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The Coble hypersurfaces

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Abstract

Let A be an indecomposable principally polarized abelian variety of dimension g. Third order theta functions embed A in a projective space $\mathbb{P}(V_3)$ of dimension $3^g - 1$, while second order theta functions embed the Kummer variety $X = A/\{\pm 1\}$ in a projective space $\mathbb{P}(V_2)$ of dimension $2^g - 1$. Coble observed that for $g = 2$ there is a unique cubic hypersurface in $\mathbb{P}(V_3)$ that is singular along A, and for $g = 3$ a unique quartic hypersurface in $\mathbb{P}(V_2)$ singular along X. We explain these facts by a simple analysis of the representations of the corresponding Heisenberg group. ***To cite this article: A. Beauville, C. R. Acad. Sci. Paris, Ser. I 337 (2003).***

Résumé

Les hypersurfaces de Coble. Soit A une variété abélienne principalement polarisée indécomposable, de dimension g. Les fonctions thêta d'ordre 3 plongent A dans un espace projectif $\mathbb{P}(V_3)$ de dimension $3^g - 1$, tandis que les fonctions thêta d'ordre 2 plongent la variété de Kummer $X = A/\{\pm 1\}$ dans un espace projectif $\mathbb{P}(V_2)$ de dimension $2^g - 1$. Coble a observé que pour $g = 2$ il existe une unique hypersurface cubique dans $\mathbb{P}(V_3)$ qui est singulière le long de A, et pour $g = 3$ une unique hypersurface quartique dans $\mathbb{P}(V_2)$ singulière le long de X. Nous expliquons ces faits par une analyse élémentaire des représentations du groupe de Heisenberg correspondant. ***Pour citer cet article : A. Beauville, C. R. Acad. Sci. Paris, Ser. I 337 (2003).***

Version française abrégée

Soit $(A, \mathcal{L})$ une variété abélienne principalement polarisée, indécomposable, de dimension g. Pour ν entier $\geqslant 1$, on pose $V_\nu = H^0(A, \mathcal{L}^\nu)$. Le morphisme naturel $\varphi_\nu : A \to \mathbb{P}(V_\nu)$ est un plongement pour $\nu \geqslant 3$; pour $\nu = 2$ il induit un plongement de la variété de Kummer $A/\{\pm 1\}$ dans $\mathbb{P}(V_2)$. Coble [3,4] a observé que pour $g = 2$ il existe une unique hypersurface cubique dans $\mathbb{P}(V_3)$ qui est singulière le long de $\varphi_3(A)$, et pour $g = 3$ une unique hypersurface quartique dans $\mathbb{P}(V_2)$ singulière le long de $\varphi_2(A)$; plus récemment ces hypersurfaces ont été interprétées en termes de fibrés vectoriels sur des courbes [10,11].

Nous allons montrer que ces faits sont une conséquence d'un résultat général et élémentaire sur les représentations du groupe de Heisenberg. Notons A_ν le noyau de la multiplication par ν dans A. Ce groupe agit sur

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A par translations en préservant le fibré $\mathcal{L}^\nu$, donc agit sur $\mathbb{P}(V_\nu)$; cette action se relève en une action sur V_ν d'une extension centrale $\widetilde{A}_\nu$ de A_ν par $\mathbb{C}^*$. Posons $n=\nu$ si ν est impair, $n=2\nu$ si ν est pair. Pour tout $\gamma\in\widetilde{A}_\nu$, l'élément γ^n appartient au centre $\mathbb{C}^*$ de $\widetilde{A}_\nu$, et l'application $\gamma\mapsto\gamma^n$ est un homomorphisme de $\widetilde{A}_\nu$ sur $\mathbb{C}^*$. Notons H_n son noyau ; c'est une extension centrale

$$1\to\boldsymbol{\mu}_n\to H_n\to A_\nu\to 0$$

de A_ν par le groupe $\boldsymbol{\mu}_n$ des racines n-ièmes de l'unité dans $\mathbb{C}$.

Notons $V=V_\nu$, et choisissons un système de coordonnées $(T_1,\ldots,T_N)$ sur $\mathbb{P}(V)$ $(N=\nu^g)$.

