

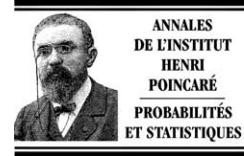


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Addendum to the article
“On ballistic diffusions in random environment”
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The object of this addendum is to clarify a step in the proof of Theorem 2.4 of [1], namely that the quantities in (2.21) of [1]

$$A_1 \stackrel{\text{def}}{=} \hat{\mathbb{P}}_x^\omega[V] \quad \text{and} \quad A_2 \stackrel{\text{def}}{=} \hat{\mathbb{P}}_y^\omega[f(X, -y, \lambda.), D = \infty] g \circ t_y$$

are independent under \mathbb{P} , as it is used below (2.21).

This fact follows from R -separation, cf. (1.6) of [1], and (1), (2) below:

- (1) A_1 is $\mathcal{H}_{\mathcal{L}(y \cdot \ell - 4R)}$ -measurable (in place of “ $\mathcal{H}_{\mathcal{L}(y \cdot \ell - 7R)}$ -measurable” stated below (2.21) of [1]),
- (2) A_2 is $\mathcal{H}_{\mathcal{R}(y \cdot \ell - 2R)}$ -measurable (in place of “ $\mathcal{H}_{\mathcal{R}(y \cdot \ell - R)}$ -measurable” stated below (2.21) of [1]).

To see (1), one simply uses, with $v = y \cdot \ell - 7R$, the following fact:

- (3) For $m \geq 1$, integer, $v \in \mathbb{R}$, $U \in \mathcal{F}_m \otimes \mathcal{S}_{m-1}$, with $U \subset \{\sup_{t \leq m} \ell \cdot X_t \leq v\}$, $\hat{\mathbb{P}}_x^\omega[U]$ is $\mathcal{H}_{\mathcal{L}(v+3R)}$ -measurable.

The statement (3) follows from

- (4) For $O \in \mathcal{F}_1$, with $O \subset \{\sup_{t \leq 1} \ell \cdot X_t \leq v\}$, $\hat{\mathbb{P}}_{z, \lambda=1}^\omega[O]$ and $\hat{\mathbb{P}}_{z, \lambda=0}^\omega[O]$ are $\mathcal{B}(\mathbb{R}^d) \otimes \mathcal{H}_{\mathcal{L}(v+3R)}$ -measurable,

together with the Markov property and standard arguments. To see (4), one simply observes that

- when $z \cdot \ell \geq v - 8R$, in view of (2.8), (2.9) of [1] and $B^z \subset \{x \in \mathbb{R}^d: x \cdot \ell > v\}$, one has $\hat{\mathbb{P}}_{z, \lambda=1}^\omega[O] = 0$, and $\hat{\mathbb{P}}_{z, \lambda=0}^\omega[O] = \frac{1}{1-\varepsilon} \mathbb{P}_z^\omega[O]$, and hence $1_{\{z \cdot \ell \geq v - 8R\}} \hat{\mathbb{P}}_{z, \lambda=i}^\omega[O]$, $i = 0, 1$, are $\mathcal{B}(\mathbb{R}^d) \otimes \mathcal{H}_{\mathcal{L}(v+3R)}$ -measurable in z, ω ,

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- when $z \cdot \ell < v - 8R$, $U^z \subset \{x \in \mathbb{R}^d: x \cdot \ell < v + 3R\}$, and hence $1_{\{z \cdot \ell < v - 8R\}} \hat{\mathbb{P}}_{z, \lambda=i}^\omega[O]$, $i = 0, 1$, are $\mathcal{B}(\mathbb{R}^d) \otimes \mathcal{H}_{\mathcal{L}(v+3R)}$ -measurable in z, ω , since the bridge-measure conditioned on $\{T_{U^z} > 1\}$, which appears in (2.8), (2.9) of [1] depends in a $\mathcal{B}(\{z: z \cdot \ell < v - 8R\}) \otimes \mathcal{H}_{\mathcal{L}(v+3R)}$ -measurable fashion on z and ω , as can be seen by expression the finite-dimensional marginals in terms of $p_{\omega, U^z}(\cdot, \cdot, \cdot)$.

The fact (2) is plain, and this completes the clarification of what is done below (2.21).

References

- [1] L. Shen, On ballistic diffusions in random environment, Ann. Inst. H. Poincaré Probab. Statist. 39 (5) (2003) 839–876.