid case.

Now we state the main results of this paper.

Theorem 1.1 *(Global existence of smooth solutions of the Euler–Maxwell system). Let $s \geqslant 3$ be an integer and $\bar{B}\in\mathbb{R}^3$ be any given constant. Under the assumptions of Proposition 1.1, there exists a constant $\delta_0>0$ independent of any given time $t>0$, such that if*

$$\left\|\left(n_\nu^0 - 1, u_\nu^0, E^0, B^0 - \bar{B}\right)\right\|_s \leqslant \delta_0, \quad \nu = e, i,$$

problem (1.1)–(1.2) admits a unique global smooth solution

$$(n_\nu - 1, u_\nu, E, B-\bar{B}) \in C^1\left([0,+\infty); H^{s-1}(\Omega)\right) \cap C\left([0,+\infty); H^s(\Omega)\right), \quad \nu = e, i.$$

Theorem 1.2 *(Long-time asymptotics of solutions of the Euler–Maxwell system). Under the assumptions of Theorem 1.1 and (1.5), the global smooth solution satisfies*

$$\lim_{t\to+\infty}\left\|\left(n_e(t)-n_i(t), u_\nu(t)\right)\right\|_{s-1} = 0, \qquad \lim_{t\to+\infty}\left\|\nabla n_\nu(t)\right\|_{s-2} = 0, \quad \forall \nu = e, i \qquad (1.8)$$

and

$$\lim_{t\to+\infty}\|E(t)\|_{s-1} = 0, \qquad \lim_{t\to+\infty}\|\nabla B(t)\|_{s-2} = 0. \qquad (1.9)$$

Moreover, for $\nu = e, i$ we have

$$\lim_{t\to+\infty}\left\|\left(n_\nu(t)-1, B(t)-\bar{B}\right)\right\|_{W^{s-2,6}(\mathbb{R}^3)} = 0, \quad \text{when } \Omega = \mathbb{R}^3, \qquad (1.10)$$

and

$$\lim_{t\to+\infty}\|n_\nu(t)-1\|_{H^{s-1}(\mathbb{T}^3)}, \qquad \lim_{t\to+\infty}\|B(t)-\bar{B}\|_{H^{s-2}(\mathbb{T}^3)} = 0, \quad \text{when } \Omega = \mathbb{T}^3. \qquad (1.11)$$

Remark 1.1. In a similar way, we may obtain estimates (1.9)–(1.11) for the smooth solution of the one-fluid Euler–Maxwell system (1.6). This yields the long-time behavior of the smooth solution for variables E and B in that case.

In comparison with the Euler–Maxwell systems, Euler–Poisson systems are another important class of equations due to their applications in semiconductors and plasma physics (see [7,22]). In the one-fluid multi-dimensional Euler–Poisson system, the global existence of smooth solutions was extensively studied by many authors. See for instance [14,1,17,15] and the references therein. We also refer to [21,29,36] for global entropy solutions in one space dimension. For the two-fluid multi-dimensional Euler–Poisson system, the only known result on the global solutions was proved in [2]. Remark that the Euler–Maxwell system and the Euler–Poisson system are essentially different due to the coupling terms and to the difference between the Poisson equation and the Maxwell equations. The rigorous derivation of the Euler–Poisson system from the Euler–Maxwell system was given in [24] via the non-relativistic limit. Finally, remark that the energy estimates used here are different from those in [2].

For simplicity, we consider the two-fluid multi-dimensional Euler–Poisson system in the periodic case in the torus $\mathbb{T}^d = (\mathbb{R}/\mathbb{Z})^d$ with $d \geqslant 1$. The system reads:

$$\begin{cases} \partial_t n_\nu + \operatorname{div}(n_\nu u_\nu) = 0, \\ \partial_t u_\nu + (u_\nu \cdot \nabla) u_\nu + \nabla h_\nu(n_\nu) = -q_\nu \nabla\phi - u_\nu, \\ -\Delta\phi = n_i - n_e, \quad \nu = e, i, \end{cases} \tag{1.12}$$

for $(t, x) \in (0, \infty) \times \mathbb{T}^d$. It can be regarded formally as a particular case of the two-fluid Euler–Maxwell system with $d = 3$, $E = -\nabla\phi$ and $B = 0$. In order that ϕ is uniquely determined, we add a restriction condition

$$m(\phi) = \int_{\mathbb{T}^d} \phi(\cdot, x)\, dx = 0. \tag{1.13}$$

By the Poincaré inequality (see Lemma 2.2), for all integers $s' \geqslant 0$ the Poisson equation in (1.12) with (1.13) gives estimate

$$\|\nabla\phi\|_{H^{s'}(\mathbb{T}^d)} \leqslant C\|n_i - n_e\|_{H^{s'}(\mathbb{T}^d)}. \tag{1.14}$$

Then, regarding $\nabla\phi$ as a function of n_e and n_i, (n_ν, u_ν) for $\nu = e, i$ still satisfy a symmetrizable hyperbolic system in which $\nabla\phi$ appearing on the right-hand side of (1.12) is a low order term. Following Kato [18], this implies that the periodic problem to (1.12) admits a unique local smooth solution, provided that the initial data (n_ν^0, u_ν^0) for $\nu = e, i$ are smooth. Moreover, estimate (1.14) implies that $\phi \in C([0, T), H^{s'+1}(\mathbb{T}^d))$ as soon as $n_e, n_i \in C([0, T), H^{s'}(\mathbb{T}^d))$ for some constant $T > 0$ and all integers $s' \geqslant 0$.

As a byproduct, here we show that our treatment for the two-fluid Euler–Maxwell system is still valid for the two-fluid Euler–Poisson system. The global existence of smooth solutions to the two-fluid Euler–Poisson equations is stated in the following theorem. Its proof follows from Corollaries 2.1, 2.2 and 2.3.

Theorem 1.3 *(Global existence of smooth solutions of the Euler–Poisson system). Let $s > 1 + d/2$ be an integer and $(n_\nu^0, u_\nu^0) \in H^s(\mathbb{T}^d)$ for $\nu = e, i$. Then there exists a constant $\delta_1 > 0$ independent of any given time $t > 0$, such that if*

$$\left\|\left(n_\nu^0 - 1, u_\nu^0\right)\right\|_{H^s(\mathbb{T}^d)} \leqslant \delta_1, \quad \nu = e, i,$$

the periodic problem to the Euler–Poisson system (1.12)–(1.13) with the initial data (n_ν^0, u_ν^0) has a unique global smooth solution (n_ν, u_ν, ϕ) satisfying

$$(n_\nu - 1, u_\nu) \in C^1\left([0, +\infty); H^{s-1}\left(\mathbb{T}^d\right)\right) \cap C\left([0, +\infty); H^s\left(\mathbb{T}^d\right)\right), \quad \nu = e, i,$$
$$\phi \in C^1\left([0, +\infty); H^s\left(\mathbb{T}^d\right)\right) \cap C\left([0, +\infty); H^{s+1}\left(\mathbb{T}^d\right)\right).$$

Moreover, the solution satisfies the long-time asymptotic property (1.8), (1.11) for n_ν, $\nu = e, i$ (with $\Omega = \mathbb{T}^d$) and

$$\lim_{t\to+\infty}\left\|\nabla\phi(t)\right\|_{H^{s-1}(\mathbb{T}^d)} = 0, \qquad \lim_{t\to+\infty}\left\|\phi(t)\right\|_{H^{s-1}(\mathbb{T}^d)} = 0. \tag{1.15}$$

This paper is organized as follows. In Section 2 we deal with the global existence of smooth solutions. The main goal is to prove Theorem 1.1 by establishing energy estimates. Section 3 is devoted to the long-time behavior of the solutions. We prove Theorem 1.2 by further energy estimates of the solutions. We also show the global existence of smooth solutions of the two-fluid Euler–Poisson equations stated in Theorem 1.3. Finally, in Appendix A we show that the Kawashima condition fails for system (1.1).

2. Global existence of smooth solutions

2.1. Preliminaries

We first introduce some notations which will be used in the sequel. Let $\Omega \subset \mathbb{R}^d$ be an open domain. When Ω is bounded, we denote by $|\Omega|$ the Lebesgue measure of Ω. Note that when $\Omega = \mathbb{T}^d$, we have $|\mathbb{T}^d| = 1$. For a multi-index $\alpha = (\alpha_1, \dots, \alpha_d) \in \mathbb{N}^d$, we denote

$$\partial_x^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \cdots \partial x_d^{\alpha_d}} \quad \text{with } |\alpha| = \alpha_1 + \cdots + \alpha_d.$$

For an integer $s > 0$ and a real number $p \geqslant 1$, $\|\cdot\|_{s,p}$ stands for the norm of the Sobolev space $W^{s,p}(\Omega)$ defined by

$$W^{s,p}(\Omega) = \left\{ f;\ \partial_x^\alpha f \in L^p(\Omega),\ \forall |\alpha| \leqslant s \right\}.$$

We denote also $H^s(\Omega) = W^{s,2}(\Omega)$, and by $\|\cdot\|$ and $\|\cdot\|_\infty$ the norms of $L^2(\Omega)$ and $L^\infty(\Omega)$, respectively.

The following lemmas are needed in the proofs of Theorems 1.1–1.3.