Proposition 0.1. *On suppose $n=3$ ou 4. Soit W un sous-H_n-module irréductible de $\mathsf{S}^{n-1}V$. Il existe une forme H_n-invariante $F\in\mathsf{S}^nV$, unique à un scalaire près, telle que $(\partial F/\partial T_1,\ldots,\partial F/\partial T_N)$ soit une base de W.*

Idée de la démonstration. La représentation de H_n sur V est l'unique représentation irréductible de H_n dans laquelle le centre $\boldsymbol{\mu}_n$ agit par homothéties. Il en résulte que la représentation de H_n sur $\mathsf{S}^{n-1}V$ est la somme de k copies de V^*, avec

$$k=\dim\mathsf{S}^{n-1}V/\dim V^*=\frac{1}{N}\binom{N+n-2}{n-1}.$$

L'espace $\mathrm{Hom}_{H_n}(V^*,\mathsf{S}^{n-1}V)$ est de dimension k ; il paramètre les sous-H_n-modules simples de $\mathsf{S}^{n-1}V$.

Considérons l'application H_n-équivariante injective

$$h:\mathsf{S}^nV\to\mathrm{Hom}(V^*,\mathsf{S}^{n-1}V)$$

donnée par $h(F)(\partial)=\partial F$ (on identifie V^* à l'espace des dérivations de degré -1 de $\mathsf{S}V$). Elle induit une injection $(\mathsf{S}^nV)^{H_n}\hookrightarrow\mathrm{Hom}_{H_n}(V^*,\mathsf{S}^{n-1}V)$ des sous-espaces H_n-invariants. Un calcul élémentaire prouve alors que ces deux espaces ont la même dimension, ce qui entraîne la proposition. □

Soit X une sous-variété de $\mathbb{P}(V)$, invariante sous A_ν, et soit $\mathcal{I}_X$ son faisceau d'idéaux dans $\mathbb{P}(V)$. Il résulte de la Proposition 0.1 que si $(F_1,\ldots,F_m)$ est une base de l'espace des formes H_n-invariantes de degré n qui sont singulières le long de X, les dérivées partielles $(\partial F_i/\partial T_j)$ forment une base de $H^0(\mathbb{P}(V),\mathcal{I}_X(n-1))$. En particulier, si $\dim H^0(\mathbb{P}(V),\mathcal{I}_X(n-1))=\nu^g$, il existe (à un scalaire près) une unique forme H_n-invariante de degré n singulière le long de X. On voit facilement que c'est le cas dans les deux exemples de Coble. De plus, dans ces deux cas, un argument simple montre qu'il n'existe pas d'autre forme de degré n singulière le long de X.

1. Introduction

The title of this Note refers to the following nice observations of Coble. Let A be a complex abelian variety, of dimension g, and $\mathcal{L}$ a line bundle on A defining a principal polarization (that is, $\mathcal{L}$ is ample and $\dim H^0(A,\mathcal{L})=1$). We will assume throughout that $(A,\mathcal{L})$ is *indecomposable*, that is, cannot be written as a product of principally polarized abelian varieties of lower dimension.

Fix an integer $\nu\geqslant 1$ and put $V_\nu=H^0(A,\mathcal{L}^\nu)$. We consider the morphism[1] $\varphi_\nu:A\to\mathbb{P}(V_\nu)$ defined by the global sections of $\mathcal{L}^\nu$. Recall that φ_ν is an embedding for $\nu\geqslant 3$, and that φ_2 induces an embedding of the Kummer variety $A/\{\pm 1\}$ in $\mathbb{P}(V_2)$. Let A_ν be the kernel of the multiplication by ν in A; the group A_ν acts on A and on $\mathbb{P}(V_\nu)$ in such a way that φ_ν is A_ν-equivariant.

[1] We use Grothendieck's notation: $\mathbb{P}(V_\nu)$ is the space of hyperplanes of V_ν.

