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Équations aux dérivées partielles/*Partial Differential Equations*

On the asymptotics of global solutions of higher-order semilinear parabolic equations in the supercritical range

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Abstract We study the asymptotic behaviour of global bounded solutions of the Cauchy problem for the semilinear $2m$th order parabolic equation $u_t = -(-\Delta)^m u + |u|^p$ in $\mathbf{R}^N \times \mathbf{R}_+$, where $m > 1$, $p > 1$, with bounded integrable initial data u_0. We prove that in the supercritical Fujita range $p > p_F = 1 + 2m/N$ any small global solution with nonnegative initial mass, $\int u_0\,dx \geqslant 0$, exhibits as $t \to \infty$ the asymptotic behaviour given by the fundamental solution of the linear parabolic operator (unlike the case $p \in]1, p_F]$ where solutions can blow-up for any arbitrarily small initial data). A discrete spectrum of other possible asymptotic patterns and the corresponding monotone sequence of critical exponents $\{p_l = 1 + 2m/(l + N),\ l = 0, 1, 2, \ldots\}$, where $p_0 = p_F$, are discussed. ***To cite this article: Yu.V. Egorov et al., C. R. Acad. Sci. Paris, Ser. I 335 (2002) 805–810.***

Sur les asymptotiques des solutions globales des équations paraboliques sémi-linéaires d'ordre supérieur dans le cas surcritique

Résumé On considère le comportement asymptotique des solutions globales bornées du problème de Cauchy pour l'équation parabolique sémi-linéaire d'ordre $2m$ $u_t = -(-\Delta)^m u + |u|^p$ in $\mathbf{R}^N \times \mathbf{R}_+$, $u(x, 0) = u_0 \in X = L^1(\mathbf{R}^N) \cap L^\infty(\mathbf{R}^N)$, où $m > 1$, $p > 1$. On vérifie que dans le cas surcritique de Fujita $p > p_F = 1 + 2m/N$ toute petite solution globale avec la donnée initiale vérifiant $\int u_0\,dx \geqslant 0$, montre le comportement asymptotique quand $t \to \infty$ défini par la solution fondamentale de l'opérateur linéaire parabolique, à la différence du cas $p \in]1, p_F]$ quand la solution peut exploser pour la donnée initiale arbitrairement petite. Le spectre discret des pistes possibles et la suite correspondante des exponents critiques $\{p_l = 1 + 2m/(l + N),\ l = 0, 1, 2, \ldots\}$, où $p_0 = p_F$, sont descriptes. ***Pour citer cet article : Yu.V. Egorov et al., C. R. Acad. Sci. Paris, Ser. I 335 (2002) 805–810.***

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On considère le comportement asymptotique des solutions globales bornées du problème de Cauchy pour l'équation parabolique sémi-linéaire d'ordre $2m$

$$u_t = -(-\Delta)^m u + |u|^p \quad \text{in } \mathbf{R}^N \times \mathbf{R}_+, \qquad u(x,0) = u_0 \in X = L^1(\mathbf{R}^N) \cap L^\infty(\mathbf{R}^N),$$

où $m > 1$, $p > 1$. On vérifie que dans le cas surcritique de Fujita $p > p_F = 1 + 2m/N$ toute petite solution globale avec la donnée initiale vérifiant $\int u_0\,dx \geqslant 0$, montre le comportement asymptotique quand $t \to \infty$ défini par la solution fondamentale de l'opérateur linéaire parabolique, à la différence du cas $p \in]1, p_F]$ quand la solution peut exploser pour les données initiales arbitrairement petites. Le résultat principal est suivant :

THÉORÈME 1. – *Soit $p > p_F$. Alors pour des données initiales telles que $\int u_0(x)\,dx \geqslant 0$ et $|u_0(x)| \leqslant Be^{-d|x|^\alpha}$, où $\alpha = 2m/(2m-1)$ et $B > 0$ est une constante suffisamment petite, il existe une solution globale $u(x,t)$ du problème de Cauchy* (1) *telle que la fonction*

$$v(y,t) = (1+t)^{N/2m} u\big(y(1+t)^{1/2m}, t\big)$$

vérifie $v(y,t) \to C_0 f(y)$ quand $t \to \infty$, uniformement dans $\mathbf{R}^N$, où la constante $C_0 > 0$ dépend des données initiales.