Lemma 2.1 *(Moser-type calculus inequalities). (See [19,20].) Let $s \geqslant 1$ be an integer and $\Omega = \mathbb{R}^d$ or $\Omega = \mathbb{T}^d$. Suppose $u \in H^s(\Omega)$, $\nabla u \in L^\infty(\Omega)$ and $v \in H^{s-1}(\Omega) \cap L^\infty(\Omega)$. Then for all multi-index $\alpha \in \mathbb{N}^d$ with $1 \leqslant |\alpha| \leqslant s$ and all smooth function f, we have $\partial_x^\alpha(uv) - u\partial_x^\alpha v \in L^2(\Omega)$, $\partial^\alpha f(u) \in L^2(\Omega)$ and*

$$\left\|\partial_x^\alpha(uv) - u\partial_x^\alpha v\right\| \leqslant C_s\left(\|\nabla u\|_\infty \left\|D^{|\alpha|-1}v\right\| + \left\|D^{|\alpha|}u\right\| \|v\|_\infty\right),$$
$$\left\|\partial^\alpha f(u)\right\| \leqslant C_\infty (1 + \|\nabla u\|_\infty)^{|\alpha|-1} \left\|D^{|\alpha|}u\right\|,$$

where the constant $C_\infty > 0$ depends on $\|u\|_\infty$ and s, and $C_s > 0$ depends only on s and

$$\left\|D^{s'}u\right\| = \sum_{|\alpha|=s'} \left\|\partial_x^\alpha u\right\|, \quad \forall s' \in \mathbb{N}.$$

Moreover, if $s > 1 + \frac{d}{2}$, then the embedding $H^{s-1}(\Omega) \hookrightarrow L^\infty(\Omega)$ is continuous and we have

$$\|uv\|_{s-1} \leqslant C_s \|u\|_{s-1} \|v\|_{s-1}, \quad \forall u, v \in H^{s-1}(\Omega),$$

and for all $u, v \in H^s(\Omega)$,

$$\left\|\partial^\alpha f(u)\right\| \leqslant C_\infty \left(1 + \|u\|_s\right)^{s-1} \|u\|_s, \qquad \left\|\partial_x^\alpha(uv) - u\partial_x^\alpha v\right\| \leqslant C_s \|u\|_s \|v\|_{s-1}, \quad \forall |\alpha| \leqslant s.$$

Lemma 2.2 *(Poincaré inequality). (See [11].) Let $1 \leqslant p \leqslant \infty$ and $\Omega \in \mathbb{R}^d$ be a bounded connected open domain with a Lipschitz boundary. Then there exists a constant $C > 0$ depending only on p and Ω such that*

$$\|u - u_\Omega\|_{L^p(\Omega)} \leqslant C\|\nabla u\|_{L^p(\Omega)}, \quad \forall u \in W^{1,p}(\Omega), \tag{2.1}$$

where

$$u_\Omega = \frac{1}{|\Omega|} \int_\Omega u(x)\,dx$$

is the average value of u over Ω.

Now for $\nu = e, i$, set

$$N_\nu = n_\nu - 1, \qquad G = B - \bar{B} \tag{2.2}$$

and

$$N = \begin{pmatrix} N_e \\ N_i \end{pmatrix}, \qquad u = \begin{pmatrix} u_e \\ u_i \end{pmatrix}, \qquad W_\nu = \begin{pmatrix} N_\nu \\ u_\nu \end{pmatrix}, \qquad W = \begin{pmatrix} W_e \\ W_i \\ E \\ G \end{pmatrix}. \tag{2.3}$$

For the two-fluid Euler–Poisson system (1.12), we keep the same notations in (2.2)–(2.3) except for W being replaced by

$$W_{EP} = \begin{pmatrix} W_e \\ W_i \end{pmatrix}. \tag{2.4}$$

With these notations, Eqs. (1.1) are written as:

$$\begin{cases} \partial_t N_\nu + \operatorname{div}\big((1+N_\nu)u_\nu\big) = 0, \\ \partial_t u_\nu + (u_\nu \cdot \nabla)u_\nu + \nabla h_\nu(1+N_\nu) = q_\nu\big(E + u_\nu \times (G+\bar{B})\big) - u_\nu, \quad \nu = e, i, \\ \partial_t E - \nabla \times G = -\big(q_e(1+N_e)u_e + q_i(1+N_i)u_i\big), \quad \operatorname{div} E = N_i - N_e, \\ \partial_t G + \nabla \times E = 0, \quad \operatorname{div} G = 0. \end{cases} \tag{2.5}$$

In (2.5) the Euler equations can be further rewritten in the form

$$\partial_t W_\nu + \sum_{j=1}^{3} A_j^\nu(W_\nu)\partial_{x_j} W_\nu = q_\nu K_1(W) + K_2(u_\nu), \quad \nu = e, i, \tag{2.6}$$

with

$$A_j^\nu(W_\nu) = \begin{pmatrix} u_{\nu j} & (1+N_\nu)e_j^t \\ h_\nu'(1+N_\nu)e_j & u_{\nu j}\mathbf{I}_3 \end{pmatrix}, \quad j = 1,2,3, \tag{2.7}$$

$$K_1(W) = \begin{pmatrix} 0 \\ E + u_\nu \times (G+\bar{B}) \end{pmatrix}, \qquad K_2(u_\nu) = \begin{pmatrix} 0 \\ -u_\nu \end{pmatrix}, \tag{2.8}$$

where (e_1, e_2, e_3) is the canonical basis of $\mathbb{R}^3$, $\mathbf{I}_3$ is the 3×3 unit matrix, and y_j denotes the jth component of $y \in \mathbb{R}^3$.

It is clear that system (2.6) for W_ν is symmetrizable hyperbolic when $n_\nu = 1 + N_\nu > 0$. More precisely, since we consider small solutions for which N_ν is close to zero, we may suppose that $\|N_\nu\|_{L^\infty((0,T)\times\Omega)} \leqslant \frac{1}{2}$. Then $\frac{1}{2} \leqslant 1 + N_\nu \leqslant \frac{3}{2}$. It follows that the matrix

$$A_0^\nu(N_\nu) = \begin{pmatrix} h_\nu'(1+N_\nu) & 0 \\ 0 & (1+N_\nu)\mathbf{I}_3 \end{pmatrix}$$

is symmetric positively definite and

$$\tilde{A}_j^\nu(W_\nu) = A_0^\nu(N_\nu)A_j^\nu(W_\nu)$$

is symmetric for all $1 \leqslant j \leqslant 3$. This choice of $A_0^\nu(N_\nu)$ will simplify energy estimates (see the proof of Lemma 2.4 in the next subsection).

According to [23], the global existence of smooth solutions follows from the local existence and uniform estimates of solutions with respect to t. By Proposition 1.1, we need to establish the uniform estimates of the local solution in $C^1([0,T); H^{s-1}(\Omega)) \cap C([0,T); H^s(\Omega))$. From (2.5), $\partial_t W$ can be expressed as functions of W and ∇W. Then, it suffices to establish the uniform estimates of the local solution in $C([0,T); H^s(\Omega))$. They are achieved in a series of the lemmas below in which the final one is Lemma 2.9. When these lemmas are obtained, the rest of the proof of Theorem 1.1 is easy.

2.2. Basic lemmas

Let $T > 0$ and W be a smooth solution of (2.5) defined on time interval $[0,T]$ with initial datum $W^0 = W(0,\cdot)$. From now on, we denote

$$\rho(T) = \sup_{0\leqslant t\leqslant T} \big\|W(t)\big\|_s, \qquad \rho_{EP}(T) = \sup_{0\leqslant t\leqslant T} \big\|W_{EP}(t)\big\|_s, \tag{2.9}$$

and by $C > 0$ various constants independent of any time t and T. From the continuous embedding $H^s(\Omega) \hookrightarrow L^\infty(\Omega)$, there is a constant $C_{em} > 0$ such that

$$\|z\|_\infty \leqslant C_{em}\|z\|_s, \quad \forall z \in H^s(\Omega).$$

If $\rho(T) \leqslant 1/2C_{em}$, from (2.9) it is easy to see that

$$\|N_\nu\|_\infty \leqslant \frac{1}{2} \quad \text{and} \quad \frac{1}{2} \leqslant n_\nu = 1 + N_\nu \leqslant \frac{3}{2}, \quad \nu = e, i.$$

Moreover, by Lemma 2.1, for any smooth function f we have

$$\sup_{0\leqslant t\leqslant T} \big\| f\big(W(t)\big)\big\|_s \leqslant C.$$

These simple estimates are used in proofs of Section 2. Note that in the proofs of Lemmas 2.3–2.9, we only suppose $\rho(T) \leqslant 1/2C_{em}$ without any smallness condition on the solution.

The first lemma concerns the zero order energy estimate.

Lemma 2.3. *Let the assumptions of Theorem 1.1 hold. If $\rho(T) \leqslant 1/2C_{2m}$, we have*

$$\big\|W(t)\big\|^2 + \int_0^t \big\|u(\tau)\big\|^2\, d\tau \leqslant C\big\|W^0\big\|^2, \quad \forall t \in [0, T]. \tag{2.10}$$

Proof. It is clear that the energy conservation of the Euler equations in (1.1) or (2.5) is

$$\partial_t\left(\frac{1}{2}n_\nu|u_\nu|^2 + H(n_\nu)\right) + \operatorname{div}\left(\frac{1}{2}n_\nu|u_\nu|^2 u_\nu + R(n_\nu)u_\nu\right)$$
$$= q_\nu n_\nu u_\nu \cdot \big(E + u_\nu \times (G + \bar{B})\big) - n_\nu|u_\nu|^2, \quad \nu = e, i, \tag{2.11}$$

where $|\cdot|$ is the Euclidean norm of $\mathbb{R}^3$,

$$H_\nu'(n) = h_\nu(n), \qquad R_\nu(n) = nh_\nu(n), \quad \nu = e, i.$$

Developing H_ν near $n_\nu = 1$ yields

$$H_\nu(n_\nu) = H_\nu(1) + h_\nu(1)N_\nu + \frac{1}{2}h_\nu'(z_\nu)N_\nu^2,$$

where z_ν is between n_ν and 1. Since

$$n_\nu u_\nu \cdot \big(u_\nu \times (G + \bar{B})\big) = 0,$$

from (2.11) and the conservation equation for n_ν, we have

$$\partial_t\left(\frac{1}{2}n_\nu|u_\nu|^2 + \frac{1}{2}h_\nu'(z_\nu)N_\nu^2\right) + \operatorname{div}\left(\frac{1}{2}n_\nu|u_\nu|^2 u_\nu + R_\nu(n_\nu)u_\nu - h_\nu(1)n_\nu u_\nu\right)$$
$$= q_\nu n_\nu u_\nu \cdot E - n_\nu|u_\nu|^2. \tag{2.12}$$

On the other hand, from

$$E \cdot \nabla \times G + G \cdot \nabla \times E = \operatorname{div}(E \times G),$$

we get also the energy conservation for the Maxwell equations as

$$\partial_t\left(\frac{1}{2}\big(|E|^2 + |G|^2\big)\right) + \operatorname{div}(E \times G) = -(q_e n_e u_e + q_i n_i u_i) \cdot E. \tag{2.13}$$

Hence, the cancellation of the term $(q_e n_e u_e + q_i n_i u_i) \cdot E$ in (2.12) and (2.13) exists. Adding Eqs. (2.13) and (2.12) for $\nu = e, i$ and integrating over Ω gives

$$\frac{d}{dt}\int_\Omega \left(\sum_{\nu=e,i}\big(n_\nu|u_\nu|^2 + h_\nu'(z_\nu)N_\nu^2\big) + |E|^2 + |G|^2\right)dx + 2\sum_{\nu=e,i}\int_\Omega n_\nu|u_\nu|^2\, dx = 0. \tag{2.14}$$

Since $n_\nu \geqslant \frac{1}{2}$ and h_ν is a strictly increasing function on $(0, +\infty)$, (2.10) follows after integrating (2.14) over $(0, t)$ with $t \in [0, T]$. □

Let us denote by W_{EP}^0 the initial datum of W_{EP}. The next estimate is concerned with the two-fluid Euler–Poisson system (1.12)–(1.13).