Proposition 1.1 (Coble). (1) *Let $g=2$. There exists a unique A_3-invariant cubic hypersurface in $\mathbb{P}(V_3)$ $(\cong \mathbb{P}^8)$ that is singular along $\varphi_3(A)$. The polars of this cubic span the space of quadrics in $\mathbb{P}(V_3)$ containing $\varphi_3(A)$.*

(2) *Let $g=3$. There exists a unique A_2-invariant quartic hypersurface in $\mathbb{P}(V_2)$ $(\cong \mathbb{P}^7)$ that is singular along $\varphi_2(A)$. The polars of this quartic span the space of cubic hypersurfaces in $\mathbb{P}(V_2)$ containing $\varphi_2(A)$.*

The proof of (2) appears in [4], and that of (1) in [3] (actually the cubic is not explicitly mentioned in that paper, but it is easily deduced from the equations for the quadrics containing $\varphi_3(A)$. I am indebted to I. Dolgachev for this reference). Both results are proved by explicit computations. These hypersurfaces have a beautiful interpretation in terms of vector bundles on curves (see [10] for the quartic and [11] for the cubic).

An analogous statement appears in [12], this time for the moduli space $\mathcal{SU}_C(2)$ of semi-stable rank 2 vector bundles with trivial determinant on a curve of genus 4 with no vanishing theta-constant (this moduli space is naturally embedded in $\mathbb{P}(V_2)$). Oxbury and Pauly prove that it is contained in a unique A_2-invariant quartic hypersurface, whose polars span the space of cubic hypersurfaces containing $\mathcal{SU}_C(2)$.

The main observation of this note is that these facts follow from a general (and elementary) result about representations of the Heisenberg group (Proposition 2.1 below). Let us just mention here a geometric consequence of that result:

Proposition 1.2. *Let $n=3$ or 4; put $\nu=3$ if $n=3$, $\nu=2$ if $n=4$. Let $(T_1,\dots,T_N)$ be a coordinate system on $\mathbb{P}(V_\nu)$. Let X be an A_ν-invariant subvariety of $\mathbb{P}(V_\nu)$. Then the space of hypersurfaces of degree $n-1$ containing X admits a basis $(\partial F_i/\partial T_j)$, where $F_1,\dots,F_m$ are forms of degree n on $\mathbb{P}(V_\nu)$, such that the hypersurfaces $F_i=0$ are A_ν-invariant (and singular along X).*

2. Heisenberg submodules of $\mathsf{S}^{n-1}V$

Let n be an integer; we put $\nu=n$ if n is odd, $\nu=n/2$ if n is even. We write for brevity V instead of V_ν. We will occasionally pick a coordinate system $(T_1,\dots,T_N)$ on $\mathbb{P}(V_\nu)$, to make some of our statements more concrete.

The action of A_ν on $\mathbb{P}(V)$ lifts to an action on V of a central extension $\widetilde{A}_\nu$ of A_ν by $\mathbb{C}^*$. For all $\gamma\in\widetilde{A}_\nu$, the element γ^n belongs to the center $\mathbb{C}^*$ of $\widetilde{A}_\nu$, and the map $\gamma\mapsto\gamma^n$ is a homomorphism of $\widetilde{A}_\nu$ onto $\mathbb{C}^*$ (this is where we need to take $n=2\nu$ instead of ν when ν is even). We denote by H_n its kernel; it is a central extension

$$1\to\boldsymbol{\mu}_n\to H_n\to A_\nu\to 0$$

of A_ν by the group $\boldsymbol{\mu}_n$ of nth roots of unity in $\mathbb{C}$.

Proposition 2.1. *Assume $n=3$ or 4. Let W be an irreducible sub-H_n-module of $\mathsf{S}^{n-1}V$. There exists a H_n-invariant form $F\in\mathsf{S}^nV$, unique up to a scalar, such that $(\partial F/\partial T_1,\dots,\partial F/\partial T_N)$ form a basis of W.*

Proof. Put $N=\dim V$ $(=\nu^g)$. The group H_n acts irreducibly on V, and this is the unique irreducible representation of H_n on which the center $\boldsymbol{\mu}_n$ acts by homotheties. It follows that the representation of H_n on $\mathsf{S}^{n-1}V$ is isomorphic to the direct sum of k copies of V^*, with

$$k=\dim\mathsf{S}^{n-1}V/\dim V^*=\frac{1}{N}\binom{N+n-2}{n-1}.$$

The space $\operatorname{Hom}_{H_n}(V^*,\mathsf{S}^{n-1}V)$ has dimension k; it parametrizes the irreducible sub-H_n-modules of $\mathsf{S}^{n-1}V$.