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1. Introduction and main results

We study the asymptotic behaviour of global solutions of higher-order semilinear evolution equations of parabolic type. Our basic example is the Cauchy problem for the semilinear $2m$th order ($m > 1$) parabolic equation

$$u_t = -(-\Delta)^m u + |u|^p \quad \text{in } \mathbf{R}^N \times \mathbf{R}_+, \qquad u(x,0) = u_0 \in X = L^1(\mathbf{R}^N) \cap L^\infty(\mathbf{R}^N), \tag{1}$$

which is a natural generalization of the classical semilinear heat equation ($m = 1$) from combustion theory. Higher-order semilinear and quasilinear parabolic equations occur in applications in thin film theory, nonlinear diffusion, lubrication theory, flame and wave propagation (the Kuramoto–Sivashinsky equation and the extended Fisher–Kolmogorov equation), phase transition at critical Lifschitz points and bi-stable systems; see a number of models and a list of references in the book [11].

It is known that $p = p_F = 1 + 2m/N$ is the *critical Fujita exponent* for this problem in the following sense:

(i) if $p \in]1, p_F]$, then any solution $u(x,t) \not\equiv 0$ with

$$\int_{\mathbf{R}^N} u_0(x)\,dx \geqslant 0, \tag{2}$$

blows up in finite time [2] (i.e., there exist arbitrarily small initial data $u_0 \in X$ leading to blow-up),

(ii) for $p > p_F$, solutions are global for any sufficiently small initial data; see [1] and [5].

These blow-up and global existence results are classical for the second-order semilinear heat equation with $m = 1$ established by H. Fujita in the 1960s. Later, using the order-preserving properties of parabolic flows, they were extended to a wide class of quasilinear equations with different types of nonlinear reaction-diffusion operators; see a list of references in Chapter 4 in [12].

In this paper we study the asymptotic behaviour of global solutions in the supercritical range in the case of higher-order diffusion operators, where the semigroup is not order-preserving. Let $b(x,t)$ be the fundamental solution satisfying $b_t = -(-\Delta)^m b$ in $\mathbf{R}^N \times \mathbf{R}_+$ with the initial function $b(x,0) = \delta(x)$ in $\mathbf{R}^N$, δ being Dirac's delta function. By the scaling invariance of the problem and uniqueness of the fundamental solution, it has the self-similar structure

$$b(x,t) = t^{-N/2m} f(y), \quad y = x/t^{1/2m}.$$

Substituting $b(x,t)$ into the heat equation, one obtains the radially symmetric profile f as a unique solution of a linear ordinary differential equation (ODE) which is the radial restriction of the elliptic equation

$$\mathbf{B} f \equiv -(-\Delta_y)^m f + \frac{1}{2m} \nabla_y f \cdot y + \frac{N}{2m} f = 0 \quad \text{in } \mathbf{R}^N, \ \int_{\mathbf{R}^N} f(y)\, dy = 1. \tag{4}$$

The rescaled kernel f satisfies a pointwise estimate [3]

$$\left| f(y) \right| \leqslant D e^{-d|y|^\alpha} > 0 \quad \text{in } \mathbf{R}^N,$$

where $\alpha = 2m/(2m-1) \in (1,2)$ and D, d are positive constants depending on m, N.

THEOREM 1. – *Let $p > p_F$. Then for initial data satisfying* (2) *from the class*

$$X_B = \left\{ u_0 \in X \colon \left| u_0(x) \right| \leqslant B e^{-d|x|^\alpha} \quad \text{in } \mathbf{R}^N \right\}, \tag{5}$$

where $B > 0$ is a sufficiently small constant, there exists a global solution $u(x,t)$ of the Cauchy problem (1) *such that the rescaled function satisfies*

$$v(y,t) = (1+t)^{N/2m} u\left(y(1+t)^{1/2m}, t \right) \to C_0 f(y) \quad \text{as } t \to \infty \tag{6}$$

uniformly in $\mathbf{R}^N$, where constant $C_0 > 0$ depends on initial data.