Corollary 2.1. *Let the assumptions of Theorem 1.3 hold. If $\rho_{EP}(T) \leqslant 1/2C_{em}$, we have*

$$\left\| W_{EP}(t)\right\|^2 + \left\|\nabla\phi(t)\right\|^2 + \int_0^t \left\|u(\tau)\right\|^2 d\tau \leqslant C\left\|W_{EP}^0\right\|^2, \quad \forall t \in [0, T]. \tag{2.15}$$

Proof. It is clear that the energy conservation (2.12) for the Euler equations in (1.12) still holds, with $E = -\nabla\phi$ and $\Omega = \mathbb{T}^d$. For the first term on the right-hand side of (2.12), using the density conservation and an integration by parts, we have

$$\begin{aligned}\int_{\mathbb{T}^d} n_\nu u_\nu E\,dx &= \int_{\mathbb{T}^d} \phi \operatorname{div}(n_\nu u_\nu)\,dx \\ &= -\int_{\mathbb{T}^d} \phi \partial_t n_\nu\,dx, \quad \nu = e, i.\end{aligned}$$

Adding this equality for $\nu = e, i$ and using the Poisson equation in (1.12) yields

$$\begin{aligned}\sum_{\nu=e,i} \int_{\mathbb{T}^d} q_\nu n_\nu u_\nu E\,dx &= \int_{\mathbb{T}^d} \phi \partial_t (n_e - n_i)\,dx \\ &= \int_{\mathbb{T}^d} \phi \partial_t \Delta\phi\,dx \\ &= -\frac{1}{2}\frac{d}{dt}\int_{\mathbb{T}^d} |\nabla\phi|\,dx.\end{aligned} \tag{2.16}$$

Let $\phi^0 = \phi(0, \cdot)$ be defined by

$$-\Delta\phi^0 = N_i^0 - N_e^0, \qquad m(\phi^0) = 0.$$

Then the Poincaré inequality implies that

$$\left\|\nabla\phi^0\right\| \leqslant C\left\|N_i^0 - N_e^0\right\| \leqslant C\left\|W_{EP}^0\right\|.$$

Together with (2.12) and (2.16), we get (2.15). □

The next lemma is a high order classical energy estimate for system (2.5) of which the proof depends also on a cancellation of the term $(q_e\partial_x^\alpha u_e + q_i\partial_x^\alpha u_i, \partial_x^\alpha E)$ between the source terms of the Euler and Maxwell energy equations.

For Euler equations, with the definition in (2.3), we set

$$r_s(t) = \left\|\nabla N(t)\right\|_{s-1} + \left\|u(t)\right\|_s. \tag{2.17}$$

Note that $r_s(t)$ doesn't contain $\|N(t)\|$, which is different from that of the one-fluid case.

Lemma 2.4. *Under the assumptions of Lemma 2.3, we have*

$$\left\|W(t)\right\|_s^2 + \int_0^t \left\|u(\tau)\right\|_s^2 d\tau \leqslant C\left\|W^0\right\|_s^2 + C\int_0^t \left\|W(\tau)\right\|_s r_s^2(\tau)\,d\tau, \quad \forall t \in [0, T]. \tag{2.18}$$

Proof. Let $\alpha \in \mathbb{N}^3$ with $1 \leqslant |\alpha| \leqslant s$. For $\nu = e, i$, differentiating Eqs. (2.6) with respect to x and multiplying the resulting equations by the symmetrizer matrix $A_0^\nu(N_\nu)$, we get

$$A_0^\nu(N_\nu)\partial_t\big(\partial_x^\alpha W_\nu\big) + \sum_{j=1}^3 \tilde{A}_j^\nu(W_\nu)\partial_{x_j}\partial_x^\alpha W_\nu = A_0^\nu(N_\nu)\partial_x^\alpha\big(q_\nu K_1(W) + K_2(u_\nu)\big) + J_\nu^\alpha, \tag{2.19}$$

where J_ν^α is defined by

$$J_\nu^\alpha = -\sum_{j=1}^3 A_0^\nu(N_\nu)\big[\partial_x^\alpha\big(A_j^\nu(W_\nu)\partial_{x_j} W_\nu\big) - A_j^\nu(W_\nu)\partial_x^\alpha(\partial_{x_j} W_\nu)\big].$$

Applying Lemma 2.1 to J_ν^α, since $\rho(T) \leqslant 1/2C_{em}$, we have

$$\begin{aligned}\|J_\nu^\alpha\| &\leqslant C\big(\big\|\nabla A_j^\nu(W_\nu)\big\|_\infty \|\partial_{x_j} W_\nu\|_{s-1} + \big\|D^s A_j^\nu(W_\nu)\big\| \|\partial_{x_j} W_\nu\|_\infty\big) \\ &\leqslant C\big(\|W_\nu\|_s + \|W_\nu\|_s^s\big) r_s(t) \leqslant C\|W_\nu\|_s r_s(t).\end{aligned} \tag{2.20}$$

Taking the inner product of Eqs. (2.19) with $\partial_x^\alpha W_\nu$ and using the fact that the matrix $\tilde{A}_j^\nu(W_\nu)$ is symmetric, we obtain the classical energy estimate

$$\begin{aligned}\frac{d}{dt}\big(A_0^\nu(N_\nu)\partial_x^\alpha W_\nu, \partial_x^\alpha W_\nu\big) &= 2\big(J_\nu^\alpha, \partial_x^\alpha W_\nu\big) + \big(\operatorname{div} A^\nu(W_\nu)\partial_x^\alpha W_\nu, \partial_x^\alpha W_\nu\big) \\ &\quad + 2\big(A_0^\nu(N_\nu)\partial_x^\alpha W_\nu, q_\nu \partial_x^\alpha K_1(W) + \partial_x^\alpha K_2(u_\nu)\big),\end{aligned} \tag{2.21}$$

where

$$\begin{aligned}\operatorname{div} A^\nu(W_\nu) &= \partial_t A_0^\nu(N_\nu) + \sum_{j=1}^3 \partial_{x_j} \tilde{A}_j^\nu(W_\nu) \\ &= \big(A_0^\nu\big)'(N_\nu)\partial_t N_\nu + \sum_{j=1}^3 \big(\tilde{A}_j^\nu\big)'(W_\nu)\partial_{x_j} W_\nu.\end{aligned}$$

From the first equation in (2.5), i.e.

$$\partial_t N_\nu = -\operatorname{div}\big((1+N_\nu)u_\nu\big)$$

and Lemma 2.1, we have

$$\|\partial_t N_\nu\|_\infty \leqslant C\|\partial_t N_\nu\|_{s-1} = C\big\|\operatorname{div}\big((1+N_\nu)u_\nu\big)\big\|_{s-1} \leqslant C\|u_\nu\|_s. \tag{2.22}$$

Then

$$\big\|\operatorname{div} A^\nu(W_\nu)\big\|_\infty \leqslant C\|W_\nu\|_s. \tag{2.23}$$

Now let us estimate each term on the right-hand side of (2.21). For the first two terms, by the Cauchy–Schwarz inequality and using estimates (2.20) and (2.23), since $|\alpha| \geqslant 1$, we have

$$\big(J_\nu^\alpha, \partial_x^\alpha W_\nu\big) + \big(\operatorname{div} A^\nu(W_\nu)\partial_x^\alpha W_\nu, \partial_x^\alpha W_\nu\big) \leqslant C\|W_\nu\|_s r_s^2(t). \tag{2.24}$$

For the last term, it follows from the definition of $A_0^\nu(N_\nu)$, $K_1(W)$ and $K_2(u_\nu)$ that

$$\begin{aligned}&2\big(A_0^\nu(N_\nu)\partial_x^\alpha W_\nu, q_\nu\partial_x^\alpha K_1(W) + \partial_x^\alpha K_2(u_\nu)\big) \\ &\quad = -2\big((1+N_\nu)\partial_x^\alpha u_\nu, \partial_x^\alpha u_\nu\big) + 2q_\nu\big(\partial_x^\alpha u_\nu, \partial_x^\alpha E\big) + 2q_\nu\big(N_\nu\partial_x^\alpha u_\nu, \partial_x^\alpha E\big) \\ &\qquad + 2q_\nu\big((1+N_\nu)\partial_x^\alpha u_\nu, \partial_x^\alpha(u_\nu \times G)\big) + 2q_\nu\big((1+N_\nu)\partial_x^\alpha u_\nu, \partial_x^\alpha(u_\nu \times \bar{B})\big).\end{aligned} \tag{2.25}$$