Consider the H_n-equivariant injective map

$$h:\mathsf{S}^nV\to\operatorname{Hom}(V^*,\mathsf{S}^{n-1}V)$$

given by $h(F)(\partial) = \partial F$ (we identify V^* with the space of degree -1 derivations of $\mathsf{S}V$). It induces an injection $(\mathsf{S}^n V)^{H_n} \hookrightarrow \mathrm{Hom}_{H_n}(V^*, \mathsf{S}^{n-1}V)$ of the H_n-invariant subspaces. The assertion of the proposition is that this map is onto, or equivalently that $\dim(\mathsf{S}^n V)^{H_n} = \frac{1}{N}\binom{N+n-2}{n-1}$.

The action of H_n on $\mathsf{S}^n V$ factors through the abelian quotient A_ν, hence is the direct sum of 1-dimensional representations V_χ corresponding to characters χ of A_ν. We claim that all non-trivial characters of A_ν appear with the same multiplicity. To see this, consider the group $\mathrm{Aut}(H_n, \boldsymbol{\mu}_n)$ of automorphisms of H_n which induce the identity on $\boldsymbol{\mu}_n$. Because of the unicity property of the representation $\rho : H_n \to GL(V)$, for every $\varphi \in \mathrm{Aut}(H_n, \boldsymbol{\mu}_n)$ the representation $\rho \circ \varphi$ is isomorphic to ρ, thus $(\mathsf{S}^n \rho) \circ \varphi$ is isomorphic to $\mathsf{S}^n \rho$. This implies that the characters appearing in the decomposition of $\mathsf{S}^n V$ are exchanged by the action of $\mathrm{Aut}(H_n, \boldsymbol{\mu}_n)$. But the action of $\mathrm{Aut}(H_n, \boldsymbol{\mu}_n)$ on A_ν factors through a surjective homomorphism $\mathrm{Aut}(H_n, \boldsymbol{\mu}_n) \to \mathrm{Sp}(A_\nu)$ (see e.g. [2], Ch. 6, Lemma 6.6). Since ν is prime, the symplectic group $\mathrm{Sp}(A_\nu)$ acts transitively on the set of nontrivial characters of A_ν, hence our claim.

Thus we have

$$\mathsf{S}^n V = \Big(\bigoplus_{\chi \neq 1} V_\chi\Big)^m \oplus (\mathsf{S}^n V)^{H_n}$$

for some integer $m \geqslant 0$. Counting dimensions yields

$$\binom{N+n-1}{n} = m(N^2-1) + \dim(\mathsf{S}^n V)^{H_n}.$$

On the other hand a simple computation gives

$$\binom{N+n-1}{n} = m(N^2-1) + \frac{1}{N}\binom{N+n-2}{n-1},$$

with $m = \frac{1}{6}(N+3)$ for $n = 3$, and $m = \frac{1}{24}(N+2)(N+4)$ for $n = 4$. Moreover we have $\dim(\mathsf{S}^n V)^{H_n} \leqslant \frac{1}{N}\binom{N+n-2}{n-1} < N^2 - 1$. Thus $\dim(\mathsf{S}^n V)^{H_n}$ and $\frac{1}{N}\binom{N+n-2}{n-1}$ are both equal to the rest of the division of $\binom{N+n-1}{n}$ by $N^2 - 1$, hence they are equal. □

Remarks. (1) Unfortunately the cases $n = 3$ and $n = 4$ seem to be the only ones for which the proposition holds. If for instance n is prime $\geqslant 5$, it is easy to check that the equality $\dim(\mathsf{S}^n V)^{H_n} = \frac{1}{N}\binom{N+n-2}{n-1}$ *never* holds.

(2) The case $n = 4$ could also easily be deduced from [7], Proposition 2.