Such results are well known for the case $m = 1$ established first in [7] and [8], where the convergence to the rescaled Gaussian kernel $f(y) = (4\pi)^{-N/2} e^{-|y|^2/4}$ was proved for nonnegative solutions of the semilinear heat equation $u_t = \Delta u - u^p$ with $p > 1 + 2/N$; see also Section 3 in [4], where the stability of other self-similar profiles $f \notin L^1(\mathbf{R}^N)$ was studied. A similar result is known for the quasilinear parabolic equations like $u_t = \nabla \cdot (u^\sigma \nabla u) \pm u^p$ with $\sigma > 0$ in the supercritical range $p > \sigma + 1 + 2/N$; see p. 236 in [12].

2. Discrete real spectrum of a non-self-adjoint operator

For any integer $m > 1$, the linear operator $\mathbf{B}$ is not symmetric. We calculate its spectrum $\sigma(\mathbf{B})$ in the weighted space $L^2_\rho(\mathbf{R}^N)$ with the exponential weight $\rho(y) = e^{a|y|^\alpha} > 0$ where $a < 2d$ is a sufficiently small positive constant. We next introduce a Hilbert space of functions $H^{2m}_\rho(\mathbf{R}^N)$ with the inner product and the norm

$$\langle v, w \rangle_\rho = \int_{\mathbf{R}^N} \rho(y) \sum_{k=0}^{2m} D^k v(y) \overline{D^k w(y)}\, dy \quad \text{and} \quad \|v\|^2_\rho = \int_{\mathbf{R}^N} \rho(y) \sum_{k=0}^{2m} \left| D^k v(y) \right|^2 dy,$$

where $D^k v$ denotes the vector $\{D^\beta v,\ |\beta| = k\}$, so that $H^{2m}_\rho(\mathbf{R}^N) \subset L^2_\rho(\mathbf{R}^N) \subset L^2(\mathbf{R}^N)$.

PROPOSITION 1. – *$\mathbf{B}$ is a bounded linear operator from $H^{2m}_\rho(\mathbf{R}^N)$ to $L^2_\rho(\mathbf{R}^N)$.*

Let $u(x,t)$ be the solution of the Cauchy problem for the heat equation with initial data $u_0 \in L^2_\rho(\mathbf{R}^N)$ given by the convolution

$$u(x,t) = b(t) * u_0 \equiv t^{-N/2m} \int_{\mathbf{R}^N} f\left((x-z) t^{-1/2m} \right) u_0(z)\, dz.$$

Introducing the rescaled variables $u(x,t) = t^{-N/2m} w(y,\tau)$, $y = x/t^{1/2m}$, $\tau = \ln t \colon \mathbf{R}_+ \to \mathbf{R}$ and performing Taylor expansion in the term $f(y - z e^{-\tau/2m})$ of the convolution operator, we arrive at the following expansion of the solution:

$$w(y,\tau) = \sum_{(\beta)} e^{-|\beta|\tau/2m} M_\beta(u_0) \psi_\beta(y),$$

where $\lambda_\beta = -|\beta|/2m$ and $\psi_\beta(y)$ given below are the eigenvalues and eigenfunctions of **B** and

$$M_\beta(u_0) = \int_{\mathbf{R}^N} z_1^{\beta_1} \cdots z_N^{\beta_N} u_0(z)\, dz.$$

LEMMA 1. – (i) *The spectrum of* **B** *consists of real eigenvalues only* $\sigma(\mathbf{B}) = \{\lambda_\beta = -|\beta|/2m,\ |\beta| = 0, 1, 2, \ldots\}$. *Eigenvalues* λ_β *have finite multiplicity with eigenfunctions*

$$\psi_\beta(y) = \frac{(-1)^{|\beta|}}{\beta!} D^\beta f(y) \equiv \frac{(-1)^{|\beta|}}{\beta!} \left(\frac{\partial}{\partial y_1}\right)^{\beta_1} \cdots \left(\frac{\partial}{\partial y_N}\right)^{\beta_N} f(y).$$

(ii) *The set of eigenfunctions* $\{\psi_\beta\}$ *is complete in* $L^2(\mathbf{R}^N)$ *and in* $L^2_\rho(\mathbf{R}^N)$.