Noting that

$$\|N_\nu\|_\infty \leqslant \frac{1}{2}, \qquad -2(1+N_\nu) \leqslant -1$$

and

$$(1+N_\nu)\partial_x^\alpha u_\nu \cdot \partial_x^\alpha(u_\nu \times \bar{B}) = (1+N_\nu)\partial_x^\alpha u_\nu \cdot \big(\partial_x^\alpha u_\nu \times \bar{B}\big) = 0,$$

we obtain

$$\begin{aligned}&2\big(A_0^\nu(N_\nu)\partial_x^\alpha W_\nu, q_\nu \partial_x^\alpha K_1(W) + \partial_x^\alpha K_2(u_\nu)\big)\\ &\quad \leqslant -\big\|\partial_x^\alpha u_\nu\big\|^2 + 2q_\nu\big(\partial_x^\alpha u_\nu, \partial_x^\alpha E\big) + 2q_\nu\big(N_\nu \partial_x^\alpha u_\nu, \partial_x^\alpha E\big) + C\|W\|_s r_s^2(t).\end{aligned} \tag{2.26}$$

Hence, (2.21), (2.24) and (2.26) give

$$\frac{d}{dt}\big(A_0^\nu(N_\nu)\partial_x^\alpha W_\nu, \partial_x^\alpha W_\nu\big) \leqslant -\big\|\partial_x^\alpha u_\nu\big\|^2 + C\|W\|_s r_s^2(t) + 2q_\nu\big(\partial_x^\alpha u_\nu, \partial_x^\alpha E\big) + 2q_\nu\big(N_\nu\partial_x^\alpha u_\nu, \partial_x^\alpha E\big). \tag{2.27}$$

Now differentiating the Maxwell equations in (2.5) with respect to x, we get

$$\begin{cases}\partial_t\big(\partial_x^\alpha E\big) - \nabla \times \big(\partial_x^\alpha G\big) = -\displaystyle\sum_{\nu=e,i} q_\nu\big(\partial_x^\alpha u_\nu + \partial_x^\alpha(N_\nu u_\nu)\big),\\ \partial_t\big(\partial_x^\alpha G\big) + \nabla \times \big(\partial_x^\alpha E\big) = 0,\\ \operatorname{div}\big(\partial_x^\alpha E\big) = \partial_x^\alpha N_i - \partial_x^\alpha N_e, \quad \operatorname{div} G = 0.\end{cases} \tag{2.28}$$

Similarly to the proof of Lemma 2.3, we get

$$\frac{d}{dt}\big(\big\|\partial_x^\alpha E\big\|^2 + \big\|\partial_x^\alpha G\big\|^2\big) = -2\sum_{\nu=e,i} q_\nu\big(\partial_x^\alpha u_\nu + \partial_x^\alpha(N_\nu u_\nu), \partial_x^\alpha E\big). \tag{2.29}$$

Due to the choice of $A_0^\nu(N)$ we see that the cancellation of the term $(q_e\partial_x^\alpha u_e + q_i\partial_x^\alpha u_i, \partial_x^\alpha E)$ in (2.25) and (2.29) exists. It follows from (2.21), (2.24), (2.25) for $\nu = e, i$ and (2.29) that

$$\begin{aligned}&\frac{d}{dt}\bigg(\sum_{\nu=e,i}\big(A_0^\nu(N_\nu)\partial_x^\alpha W_\nu, \partial_x^\alpha W_\nu\big) + \big\|\partial_x^\alpha E\big\|^2 + \big\|\partial_x^\alpha G\big\|^2\bigg)\\ &\quad \leqslant \sum_{\nu=e,i}\big(-\big\|\partial_x^\alpha u_\nu\big\|^2 + 2q_\nu\big(N_\nu\partial_x^\alpha u_\nu - \partial_x^\alpha(N_\nu u_\nu), \partial_x^\alpha E\big)\big) + C\|W\|_s r_s^2(t).\end{aligned} \tag{2.30}$$

Moreover, by Lemma 2.1 we have

$$\begin{aligned}2q_\nu\big(N_\nu\partial_x^\alpha u_\nu - \partial_x^\alpha(N_\nu u_\nu), \partial_x^\alpha E\big) &\leqslant C\big(\|\nabla N_\nu\|_\infty\big\|D^{|\alpha|-1}u_\nu\big\| + \big\|D^{|\alpha|}N_\nu\big\|\|u_\nu\|_\infty\big)\big\|\partial_x^\alpha E\big\|\\ &\leqslant C\big\|W(t)\big\|_s r_s^2(t).\end{aligned} \tag{2.31}$$

Since $A_0^\nu(N_\nu)$ is positively definite, integrating (2.30) over $[0,t]$ with $t \in [0,T]$ and using (2.31), we get

$$\big\|\partial_x^\alpha W(t)\big\|^2 + \int_0^t \big\|\partial_x^\alpha u(\tau)\big\|^2 d\tau \leqslant C\big\|W^0\big\|_s^2 + C\int_0^t \big\|W(\tau)\big\|_s r_s^2(\tau)\,d\tau, \quad \forall t \in [0,T]. \tag{2.32}$$

Summing (2.32) for all α with $1 \leqslant |\alpha| \leqslant s$, together with (2.10) we obtain (2.18). □

The next estimate is concerned with the two-fluid Euler–Poisson system (1.12)–(1.13).

Corollary 2.2. *Under the assumptions of Corollary 2.1, we have*

$$\begin{aligned}&\big\|W_{EP}(t)\big\|_s^2 + \big\|\nabla\phi(t)\big\|_s^2 + \int_0^t \big\|u(\tau)\big\|_s^2 d\tau\\ &\quad \leqslant C\big\|W_{EP}^0\big\|^2 + C\int_0^t \big\|W_{EP}(\tau)\big\|_s r_s^2(\tau)\,d\tau, \quad \forall t \in [0,T].\end{aligned} \tag{2.33}$$

Proof. As in the proof of Corollary 2.1, estimates (2.21), (2.24) and (2.25) for the Euler equations are still valid for all $\alpha \in \mathbb{N}^d$ with $1 \leqslant |\alpha| \leqslant s$. Since $B = 0$ in the present case, similarly to (2.27) we obtain

$$\frac{d}{dt}\big(A_0^\nu(N_\nu)\partial_x^\alpha W_\nu, \partial_x^\alpha W_\nu\big) \leqslant -\big\|\partial_x^\alpha u_\nu\big\|^2 + C\|W_{EP}\|_s r_s^2(t) + 2q_\nu\big(\partial_x^\alpha u_\nu, \partial_x^\alpha E\big) + 2q_\nu\big(N_\nu \partial_x^\alpha u_\nu, \partial_x^\alpha E\big). \tag{2.34}$$

Now recall $E = -\nabla\phi$. For the last two terms in (2.34), we use the density conservations and the Poisson equation in (1.12). Then

$$-\partial_t \operatorname{div}\big(\partial_x^\alpha \nabla\phi\big) = \partial_t\big(\partial_x^\alpha N_i - \partial_x^\alpha N_e\big) = -\sum_{\nu=e,i} q_\nu \operatorname{div}\big(\partial_x^\alpha u_\nu + \partial_x^\alpha(N_\nu u_\nu)\big).$$

Multiplying this equality by $\partial_x^\alpha \phi$ and integrating over $\mathbb{T}^d$, we get

$$\frac{d}{dt}\int_{\mathbb{T}^d} \big|\partial_x^\alpha E\big|^2 dx = -2\sum_{\nu=e,i} q_\nu\big(\partial_x^\alpha u_\nu + \partial_x^\alpha(N_\nu u_\nu), \partial_x^\alpha E\big).$$

Together with (2.34) yields

$$\frac{d}{dt}\bigg(\sum_{\nu=e,i}\big(A_0^\nu(N_\nu)\partial_x^\alpha W_\nu, \partial_x^\alpha W_\nu\big) + \big\|\partial_x^\alpha E\big\|^2\bigg) \leqslant \sum_{\nu=e,i}\Big(-\big\|\partial_x^\alpha u_\nu\big\|^2 + 2q_\nu\big(N_\nu\partial_x^\alpha u_\nu - \partial_x^\alpha(N_\nu u_\nu), \partial_x^\alpha E\big)\Big) + C\|W_{EP}\|_s r_s^2(t).$$

Finally, applying the Poincaré inequality to the Poisson equation, we have

$$\big\|\partial_x^\alpha E\big\| \leqslant C\big\|\partial_x^\alpha N_i - \partial_x^\alpha N_e\big\| \leqslant C\|W_{EP}\|_s.$$

Thus, noting (2.31) we get

$$2q_\nu\big(N_\nu\partial_x^\alpha u_\nu - \partial_x^\alpha(N_\nu u_\nu), \partial_x^\alpha E\big) \leqslant C\|W_{EP}\|_s r_s^2(t),$$

which implies (2.33) together with Corollary 2.1. □

Lemma 2.4 is not sufficient to prove the global existence of solutions since ∇N_ν appearing on the right-hand side of (2.18) is not controlled in the $L^2(0,T;H^{s-1}(\Omega))$ norm. For overcoming this, we require to explore the relation between N_ν and u_ν in the Euler equations (2.6). Using the fact that p_ν is a strictly increasing function, we establish the $L^2(0,T;H^{s-1}(\Omega))$ norm of ∇N_ν.

In order to prove the final estimate in Lemma 2.9, we need the following Lemmas 2.5–2.8. Remark that the proofs of Lemmas 2.5–2.7 only employ the conservation equations of densities and that of Lemma 2.8 employs the same equations and Lemma 2.4. All these results are also valid for the two-fluid Euler–Poisson equations after slight modifications in the proof of Lemma 2.8 by applying Corollary 2.2 in the place of Lemma 2.4.