(3) The result holds more generally in characteristic $\neq \nu$, with the same proof (the representation theory of a p-group in characteristic $\neq p$ is isomorphic to its theory of complex representations). Therefore the results below hold in characteristic $\neq \nu$, with the possible exception of Proposition 3.1 which uses a result of Donagi whose proof requires the characteristic to be zero.

Corollary 2.2. *Let X be a subvariety of $\mathbb{P}(V)$, invariant under the action of A_ν; denote by $\mathcal{I}_X$ the ideal sheaf of X in $\mathbb{P}(V)$. Let $(F_1, \dots, F_m)$ be a basis of the space of H_n-invariant forms in $\mathsf{S}^n V$ which are singular along X. Then the partial derivatives $(\partial F_i / \partial T_j)$ form a basis of $H^0(\mathbb{P}(V), \mathcal{I}_X(n-1))$. In particular, if $\dim H^0(\mathbb{P}(V), \mathcal{I}_X(n-1)) = \nu^g$, there exists a unique H_n-invariant form in $\mathsf{S}^n V$ which is singular along X.*

Indeed $H^0(\mathbb{P}(V), \mathcal{I}_X(n-1))$ is a sub-H_n-module of $H^0(\mathbb{P}(V), \mathcal{O}_{\mathbb{P}}(n-1)) = \mathsf{S}^{n-1}V$, and therefore isomorphic to a direct sum of simple modules. □

In the next section we will apply the corollary to the abelian variety A embedded in $\mathbb{P}(V_\nu)$. Another interesting case is when X is the moduli space of vector bundles of rank 2 and trivial determinant on a curve C of genus 4 with no vanishing theta-constant. Let A be the Jacobian of C; then X has a natural A_2-equivariant embedding in $\mathbb{P}(V_2)$, and Oxbury and Pauly prove the equality $\dim H^0(\mathbb{P}(V_2), \mathcal{I}_X(3)) = 8$ [12]. Therefore there exists a unique H_4-invariant quartic hypersurface singular along X.

3. Application: equations for abelian varieties

(3.1) Let us apply Corollary 2.2 to $X=\varphi_\nu(A)$ embedded in $\mathbb{P}(V_\nu)$. If $n=4$ we will assume that $(A,\mathcal{L})$ *has no vanishing theta-constant* (that is, no symmetric theta divisor singular at 0 – if $g=3$ this simply means that $(A,\mathcal{L})$ is the Jacobian of a non-hyperelliptic curve). This implies that the Kummer variety $\varphi_2(A)\subset\mathbb{P}(V_2)$ is projectively normal, while $\varphi_3(A)$ is always projectively normal in $\mathbb{P}(V_3)$ [8]. Thus the natural map $H^0(\mathbb{P}(V_\nu),\mathcal{O}_{\mathbb{P}}(n-1))\to H^0(X,\mathcal{O}_X(n-1))$ is surjective, and this allows us to compute the dimension of its kernel. We find that *the space of H_n-invariant forms in S^nV singular along X has dimension $m_n(g)$ given by*

$$m_3(g)=\tfrac{1}{2}(3^g-2^{g+1}+1),\qquad m_4(g)=\tfrac{1}{6}\big(2^g(2^g+3)-3^{g+1}-1\big);$$

for any basis $(F_1,\dots,F_{m_n(g)})$ of this space, the derivatives $(\partial F_i/\partial T_j)$ form a basis of the space of forms of degree $n-1$ vanishing along X.

(3.2) Let us consider in particular the case $g=n-1$ considered by Coble. Since $m_3(2)=m_4(3)=1$ we recover Coble's result: there is a unique H_n-invariant hypersurface of degree n singular along $\varphi_\nu(A)$. In fact we have a slightly better result:

Proposition 3.1. *Assume $g=n-1$. The Coble hypersurface in $\mathbb{P}(V_\nu)$ is the unique hypersurface of degree n singular along $\varphi_\nu(A)$.*

Proof. The case of the Coble quartic is explained in [9], and the proof works equally well for the cubic. Let us recall briefly the argument. Let $F=0$ be the Coble hypersurface. The derivatives $\partial F/\partial T_1,\dots,\partial F/\partial T_N$ span the space I_{n-1} of forms of degree $n-1$ vanishing along $\varphi_\nu(A)$; the action of H_n on I_{n-1} is irreducible.