Denote $\widetilde{\mathbf{B}} = \mathbf{B} - I$ and consider the problem $\widetilde{\mathbf{B}} w = g$, where $g = \rho^{-1/2}\tilde{g}$ and $\tilde{g} \in L^2(\mathbf{R}^N)$. Using the descent method [3], we arrive at the following integral operator:

$$\big(\widetilde{\mathbf{B}}^{-1}\tilde{g}\big)(y) \equiv \int_{\mathbf{R}^N} K(y,\zeta)\tilde{g}(\zeta)\, d\zeta \quad \text{with the kernel}$$

$$K(y,\zeta) = -e^{-\frac{a}{2}|\zeta|^\alpha} \int_0^1 (1-z)^{-N/2m} f\big[\big(y - \zeta z^{1/2m}\big)(1-z)^{-1/2m}\big]\, dz.$$

The following estimates establish that $\widetilde{\mathbf{B}}^{-1}$ is a compact operator with a discrete spectrum accumulating at 0. The proof is based on Eidelman's estimate, see Lemma 5.1 in [3].

PROPOSITION 2. – *There holds*

$$K \in L^2\big(\mathbf{R}^N \times \mathbf{R}^N\big), \quad N < 2m; \qquad K \in L^q\big(\mathbf{R}^N \times \mathbf{R}^N\big), \quad N \geqslant 2m \text{ with a } q \in (1, p_F).$$

3. Spectrum and polynomial eigenfunctions of the adjoint operator

We now describe the spectrum and the eigenfunctions of the adjoint operator

$$\mathbf{B}^* = -(-\Delta)^m - \frac{1}{2m} y \cdot \nabla.$$

We consider $\mathbf{B}^*$ in the weighted space $L^2_{\rho^*}(\mathbf{R}^N)$ with the exponentially decaying weight function $\rho^*(y) \equiv 1/\rho(y) = e^{-a|y|^\alpha} > 0$, and ascribe to $\mathbf{B}^*$ the domain $H^{2m}_{\rho^*}(\mathbf{R}^N)$ dense in $L^2_{\rho^*}(\mathbf{R}^N)$. We show that $\mathbf{B}^* : H^{2m}_{\rho^*}(\mathbf{R}^N) \to L^2_{\rho^*}(\mathbf{R}^N)$ is a bounded linear operator and $\langle \mathbf{B}v, w\rangle = \langle v, \mathbf{B}^* w\rangle$ for any $v \in H^{2m}_\rho(\mathbf{R}^N)$ and $w \in H^{2m}_{\rho^*}(\mathbf{R}^N)$.

LEMMA 2. – (i) $\sigma(\mathbf{B}^*) = \sigma(\mathbf{B})$ *and eigenfunctions* $\{\psi^*_\beta(y)\}$ *are polynomials of order* $|\beta|$,

$$\psi^*_\beta(y) = y^\beta + \sum_{j=1}^{[|\beta|/2m]} \frac{1}{j!} (-\Delta)^{mj} y^\beta.$$

(ii) *The subset* $\{\psi^*_\beta\}$ *is complete in* $L^2_{\rho^*}(\mathbf{R}^N)$.

Let l^2_ρ be a Hilbert space of functions $v = \sum a_\beta \psi_\beta$ with expansion coefficients satisfying

$$\sum |a_\beta|^2 < \infty, \tag{7}$$

and the scalar product and the induced norm

$$(v, w)_0 = \sum a_\beta c_\beta, \quad w = \sum c_\beta \psi_\beta \in l^2_\rho, \text{ and } \|v\|_0^2 = (v, v)_0. \tag{8}$$

Then l^2_ρ is a subspace of $L^2_\rho(\mathbf{R}^N)$. It is not difficult to see that the operator **B** (being closed and densely defined) is a sectorial operator in l^2_ρ.

