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Addendum to the article "On ballistic diffusions in random environment" Annales de l'Institut Henri Poincaré Probabilités et statistiques 39 (5) (2003) 839–876 ☆

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The object of this addendum is to clarify a step in the proof of Theorem 2.4 of [1], namely that the quantities in (2.21) of [1]

$$A_1 \stackrel{\text{def}}{=} \hat{\mathsf{P}}_x^{\omega}[V] \quad \text{and} \quad A_2 \stackrel{\text{def}}{=} \hat{\mathsf{P}}_y^{\omega}\big[f(X_\cdot - y, \lambda_\cdot), D = \infty\big]\, g \circ t_y$$

are independent under $\mathbb{P}$, as it is used below (2.21).

This fact follows from R-separation, cf. (1.6) of [1], and (1), (2) below:

(1) A_1 is $\mathcal{H}_{\mathcal{L}(y\cdot\ell-4R)}$-measurable (in place of "$\mathcal{H}_{\mathcal{L}(y\cdot\ell-7R)}$-measurable" stated below (2.21) of [1]),
(2) A_2 is $\mathcal{H}_{\mathcal{R}(y\cdot\ell-2R)}$-measurable (in place of "$\mathcal{H}_{\mathcal{R}(y\cdot\ell-R)}$-measurable" stated below (2.21) of [1]).

To see (1), one simply uses, with $v = y\cdot\ell - 7R$, the following fact:

(3) For $m \geqslant 1$, integer, $v \in \mathbb{R}$, $U \in \mathcal{F}_m \otimes \mathcal{S}_{m-1}$, with $U \subset \{\sup_{t\leqslant m} \ell\cdot X_t \leqslant v\}$, $\hat{\mathsf{P}}_x^{\omega}[U]$ is $\mathcal{H}_{\mathcal{L}(v+3R)}$-measurable.

The statement (3) follows from

(4) For $O \in \mathcal{F}_1$, with $O \subset \{\sup_{t\leqslant 1} \ell\cdot X_t \leqslant v\}$, $\hat{\mathsf{P}}^{\omega}_{z,\lambda=1}[O]$ and $\hat{\mathsf{P}}^{\omega}_{z,\lambda=0}[O]$ are $\mathcal{B}(\mathbb{R}^d)\otimes\mathcal{H}_{\mathcal{L}(v+3R)}$-measurable,

together with the Markov property and standard arguments. To see (4), one simply observes that

- when $z\cdot\ell \geqslant v - 8R$, in view of (2.8), (2.9) of [1] and $B^z \subset \{x \in \mathbb{R}^d\colon\ x\cdot\ell > v\}$, one has $\hat{\mathsf{P}}^{\omega}_{z,\lambda=1}[O] = 0$, and $\hat{\mathsf{P}}^{\omega}_{z,\lambda=0}[O] = \frac{1}{1-\varepsilon}\mathsf{P}_z^{\omega}[O]$, and hence $1_{\{z\cdot\ell\geqslant v-8R\}}\hat{\mathsf{P}}^{\omega}_{z,\lambda=i}[O]$, $i = 0, 1$, are $\mathcal{B}(\mathbb{R}^d)\otimes\mathcal{H}_{\mathcal{L}(v+3R)}$-measurable in z, ω,

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- when $z \cdot \ell < v - 8R$, $U^z \subset \{x \in \mathbb{R}^d\colon\ x \cdot \ell < v + 3R\}$, and hence $1_{\{z \cdot \ell < v - 8R\}} \hat{\mathsf{P}}^{\omega}_{z,\lambda=i}[O]$, $i = 0, 1$, are $\mathcal{B}(\mathbb{R}^d) \otimes \mathcal{H}_{\mathcal{L}(v+3R)}$-measurable in z, ω, since the bridge-measure conditioned on $\{T_{U^z} > 1\}$, which appears in (2.8), (2.9) of [1] depends in a $\mathcal{B}(\{z\colon\ z \cdot \ell < v - 8R\}) \otimes \mathcal{H}_{\mathcal{L}(v+3R)}$-measurable fashion on z and ω, as can be seen by expression the finite-dimensional marginals in terms of $p_{\omega, U^z}(\cdot\,, \cdot\,, \cdot)$.

The fact (2) is plain, and this completes the clarification of what is done below (2.21).

References

[1] L. Shen, On ballistic diffusions in random environment, Ann. Inst. H. Poincaré Probab. Statist. 39 (5) (2003) 839–876.