Lemma 2.5. *Under the assumptions of Lemma 2.3, for $\nu = e, i$ we have*

$$-(u_\nu, \nabla N_\nu) \leqslant -\frac{d}{dt}\int_\Omega \big(N_\nu - \log(1+N_\nu)\big)\,dx + C\|W_\nu\|_s r_s^2(t), \quad \forall t \in [0,T]. \tag{2.35}$$

Proof. Omit the subscript ν in this proof. From the first equation of (2.5), we have

$$\operatorname{div} u = -\frac{\partial_t N + u\cdot\nabla N}{1+N}. \tag{2.36}$$

Then

$$-(u,\nabla N)=(\operatorname{div} u, N)=-\left(\frac{\partial_t N+u\cdot\nabla N}{1+N}, N\right)$$
$$=-\left(\frac{\partial_t N}{1+N}, N\right)-\left(\frac{u\cdot\nabla N}{1+N}, N\right).$$

Noting that

$$\frac{N\partial_t N}{1+N}=\frac{d}{dt}\big(N-\log(1+N)\big)$$

and

$$-\left(\frac{u\cdot\nabla N}{1+N}, N\right)\leqslant C\|W\|_s r_s^2(t),$$

inequality (2.35) follows. □

Lemma 2.6. *Under the assumptions of Lemma 2.3, for $\nu=e,i$ and for all $\beta\in\mathbb{N}^3$ with $1\leqslant|\beta|\leqslant s-1$, we have*

$$-\big(\partial_x^\beta u_\nu, \partial_x^\beta\nabla N_\nu\big)\leqslant-\frac{1}{2}\frac{d}{dt}\left(\frac{1}{1+N_\nu}\partial_x^\beta N_\nu, \partial_x^\beta N_\nu\right)+C\|W_\nu\|_s r_s^2(t),\quad \forall t\in[0,T]. \tag{2.37}$$

Proof. Omit the subscript in this proof. Using still (2.36), we may write

$$\begin{aligned}
-\big(\partial_x^\beta u, \partial_x^\beta\nabla N\big)&=\big(\partial_x^\beta \operatorname{div} u, \partial_x^\beta N\big)\\
&=-\left(\partial_x^\beta\left(\frac{\partial_t N+u\cdot\nabla N}{1+N}\right), \partial_x^\beta N\right)\\
&=-\left(\frac{\partial_t\partial_x^\beta N}{1+N}, \partial_x^\beta N\right)-\left(\frac{u\cdot\partial_x^\beta\nabla N}{1+N}, \partial_x^\beta N\right)\\
&\quad-\left(\partial_x^\beta\left(\frac{\partial_t N}{1+N}\right)-\frac{\partial_x^\beta\partial_t N}{1+N}, \partial_x^\beta N\right)-\left(\partial_x^\beta\left(\frac{u\cdot\nabla N}{1+N}\right)-\frac{u\cdot\partial_x^\beta\nabla N}{1+N}, \partial_x^\beta N\right),
\end{aligned} \tag{2.38}$$

with

$$\begin{aligned}
-\left(\frac{\partial_t\partial_x^\beta N}{1+N}, \partial_x^\beta N\right)&=-\frac{1}{2}\frac{d}{dt}\left(\frac{1}{1+N}\partial_x^\beta N, \partial_x^\beta N\right)+\left(\partial_t\left(\frac{1}{1+N}\right)\partial_x^\beta N, \partial_x^\beta N\right)\\
&=-\frac{1}{2}\frac{d}{dt}\left(\frac{1}{1+N}\partial_x^\beta N, \partial_x^\beta N\right)+\left(\frac{1}{(1+N)^2}\partial_t N\partial_x^\beta N, \partial_x^\beta N\right).
\end{aligned} \tag{2.39}$$

Obviously,

$$\begin{aligned}
\left|\left(\frac{u\cdot\nabla\partial_x^\beta N}{1+N}, \partial_x^\beta N\right)\right|&\leqslant C\|N\|_{s-1}\|u\|_s\|\nabla N\|_{s-1}\\
&\leqslant C\|W\|_s r_s^2(t).
\end{aligned} \tag{2.40}$$

Using (2.22) and $|\beta|\geqslant 1$, we obtain

$$\begin{aligned}
\left|\left(\frac{1}{(1+N)^2}\partial_t N\partial_x^\beta N, \partial_x^\beta N\right)\right|&\leqslant C\|\partial_t N\|_\infty\big\|\partial_x^\beta N\big\|^2\\
&\leqslant C\big(1+\|N\|_s\big)\|u\|_s\big\|\partial_x^\beta N\big\|^2\\
&\leqslant C\|W\|_s r_s^2(t).
\end{aligned} \tag{2.41}$$

By Lemma 2.1 and the continuous embedding $H^{s-1}(\Omega)\hookrightarrow L^\infty(\Omega)$, we have

$$\begin{aligned}\left\|\partial_x^\beta\left(\frac{\partial_t N}{1+N}\right)-\frac{\partial_x^\beta\partial_t N}{1+N}\right\| &\leqslant C\left(\left\|\nabla\left(\frac{1}{1+N}\right)\right\|_\infty\|\partial_t N\|_{s-2}+\left\|D^{|\beta|}\left(\frac{1}{1+N}\right)\right\|\|\partial_t N\|_\infty\right)\\ &\leqslant C\big(\|\nabla N\|_{s-1}\|\partial_t N\|_{s-2}+\|\nabla N\|_{s-2}\|\partial_t N\|_{s-1}\big)\\ &\leqslant C\|\nabla N\|_{s-1}\|\partial_t N\|_{s-1}\\ &\leqslant C\big(1+\|N\|_s\big)\|u\|_s\|\nabla N\|_{s-1}.\end{aligned}$$

Then, since $\rho(T)\leqslant 1/2C_{em}$,

$$\left|\left(\partial_x^\beta\left(\frac{\partial_t N}{1+N}\right)-\frac{\partial_x^\beta\partial_t N}{1+N},\partial_x^\beta N\right)\right|\leqslant C\|W\|_s r_s^2(t). \tag{2.42}$$

Similarly,

$$\begin{aligned}\left\|\partial_x^\beta\left(\frac{u\cdot\nabla N}{1+N}\right)-\frac{u\cdot\partial_x^\beta\nabla N}{1+N}\right\| &\leqslant C\left(\left\|\nabla\left(\frac{u}{1+N}\right)\right\|_\infty\|\nabla N\|_{s-2}+\left\|D^{|\beta|}\left(\frac{u}{1+N}\right)\right\|\|\nabla N\|_\infty\right)\\ &\leqslant C\|W\|_s\|\nabla N\|_{s-1}.\end{aligned}$$

Therefore,

$$\left|\left(\partial_x^\beta\left(\frac{u\cdot\nabla N}{1+N}\right)-\frac{u\cdot\partial_x^\beta\nabla N}{1+N},\partial_x^\beta N\right)\right|\leqslant C\|W\|_s r_s^2(t). \tag{2.43}$$

Thus, combining (2.38)–(2.43), we get (2.37). □

Lemma 2.7. *Under the assumptions of Lemma 2.3, for $\nu=e,i$ we have*

$$-\int_0^t\sum_{|\beta|\leqslant s-1}\big(\partial_x^\beta u_\nu(\tau),\partial_x^\beta\nabla N_\nu(\tau)\big)\,d\tau\leqslant C\big\|W^0\big\|_s^2+C\int_0^t\big\|W_\nu(\tau)\big\|_s r_s^2(\tau)\,d\tau,\quad\forall t\in[0,T]. \tag{2.44}$$

Proof. We first note that function $f(N)=N-\log(1+N)$ satisfies

$$f(0)=0,\qquad f'(0)=0,\qquad f''(N)=\frac{1}{(1\pm N)^2}\geqslant\frac{4}{9},\quad\forall N\in\left[-\frac{1}{2},\frac{1}{2}\right].$$

Then

$$\int_\Omega f(N)\,dx\geqslant\frac{4}{9}\|N\|^2.$$

It follows that

$$\sum_{1\leqslant|\beta|\leqslant s-1}\left(\frac{1}{1+N}\partial_x^\beta N,\partial_x^\beta N\right)+\int_\Omega f(N)\,dx$$

is equivalent to $\|N\|_{s-1}^2$. Adding (2.35) and (2.37) for all β with $1\leqslant|\beta|\leqslant s-1$ and integrating over $[0,t]$, we obtain (2.44). □

Lemma 2.8. *Under the assumptions of Lemma 2.3, for $\nu=e,i$ and for all $\beta\in\mathbb{N}^3$ with $|\beta|\leqslant s-1$, we have*

$$\left|\int_0^t\big(\partial_x^\beta\partial_t u_\nu(\tau),\partial_x^\beta\nabla N_\nu(\tau)\big)\,d\tau\right|\leqslant C\big\|W^0\big\|_s^2+C\int_0^t\big\|W(\tau)\big\|_s r_s^2(\tau)\,d\tau,\quad\forall t\in[0,T]. \tag{2.45}$$

Proof. We have

$$-\left(\partial_x^\beta \partial_t u_\nu, \partial_x^\beta \nabla N_\nu\right) = \left(\partial_x^\beta \partial_t \operatorname{div} u_\nu, \partial_x^\beta N_\nu\right)$$
$$= \frac{d}{dt}\left(\partial_x^\beta \operatorname{div} u_\nu, \partial_x^\beta N_\nu\right) - \left(\partial_x^\beta \operatorname{div} u_\nu, \partial_x^\beta \partial_t N_\nu\right).$$

Integrating it over $[0, t]$ yields

$$-\int_0^t \left(\partial_x^\beta \partial_t u_\nu(\tau), \partial_x^\beta \nabla N_\nu(\tau)\right) d\tau = \left(\partial_x^\beta \operatorname{div} u_\nu(t), \partial_x^\beta N_\nu(t)\right) - \left(\partial_x^\beta \operatorname{div} u_\nu^0, \partial_x^\beta N_\nu^0\right)$$
$$- \int_0^t \left(\partial_x^\beta \operatorname{div} u_\nu(\tau), \partial_x^\beta \partial_t N_\nu(\tau)\right) d\tau.$$

Obviously,

$$\left|\left(\partial_x^\beta \operatorname{div} u_\nu(t), \partial_x^\beta N_\nu(t)\right)\right| \leqslant \left\|u_\nu(t)\right\|_s \left\|N_\nu(t)\right\|_{s-1} \leqslant \left\|W(t)\right\|_s^2$$

and

$$\left|\left(\partial_x^\beta \operatorname{div} u_\nu^0, \partial_x^\beta N_\nu^0\right)\right| \leqslant \left\|u_\nu^0\right\|_s \left\|N_\nu^0\right\|_{s-1} \leqslant \left\|W^0\right\|_s^2.$$

Moreover, from (2.22) we get

$$\left|\left(\partial_x^\beta \operatorname{div} u_\nu, \partial_x^\beta \partial_t N_\nu\right)\right| \leqslant \left\|\partial_x^\beta \operatorname{div} u_\nu\right\| \left\|\partial_x^\beta \partial_t N_\nu\right\|$$
$$\leqslant \|u_\nu\|_s \|\partial_t N_\nu\|_{s-1}$$
$$\leqslant \left(1 + \|N_\nu\|_s\right)\|u_\nu\|_s^2$$
$$\leqslant C\|u_\nu\|_s^2.$$

Thus, (2.45) follows from Lemma 2.4. □

2.3. Proof of Theorem 1.1

We first show Lemma 2.9 below which is sufficient to prove the global existence of solutions. Remark that in the proof of the lemma, we only employ the Euler equations, the constraint equation $\operatorname{div} E = N_i - N_e$ in (2.5) and Lemmas 2.4, 2.7 and 2.8.