For the adjoint operator $\mathbf{B}^*$ we define a Hilbert space $l^{2*}_{\rho^*}$ of functions $v^* = \sum a_\beta \psi^*_\beta \in L^2_{\rho^*}(\mathbf{R}^N)$ with coefficients satisfying (7) and the inner product similar to (8). We show that $\mathbf{B}^*$ is sectorial in $l^{2*}_{\rho^*}$.

4. Invariant region in the supercritical case

We return to the problem (1) in the supercritical range $p > p_F$. We take initial data from (5), where $d > 0$ is as defined in (5) and $B > 0$ is a constant to be specified. We introduce a solution subset

$$Y_A = \left\{u\colon \left|u(x,t)\right| \leqslant A(1+t)^{-N/2m} \exp\left\{-k|x|^\alpha/(1+t)^{\alpha/2m}\right\} \text{ in } \mathbf{R}^N \times \mathbf{R}_+\right\},$$

where $A > 0$ is a constant depending on B and $k = d/p$.

THEOREM 2. – *Let $p > p_F$. Then for any sufficiently small $B > 0$, there exists an $A > 0$ such that the solution of* (1) *satisfies $u_0 \in X_B \Longrightarrow u(x,t) \in Y_A$ for any $t > 0$.*

5. Perturbed dynamical system: proof of Theorem 1

The rescaled solution (6) satisfies the following semilinear parabolic equation:

$$v_\tau = \mathbf{C}(v,\tau) \equiv \mathbf{B}v + e^{-\gamma\tau}|v|^p \quad \text{in } \mathbf{R}^N \times \mathbf{R}_+. \tag{9}$$

The parameter $\gamma = (p-1)\frac{N}{2m} - 1 \equiv \frac{N}{2m}(p - p_F)$ is positive for $p > p_F$. The initial function is the same, $v_0(y) \equiv u_0(y)$ in $\mathbf{R}^N$. We see that after rescaling the nonlinear term $|u|^p$ in (1) forms a small perturbation in (9) with the exponential decay rate $e^{-\gamma\tau} \to 0$ as $\tau \to \infty$ on bounded orbits. It follows from Theorem 2 that $|v(y,\tau)| \leqslant \Phi_A(y) \equiv Ae^{-k|y|^\alpha}$ for any $v_0 \in X_B$. We consider the dynamical system (9) as an asymptotically small perturbation of the *autonomous* dynamical system

$$v_\tau = \mathbf{B}v \quad \text{for } \tau > 0, \tag{10}$$

and will apply the stability result from [6], Theorem 3 in Section 3. Let $\mathcal{L}$ be the set of solutions $v \in C([0,\infty[\,, Y) \cap Y_A$ of (5) with initial data $v_0 \in X$ satisfying (2). We then need to check the following three hypotheses under which we can compare the ω-limit sets of two dynamical systems (9) and (10).

(H1) *Compactness of the orbits.*

(H2) *Convergence.* We check that $\mathbf{C}$ is a small perturbation of $\mathbf{B}$ in the following sense: given a solution $v \in \mathcal{L}$, if for a sequence $\{\tau_j\} \to \infty$ the bounded sequence $\{v(\tau_j + s)\}$ converges in $L^\infty_{\text{loc}}([0,\infty) : Y)$ as $j \to \infty$ to a function $v(s)$ then v is an Y-valued solution of (9) (in the class $\mathcal{L}$).

(H3) *Uniform stability for the unperturbed equation.* It means for for every $\varepsilon > 0$ there exists a $\delta = \delta(\varepsilon) > 0$ such that for any solution $v(\tau)$ of (10) in $\mathcal{L}$,

$$d(v_0, \Omega_*) \leqslant \delta \implies d\big(v(\tau), \Omega_*\big) \leqslant \varepsilon \text{ for every } \tau > 0.$$

Here Ω_* is the reduced ω-limit set of (10) which is the closure of the set of all possible ω-limits for initial data from X_B, so that $\Omega_* \subseteq \{cf(y),\ 0 \leqslant c \leqslant C_1\}$, where $C_1 > 0$ is a constant.