Let W be the space of forms of degree n which are singular along $\varphi_\nu(A)$; it is a sub-H_n-module of S^nV, hence a sum of one-dimensional representations W_χ. Let $G\neq 0$ in W_χ. The derivatives $\partial G/\partial T_1,\dots,\partial G/\partial T_N$ vanish on $\varphi_\nu(A)$, hence span a subspace of I_{n-1}; since this subspace is stable under H_n, it is equal to I_{n-1}. By [5], §1, this implies that there exists an automorphism T of V_ν such that $G=F\circ T$.

Now the singular locus of the Coble hypersurface is exactly $\varphi_\nu(A)$ (see (3.3) below); thus T must preserve $\varphi_\nu(A)$. In the group of automorphisms of V_ν preserving $\varphi_\nu(A)$, the Heisenberg group H_n is normal – because the group of translations of A is normal inside the group of all automorphisms. Thus T normalizes H_n; this implies that the form $G=F\circ T$ is H_n-invariant, and therefore proportional to F by Coble's result. □

(3.3) For $g=2$, Coble states in [3] that $\varphi_3(A)$ is the set-theoretical intersection of the quadrics that contain it – in other words, $\varphi_3(A)$ is the singular locus of the Coble cubic; this is proved even scheme-theoretically in [1]. When $g=3$ and $(A,\mathcal{L})$ has no vanishing theta-constant, Narasimhan and Ramanan have proved that the Kummer variety $\varphi_2(A)$ is set-theoretically the singular locus of the Coble quartic [10]; this holds also scheme-theoretically by [9]. It is tempting to conjecture that both statements hold in higher dimension as well, namely that the abelian variety $\varphi_3(A)$ is a scheme-theoretical intersection of quadrics and that the Kummer variety $\varphi_2(A)$ is a scheme-theoretical intersection of cubics. However these quadrics or cubics cannot generate the full ideal of $\varphi_\nu(A)$:

Proposition 3.2. *The graded ideal I of $\varphi_\nu(A)$ in $\mathbb{P}(V_\nu)$ is* not *generated by its elements of degree $\leqslant n-1$.*

(Recall that I is generated by its elements of degree $\leqslant n$, see [2], Ch. 7 and [6].)

Note that the proposition is immediate in the case $g=n-1$ considered by Coble, because then $\dim(V\otimes I_{n-1})<\dim I_n$. However this inequality does not hold any more in higher genus.

Proof. We will prove the inequality $\dim(V \otimes I_{n-1})^{H_n} < \dim(I_n)^{H_n}$, which implies that the multiplication map $V \otimes I_{n-1} \to I_n$ cannot be surjective. Let us treat first the case $n = 3$. From the exact sequence $0 \to I_3 \to \mathsf{S}^3 V \to H^0(A, \mathcal{L}^9) \to 0$ (3.1) we get

$$\dim I_3 = \binom{N+2}{3} - N^2 = \frac{N-3}{6}(N^2-1) + \frac{N-1}{2};$$

as in Proposition 2.1 we conclude that $\dim(I_3)^{H_3} = (N-1)/2$.

Let $K \subset \mathsf{S}^3 V$ be the space of H_3-invariant cubic forms singular along $\varphi_3(A)$; by the proposition the natural map $V^* \otimes K \to I_2$ is an isomorphism. The action of H_3 on K is trivial, and the H_3-module $V \otimes V^*$ is the direct sum of a one-dimensional factor for each character of A_3; thus

$$\dim(V \otimes I_2)^{H_3} = \dim K = \tfrac{1}{2}(3^g - 2^{g+1} + 1) < \tfrac{1}{2}(N-1)$$

(3.1), hence the result.

For $n = 4$ the same method gives $\dim(I_4)^{H_2} = \frac{1}{6}(N-1)(N-2)$, which is larger than $\dim(V \otimes I_3)^{H_2} = \frac{1}{6}(N(N+3) - 3^{g+1} - 1)$. □

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