Lemma 2.9. *Under the assumptions of Lemma 2.3, there are constants $C_1 > 0$ and $C_2 > 0$, independent of t and T, such that*

$$\left\|W(t)\right\|_s^2 + \int_0^t \left(\left\|N_e(\tau) - N_i(\tau)\right\|_s^2 + r_s^2(\tau)\right) d\tau$$
$$\leqslant C_1\left\|W^0\right\|_s^2 + C_2 \int_0^t \left\|W(\tau)\right\|_s r_s^2(\tau)\, d\tau, \quad \forall t \in [0, T]. \tag{2.46}$$

Proof. Let $\beta \in \mathbb{N}^3$ with $|\beta| \leqslant s - 1$. Differentiating the second equations of (2.5) with respect to x and taking the inner product of the resulting equations with $\partial_x^\beta \nabla N_\nu$, we get

$$\left(h_\nu'(1 + N_\nu)\partial_x^\beta \nabla N_\nu - q_\nu \partial_x^\beta E, \partial_x^\beta \nabla N_\nu\right)$$
$$= -\left(\partial_x^\beta\left(h_\nu'(1 + N_\nu)\nabla N_\nu\right) - h_\nu'(1 + N_\nu)\partial_x^\beta \nabla N_\nu, \partial_x^\beta \nabla N_\nu\right) - \left(\partial_x^\beta \partial_t u_\nu, \partial_x^\beta \nabla N_\nu\right)$$
$$+ \left(\partial_x^\beta\left(q_\nu u_\nu \times (G + \bar{B}) - u_\nu \cdot \nabla u_\nu\right), \partial_x^\beta \nabla N_\nu\right) - \left(\partial_x^\beta u_\nu, \partial_x^\beta \nabla N_\nu\right). \tag{2.47}$$

Let us estimate each term in (2.47). First, noting that $1+N_\nu \geqslant \frac{1}{2}$ and h is a strictly increasing function on $(0,+\infty)$, we have $h'_\nu(1+N_\nu) \geqslant C^{-1}$, so that

$$\begin{aligned}
&\big(h'_\nu(1+N_\nu)\partial_x^\beta \nabla N_\nu - q_\nu \partial_x^\beta E, \partial_x^\beta \nabla N_\nu\big)\\
&\quad = \big(h'_\nu(1+N_\nu)\partial_x^\beta \nabla N_\nu, \partial_x^\beta \nabla N_\nu\big) + q_\nu\big(\partial_x^\beta \operatorname{div} E, \partial_x^\beta N_\nu\big)\\
&\quad = \big(h'_\nu(1+N_\nu)\partial_x^\beta \nabla N_\nu, \partial_x^\beta \nabla N_\nu\big) + \big(\partial_x^\beta (N_i - N_e), \partial_x^\beta (q_\nu N_\nu)\big)\\
&\quad \geqslant C^{-1}\big\|\partial_x^\beta \nabla N_\nu\big\|^2 + \big(\partial_x^\beta (N_i - N_e), \partial_x^\beta (q_\nu N_\nu)\big),
\end{aligned}$$

in which we have used equation $\operatorname{div} E = N_i - N_e$. Hence,

$$\sum_{\nu=e,i} \big(h'_\nu(1+N_\nu)\partial_x^\beta \nabla N_\nu - q_\nu \partial_x^\beta E, \partial_x^\beta \nabla N_\nu\big) \geqslant C^{-1}\big\|\partial_x^\beta \nabla N\big\|^2 + \big\|\partial_x^\beta (N_i - N_e)\big\|^2. \tag{2.48}$$

By Lemma 2.1, we have

$$\begin{aligned}
&\big\|\partial_x^\beta \big(h'_\nu(1+N_\nu)\nabla N_\nu\big) - h'_\nu(1+N_\nu)\partial_x^\beta \nabla N_\nu\big\|\\
&\quad \leqslant \big\|\nabla h'_\nu(1+N_\nu)\big\|_\infty \|\nabla N_\nu\|_{s-2} + \big\|D^{|\beta|} h'_\nu(1+N_\nu)\big\| \|\nabla N_\nu\|_\infty \leqslant C\|W_\nu\|_s r_s(t).
\end{aligned}$$

Then,

$$\big|\big(\partial_x^\beta \big(h'_\nu(1+N_\nu)\nabla N_\nu\big) - h'_\nu(1+N_\nu)\partial_x^\beta \nabla N_\nu, \partial_x^\beta \nabla N_\nu\big)\big| \leqslant C\|W\|_s r_s^2(t). \tag{2.49}$$

Since $\rho(T) \leqslant 1/2C_{em}$, it follows from (2.9) that

$$\begin{aligned}
&\big|\big(\partial_x^\beta \big(q_\nu u_\nu \times (G+\bar{B}) - u_\nu \cdot \nabla u_\nu\big), \partial_x^\beta \nabla N_\nu\big)\big|\\
&\quad \leqslant \big(\big\|\partial_x^\beta (u_\nu \times G)\big\| + \big\|\partial_x^\beta u_\nu \times \bar{B}\big\| + \big\|\partial_x^\beta (u_\nu \cdot \nabla u_\nu)\big\|\big)\big\|\partial_x^\beta \nabla N_\nu\big\|\\
&\quad \leqslant C\big(\|u_\nu\|_{s-1}\|G\|_{s-1} + \|u_\nu\|_{s-1} + \|u_\nu\|_{s-1}\|u_\nu\|_s\big)\|\nabla N_\nu\|_{s-1}\\
&\quad \leqslant C\|W\|_s r_s^2(t) + \varepsilon\|\nabla N_\nu\|_{s-1}^2 + C_\varepsilon \|u_\nu\|_s^2.
\end{aligned} \tag{2.50}$$

Thus, combining (2.47)–(2.50), we get

$$\begin{aligned}
C^{-1}\big\|\partial_x^\beta \nabla N\big\|^2 + \big\|\partial_x^\beta (N_i - N_e)\big\|^2 &\leqslant C\|W\|_s r_s^2(t) + \varepsilon\|\nabla N\|_{s-1}^2 + C_\varepsilon \|u\|_s^2\\
&\quad - \sum_{\nu=e,i}\big(\big(\partial_x^\beta \partial_t u_\nu, \partial_x^\beta \nabla N_\nu\big) + \big(\partial_x^\beta u_\nu, \partial_x^\beta \nabla N_\nu\big)\big).
\end{aligned} \tag{2.51}$$

Sum up this inequality for all $|\beta| \leqslant s-1$ and take $\varepsilon > 0$ so small that the term $\varepsilon\|\nabla N\|_{s-1}^2$ can be controlled by the left-hand side. Integrating (2.51) over $[0,t]$, applying Lemma 2.4, 2.7 and 2.8 to its last two terms, we obtain

$$\begin{aligned}
&\big\|W(t)\big\|_s^2 + \int_0^t \big(\big\|N_e(\tau) - N_i(\tau)\big\|_{s-1}^2 + r_s^2(\tau)\big)\,d\tau\\
&\quad \leqslant C_1\big\|W^0\big\|_s^2 + C_2\int_0^t \big\|W(\tau)\big\|_s r_s^2(\tau)\,d\tau, \quad \forall t \in [0,T].
\end{aligned} \tag{2.52}$$

Finally, from

$$\big\|N_e(t) - N_i(t)\big\|_s^2 \leqslant \big\|N_e(t) - N_i(t)\big\|_{s-1}^2 + Cr_s^2(t), \quad \forall t \in [0,T],$$

(2.46) follows. □

The next estimate is concerned with the two-fluid Euler–Poisson system (1.12)–(1.13).

Corollary 2.3. *Under the assumptions of Corollary 2.1, we have*

$$\|W_{EP}(t)\|_s^2 + \|\nabla\phi(t)\|_s^2 + \int_0^t \big(\|N_e(\tau) - N_i(\tau)\|_s^2 + r_s^2(\tau)\big)\,d\tau$$
$$\leqslant C\|W_{EP}^0\|_s^2 + C\int_0^t \|W_{EP}(\tau)\|_s r_s^2(\tau)\,d\tau, \quad \forall t \in [0,T]. \tag{2.53}$$

Proof. In the proofs of Lemma 2.9, we employ Lemma 2.4 but not Maxwell equations. Then, applying Corollary 2.2 in the place of Lemma 2.4 and adapting these proofs for the Euler–Poisson equations (1.12)–(1.13), we obtain (2.53). The details of the proof are omitted here. □

Proof of Theorem 1.1. By Lemma 2.9, we deduce that if $C_2\rho(T) < 1$, the integral term on the right-hand side of (2.46) can be controlled by that of the left-hand side. It follows that

$$\|W(t)\|_s \leqslant \sqrt{C_1}\|W^0\|_s, \quad \forall t \in [0,T].$$

Thus, it suffices to take a constant $\delta_0 > 0$ sufficiently small such that

$$\sqrt{C_1}\delta_0 < \min\left(\frac{1}{2C_{em}}, \frac{1}{C_2}\right),$$

which guarantees both $\rho(T) \leqslant 1/2C_{em}$ and $C_2\rho(T) < 1$. Finally, the global existence of smooth solutions follows from the local existence result given in Proposition 1.1 and a standard argument on the continuous extension of local solutions. See for instance [23] for details. □

3. Long-time behavior of smooth solutions

3.1. Further energy estimates

The long-time behavior of smooth solutions follows from uniform energy estimates of N, u, E and ∇B with respect to T in $L^2(0,T;H^{s'}(\Omega))$ for suitable integers $s' \geqslant 1$. They are established in Lemmas 3.1–3.2, respectively.