PROPOSITION 3. – *There exists a constant $c_* = c_*(m,N) > 1$ for $m > 1$ such that, given two initial data $v_{10}, v_{20} \in X$, the L_1-norms of the solutions of* (10) *satisfy $\|v_1(\tau) - v_2(\tau)\|_1 \leqslant c_*\|v_{10} - v_{20}\|_1$ for $\tau > 0$.*

PROPOSITION 4. – *For any orbit in $\mathcal{L}$, $\omega(v_0) \subseteq \Omega_*$.*

As the last step, we establish that $\omega(v_0)$ consists of a unique nontrivial profile.

PROPOSITION 5. – *For any orbit in $\mathcal{L}$, there exists a constant $C_0 > 0$ such that $\omega(v_0) = \{C_0 f\}$.*

6. Spectra of asymptotic patterns and critical exponents

We now consider arbitrary initial data $u_0 \in X_B$ which are not supposed to satisfy (5). Then there exists a global solution which is sufficiently small in the sense that $u(x,t) \in Y_A$. Since

$$\frac{d}{d\tau}\int v(y,\tau)\,dy = e^{-\gamma t}\int \left|v(y,\tau)\right|^p dy > 0,$$

there exists a finite $\lim_{\tau\to\infty}\int v(y,\tau)\,dy = C_0$. If $C_0 \neq 0$, then $a_0(\tau) \to C_0$ and $v(\tau) \to C_0 f$ as $\tau \to \infty$.

Let now $a_0(\tau) \to 0$ and $\int v(y,\tau)\,dy \to 0$ as $\tau \to \infty$. Obviously, such patterns can be generated by the eigenfunctions of the linear operator **B**.

PROPOSITION 6. – *If Eq.* (9) *admits a solution with the linearized behaviour*

$$v(y,\tau) = Ce^{-l\tau/2m}\big[\varphi_l(y) + \mathrm{o}(1)\big] \quad \textit{as } \tau \to \infty \textit{ uniformly},$$

where $l \geqslant 0$ is an integer, $\mathbf{B}\varphi_l = -(l/2m)\varphi_l$, and $C \neq 0$, then $p > p_l = 1 + 2m/(l+N)$.

In order to get asymptotic patterns in critical cases $p = p_l$, we perform the rescaling $u(x,t) = (1+t)^{-(l+N)/2m} v(y,\tau)$, $y = x/(1+t)^{1/2m}$ and $\tau = \ln(1+t)$. The rescaled solution $v(y,\tau)$ satisfies the perturbed equation $v_\tau = (\mathbf{B} + \frac{l}{2m}I)v + e^{-\gamma_l\tau}|v|^p$, with the exponent $\gamma_l = (p-1)(l+N)/2m - 1$. If $p = p_l$, then $\gamma_l = 0$ and we obtain the autonomous equation

$$v_\tau = \left(\mathbf{B} + \frac{l}{2m}I\right)v + |v|^p.$$

Consider the behaviour close to the centre subspace of $\mathbf{B} + \frac{l}{2m}I$ assuming that in $H^{2m}_\rho(\mathbf{R}^N)$

$$v(\tau) = a_l(\tau)\big[\varphi_l + \mathrm{o}(1)\big] \quad \text{as } \tau \to \infty,$$

where $\varphi_l \in \{\psi_\beta\}$. Then by standard eigenfunction expansion $\dot a_l = |a_l|^{p_l}[c_l + \mathrm{o}(1)]$ for $\tau \gg 1$, where $c_l = \langle |\varphi_l|^{p_l}, \varphi_l^*\rangle$. Let, without loss of generality, $c_l > 0$. Then $\dot a_l \geqslant 0$ and stable behaviour corresponds to negative expansion coefficients satisfying $a_l(\tau) = -[(p_l - 1)c_l\tau]^{-1/(p_l-1)}(1+\mathrm{o}(1))$ as $\tau \to \infty$. In the original variables this gives the behaviour with an extra logarithmic factor

$$u(x,t) = -C_l(t\ln t)^{-(l+N)/2m}\big[\varphi_l\big(x/t^{1/2m}\big) + \mathrm{o}(1)\big] \quad \text{as } t \to \infty,$$

where the constant $C_l > 0$ does not depend on initial data, $C_l = [2mc_l/(l+N)]^{-(l+N)/2m}$.

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