Lemma 3.1. *Under the assumptions of Lemma 2.3, for any constant $\varepsilon > 0$ and all $t \in [0,T]$ we have*

$$\int_0^t \|E(\tau)\|_{s-1}^2\,d\tau \leqslant C_\varepsilon\|W^0\|_s^2 + \varepsilon\int_0^t \|\nabla G(\tau)\|_{s-2}^2\,d\tau + C_\varepsilon\int_0^t \|W(\tau)\|_s r_s^2(\tau)\,d\tau. \tag{3.1}$$

Proof. Let $\beta \in \mathbb{N}^3$ with $|\beta| \leqslant s-1$. From the second equation of (2.5), we have

$$q_\nu\partial_x^\beta E = \partial_t\partial_x^\beta u_\nu + \partial_x^\beta\big((u_\nu\cdot\nabla)u_\nu + \nabla h_\nu(1+N_\nu) - q_\nu u_\nu \times (G+\bar{B}) + u_\nu\big).$$

Since $q_\nu = \pm 1$, omitting the subscript ν, we obtain

$$\|\partial_x^\beta E\|^2 = \big(\partial_t\partial_x^\beta u, q\partial_x^\beta E\big) + \big(\partial_x^\beta\big((u\cdot\nabla)u + \nabla h(1+N) - qu\times(G+\bar{B}) + u\big), q\partial_x^\beta E\big).$$

It follows from the definition of $r_s(t)$ that

$$\|\partial_x^\beta E\|^2 \leqslant \big(\partial_t\partial_x^\beta u, q\partial_x^\beta E\big) + \frac{1}{2}\|\partial_x^\beta E\|^2 + Cr_s^2(t).$$

Then

$$\|\partial_x^\beta E\|^2 \leqslant 2\big(\partial_t\partial_x^\beta u, q\partial_x^\beta E\big) + Cr_s^2(t).$$

Next, from the Maxwell equation in (2.5), we have

$$\begin{aligned}\big(\partial_t\partial_x^\beta u, q\partial_x^\beta E\big) &= \frac{d}{dt}\big(\partial_x^\beta u, q\partial_x^\beta E\big) - \big(\partial_x^\beta u, q\partial_x^\beta(\partial_t E)\big)\\ &= \frac{d}{dt}\big(\partial_x^\beta u, q\partial_x^\beta E\big) - \big(\partial_x^\beta u, q\partial_x^\beta(\nabla\times G)\big) - \bigg(\partial_x^\beta u, q\sum_{\nu=e,i} q_\nu\partial_x^\beta\big((1+N_\nu)u_\nu\big)\bigg).\end{aligned}$$

Obviously,

$$\bigg|\bigg(\partial_x^\beta u, q\sum_{\nu=e,i} q_\nu\partial_x^\beta\big((1+N_\nu)u_\nu\big)\bigg)\bigg| \leqslant Cr_s^2(t).$$

For the second term on the right-hand side of the above equality with $\beta=0$, for any constant $\varepsilon>0$, we have

$$\big|(u, q\nabla\times G)\big| \leqslant \varepsilon^2\|\nabla G\|^2 + C_\varepsilon r_s^2(t).$$

For $1\leqslant|\beta|\leqslant s-1$, we use formula

$$f\cdot(\nabla\times g) = (\nabla\times f)\cdot g - \operatorname{div}(f\times g), \tag{3.2}$$

so that

$$\big|\big(\partial_x^\beta u, q\partial_x^\beta(\nabla\times G)\big)\big| = \big|\big(\partial_x^\beta(\nabla\times u), \partial_x^\beta G\big)\big| \leqslant \varepsilon^2\|\nabla G\|_{s-2}^2 + C_\varepsilon r_s^2(t).$$

Hence,

$$\big(\partial_t\partial_x^\beta u_\nu, q\partial_x^\beta E\big) \leqslant \frac{d}{dt}\big(\partial_x^\beta u_\nu, q\partial_x^\beta E\big) + \varepsilon^2\|\nabla G\|_{s-2}^2 + C_\varepsilon r_s^2(t),$$

which implies that

$$\big\|\partial_x^\beta E\big\|^2 \leqslant 2\frac{d}{dt}\big(\partial_x^\beta u_\nu, q\partial_x^\beta E\big) + \varepsilon^2\|\nabla G\|_{s-2}^2 + C_\varepsilon r_s^2(t). \tag{3.3}$$

Note that for all $t\in[0,T]$,

$$\big|\big(\partial_x^\beta u(t), q\partial_x^\beta E(t)\big)\big| \leqslant \big\|W(t)\big\|_s^2, \quad \forall|\beta|\leqslant s-1.$$

Let $\varepsilon>0$ be sufficiently small. Integrating (3.3) over $[0,t]$ and summing for all $|\beta|\leqslant s-1$, together with Lemma 2.9, we obtain (3.1). □

Lemma 3.2. *Under the assumptions of Lemma 2.3, for all $t\in[0,T]$ we have*

$$\int_0^t \big(\big\|E(\tau)\big\|_{s-1}^2 + \big\|\nabla G(\tau)\big\|_{s-2}^2\big)\,d\tau \leqslant C\big\|W^0\big\|_s^2 + C\int_0^t \big\|W(\tau)\big\|_s r_s^2(\tau)\,d\tau. \tag{3.4}$$

Proof. We first prove that

$$\int_0^t \big\|\nabla\times G(\tau)\big\|_{s-2}^2\,d\tau \leqslant C\big\|W^0\big\|_s^2 + C\int_0^t \big(\big\|E(\tau)\big\|_{s-1}^2 + \big\|W(\tau)\big\|_s r_s^2(\tau)\big)\,d\tau. \tag{3.5}$$

Let $\beta\in\mathbb{N}^3$ with $|\beta|\leqslant s-2$. From the third equation of (2.5), we have

$$\partial_x^\beta(\nabla\times G) = \partial_t\partial_x^\beta E + \sum_{\nu=e,i} q_\nu\partial_x^\beta\big((1+N_\nu)u_\nu\big).$$

Then

$$\big\|\partial_x^\beta(\nabla\times G)\big\|^2 = \big(\partial_t\partial_x^\beta E, \partial_x^\beta(\nabla\times G)\big) + \sum_{\nu=e,i} q_\nu\big(\partial_x^\beta\big((1+N_\nu)u_\nu\big), \partial_x^\beta(\nabla\times G)\big). \tag{3.6}$$

Obviously,

$$\left|\sum_{\nu=e,i} q_\nu\big(\partial_x^\beta\big((1+N_\nu)u_\nu\big), \partial_x^\beta(\nabla\times G)\big)\right| \leqslant \frac{\varepsilon}{2}\big\|\partial_x^\beta(\nabla\times G)\big\|^2 + Cr_s^2(t).$$

For the second term on the right-hand side of (3.6), it follows from (3.2) and the fourth equation of (2.5) that

$$\begin{aligned}\big(\partial_t\partial_x^\beta E, \partial_x^\beta(\nabla\times G)\big) &= \frac{d}{dt}\big(\partial_x^\beta E, \partial_x^\beta(\nabla\times G)\big) - \big(\partial_x^\beta E, \partial_x^\beta(\nabla\times\partial_t G)\big)\\ &= \frac{d}{dt}\big(\partial_x^\beta E, \partial_x^\beta(\nabla\times G)\big) + \big(\partial_x^\beta(\nabla\times E), \partial_x^\beta(\nabla\times E)\big)\\ &= \frac{d}{dt}\big(\partial_x^\beta E, \partial_x^\beta(\nabla\times G)\big) + \big\|\partial_x^\beta(\nabla\times E)\big\|^2.\end{aligned}$$

This implies that

$$\big\|\partial_x^\beta(\nabla\times G)\big\|^2 \leqslant 2\frac{d}{dt}\big(\partial_x^\beta E, \partial_x^\beta(\nabla\times G)\big) + 2\big\|\partial_x^\beta(\nabla\times E)\big\|^2 + Cr_s^2(t). \tag{3.7}$$

Note that for all $|\beta| \leqslant s-2$ and $t\in[0,T]$, we have

$$\big|\big(\partial_x^\beta E(t), \partial_x^\beta\big(\nabla\times G(t)\big)\big)\big| \leqslant C\big\|W(t)\big\|_s^2.$$

Integrating (3.7) over $[0,t]$ and summing for all $|\beta|\leqslant s-2$, together with Lemma 2.9, we obtain (3.5).

Now remark that $\operatorname{div} G = 0$. Then $\|\nabla\times G\|_{s-2}$ is equivalent to $\|\nabla G\|_{s-2}$. Finally, taking $\varepsilon>0$ sufficiently small and using (3.1) and (3.5), we get

$$\int_0^t \big\|\nabla G(\tau)\big\|_{s-2}^2\,d\tau \leqslant C\big\|W^0\big\|_s^2 + C\int_0^t \big\|W(\tau)\big\|_s r_s^2(\tau)\,d\tau,$$

which yields (3.4) together with (3.1). □

3.2. Proofs of Theorems 1.2 and 1.3

In the proofs of Theorems 1.2 and 1.3 we need a simple argument on the decay property at $+\infty$ of a uniform continuous function in $L^1(0,+\infty)$. For the long-time behavior of n_ν and B, besides the Poincaré inequality in the case $\Omega=\mathbb{T}^3$, we also use the Sobolev inequality in the case $\Omega=\mathbb{R}^3$. These results are stated below.

Lemma 3.3. *Let $f:(0,+\infty)\to\mathbb{R}$ be a uniformly continuous function such that $f\in L^1(0,+\infty)$. Then $\lim_{t\to+\infty} f(t)=0$. In particular, the conclusion holds when $f\in L^1(0,+\infty)\cap W^{1,\infty}(0,+\infty)$.*

Lemma 3.4 *(Sobolev inequality). (See [11].) Let $1\leqslant p<d$ and $\frac{1}{p^*}=\frac{1}{p}-\frac{1}{d}$. Then there exists a constant $C>0$ depending only on p and d such that*

$$\|u\|_{L^{p^*}(\mathbb{R}^d)} \leqslant C\|\nabla u\|_{L^p(\mathbb{R}^d)}, \quad \forall u\in W^{1,p}\big(\mathbb{R}^d\big). \tag{3.8}$$

Proof of Theorem 1.2. By Lemma 2.9, there is a constant $\delta_0>0$ such that if $\rho(T)<\delta_0$, we have

$$\big\|W(t)\big\|_s^2 + \int_0^t \big(\big\|N_e(\tau)-N_i(\tau)\big\|_s^2 + \big\|\nabla N(\tau)\big\|_{s-1}^2 + \big\|u(\tau)\big\|_s^2\big)\,d\tau \leqslant C\big\|W^0\big\|_s^2. \tag{3.9}$$

Since $N_e-N_i=n_e-n_i$ and $\nabla N_\nu=\nabla n_\nu$, this implies that

$$n_e-n_i, u_\nu \in L^2\big((0,+\infty); H^s(\Omega)\big)\cap L^\infty\big((0,+\infty); H^s(\Omega)\big), \quad \forall \nu=e,i,$$

and

$$\nabla n_\nu \in L^2\big((0,+\infty); H^{s-1}(\Omega)\big) \cap L^\infty\big((0,+\infty); H^{s-1}(\Omega)\big), \quad \forall \nu = e, i.$$

Using the first and the second equations of (2.5), we get

$$\partial_t n_\nu, \partial_t(n_e - n_i), \partial_t u_\nu \in L^\infty\big((0,+\infty); H^{s-1}(\Omega)\big), \quad \forall \nu = e, i,$$

and

$$\partial_t \nabla n_\nu \in L^\infty\big((0,+\infty); H^{s-2}(\Omega)\big), \quad \forall \nu = e, i.$$

Therefore,

$$n_e - n_i, u_\nu \in L^2\big((0,+\infty); H^{s-1}(\Omega)\big) \cap W^{1,\infty}\big((0,+\infty); H^{s-1}(\Omega)\big), \quad \forall \nu = e, i,$$

and

$$\nabla n_\nu \in L^2\big((0,+\infty); H^{s-2}(\Omega)\big) \cap W^{1,\infty}\big((0,+\infty); H^{s-2}(\Omega)\big), \quad \forall \nu = e, i,$$

which imply (1.8) by Lemma 3.3.

Similarly, the estimates of Lemma 3.2 and (3.9) show that

$$E \in L^2\big((0,+\infty); H^{s-1}(\Omega)\big) \cap L^\infty\big((0,+\infty); H^{s}(\Omega)\big),$$
$$G \in L^\infty\big((0,+\infty); H^{s}(\Omega)\big), \qquad \nabla G \in L^2\big((0,+\infty); H^{s-2}(\Omega)\big).$$

Then

$$\nabla G \in L^2\big((0,+\infty); H^{s-2}(\Omega)\big) \cap L^\infty\big((0,+\infty); H^{s-1}(\Omega)\big).$$

It follows from the Maxwell equations in (2.5) that

$$\partial_t E \in L^2\big((0,+\infty); H^{s-2}(\Omega)\big) \cap L^\infty\big((0,+\infty); H^{s-1}(\Omega)\big).$$

Therefore,

$$E \in L^2\big((0,+\infty); H^{s-1}(\Omega)\big) \cap W^{1,\infty}\big((0,+\infty); H^{s-1}(\Omega)\big),$$

which implies the first limit of (1.9). We further deduce that

$$\partial_t G = -\nabla \times E \in L^2\big((0,+\infty); H^{s-2}(\Omega)\big) \cap L^\infty\big((0,+\infty); H^{s-1}(\Omega)\big).$$

Then

$$\partial_t(\nabla G) \in L^2\big((0,+\infty); H^{s-3}(\Omega)\big) \cap L^\infty\big((0,+\infty); H^{s-2}(\Omega)\big).$$

This yields

$$\nabla G \in L^2\big((0,+\infty); H^{s-2}(\Omega)\big) \cap W^{1,\infty}\big((0,+\infty); H^{s-2}(\Omega)\big),$$

which implies the second limit of (1.9) by Lemma 3.3.

When $\Omega = \mathbb{R}^3$, applying the Sobolev inequality to $n_\nu - 1$ with $p = 2$ and $d = 3$, then $p^* = 6$. Since $s \geqslant 3$, we obtain

$$\big\|n_\nu(t) - 1\big\|_{W^{s-2,6}(\mathbb{R}^3)} \leqslant C\big\|\nabla n_\nu(t)\big\|_{H^{s-2}(\mathbb{R}^3)}.$$

Together with (1.8) yields (1.10) for n_ν, $\nu = e, i$.

When $\Omega = \mathbb{T}^3$, from equations

$$\partial_t n_\nu + \operatorname{div}(n_\nu u_\nu) = 0,$$

we deduce

$$\frac{d}{dt}\int_{\mathbb{T}^3} n_\nu(t,x)\,dx = 0.$$

Since $|\mathbb{T}^3| = 1$, from (1.5) we have

$$\int_{\mathbb{T}^3} n_\nu(t,x)\,dx = \int_{\mathbb{T}^3} n_\nu^0(x)\,dx = 1.$$

Using

$$\nabla n_\nu \in L^2\big((0,+\infty); H^{s-1}(\mathbb{T}^3)\big) \cap L^\infty\big((0,+\infty); H^{s-1}(\mathbb{T}^3)\big)$$

and applying the Poincaré inequality to $n_\nu - 1$, we obtain

$$n_\nu - 1 \in L^2\big((0,+\infty); H^{s-1}(\mathbb{T}^3)\big) \cap L^\infty\big((0,+\infty); H^{s-1}(\mathbb{T}^3)\big).$$

Together with the discussion above yields

$$n_\nu - 1 \in L^2\big((0,+\infty); H^{s-1}(\mathbb{T}^3)\big) \cap W^{1,\infty}\big((0,+\infty); H^{s-1}(\mathbb{T}^3)\big).$$

This proves (1.11) for n_ν, $\nu = e, i$. The proof of (1.10)–(1.11) for B is similar. □

Proof of Theorem 1.3. Similarly to the proofs of Theorems 1.1 and 1.2. The existence of smooth solutions (n_ν, u_ν, ϕ) and the long-time asymptotic property (1.8) follows from Corollary 2.3. Since $n_i - n_e = N_i - N_e$, (1.15) follows from (1.8) and (1.14) together with (1.13) and the Poincaré inequality. Finally, (1.11) for n_ν, $\nu = e, i$, follows from the same argument as above. This ends the proof of Theorem 1.3. □

Appendix A. On the Kawashima condition for system (1.1)

In this appendix we show that system (2.5) (or (1.1)) does not satisfy the Kawashima stability condition at equilibrium state $W = 0$. For this purpose, we rewrite system (2.5) in the form:

$$\partial_t W + \sum_{j=1}^{3} A_j(W)\partial_{x_j} W = K(W),$$

where

$$A_j(W) = \begin{pmatrix} A_j^e(W_e) & 0 & 0 \\ 0 & A_j^i(W_i) & 0 \\ 0 & 0 & A_j^M \end{pmatrix}, \qquad K(W) = \begin{pmatrix} 0 \\ -u_e - E - u_e \times (G + \bar{B}) \\ 0 \\ -u_i + E + u_i \times (G + \bar{B}) \\ (1+N_e)u_e - (1+N_i)u_i \\ 0 \end{pmatrix},$$

with $A_j^\nu(W_\nu)$ being defined in (2.7) and

$$A_j^M = \begin{pmatrix} 0 & L_j \\ L_j^t & 0 \end{pmatrix}, \quad j = 1,2,3,$$

$$L_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \qquad L_2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad L_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Let S^2 be the unit sphere in $\mathbb{R}^3$. For $\omega = (\omega_1, \omega_2, \omega_3)^t \in S^2$, we denote

$$A(\omega) = \sum_{j=1}^{3} \omega_j A_j(0) = \begin{pmatrix} A^e(\omega) & 0 & 0 \\ 0 & A^i(\omega) & 0 \\ 0 & 0 & A^M(\omega) \end{pmatrix},$$

with

$$A^\nu(\omega) = \sum_{j=1}^{3} \omega_j A_j^\nu(0) = \begin{pmatrix} 0 & \omega^t \\ h'_\nu(1)\omega & 0 \end{pmatrix}, \quad \nu = e, i,$$

$$A^M(\omega) = \sum_{j=1}^{3} \omega_j A_j^M = \begin{pmatrix} 0 & \sum_{j=1}^{3} \omega_j L_j \\ \sum_{j=1}^{3} \omega_j L_j^t & 0 \end{pmatrix}.$$

The Kawashima condition means that no eigenvector of $A(\omega)$ is in the kernel of $K'(0)$ for all $\omega \in S^2$. Since $K(W)$ does not contain the linear term of G, the last three columns of $K'(0)$ vanish. Therefore, for any $\eta = (\eta_1, \eta_2, \eta_3)^t \in \mathbb{R}^3$, $\eta_* = (0, \eta^t)^t \in \mathbb{R}^{10}$ is in the kernel of $K'(0)$. On the other hand, a straightforward computation gives

$$A(\omega)\eta_* = \begin{pmatrix} 0_8 \\ \sum_{j=1}^{3} \omega_j L_j \eta \\ 0_3 \end{pmatrix} = \begin{pmatrix} 0_8 \\ \eta \times \omega \\ 0_3 \end{pmatrix},$$

with $0_d \in \mathbb{R}^d$. Thus, for $\eta = \omega \neq 0$, η_* is an eigenvector of $A(\omega)$ associated to the zero eigenvalue. This shows that the Kawashima condition is not satisfied. Hence, the two-fluid Euler–Maxwell system (1.1) does not belong to the class of equations treated in [16,34].

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